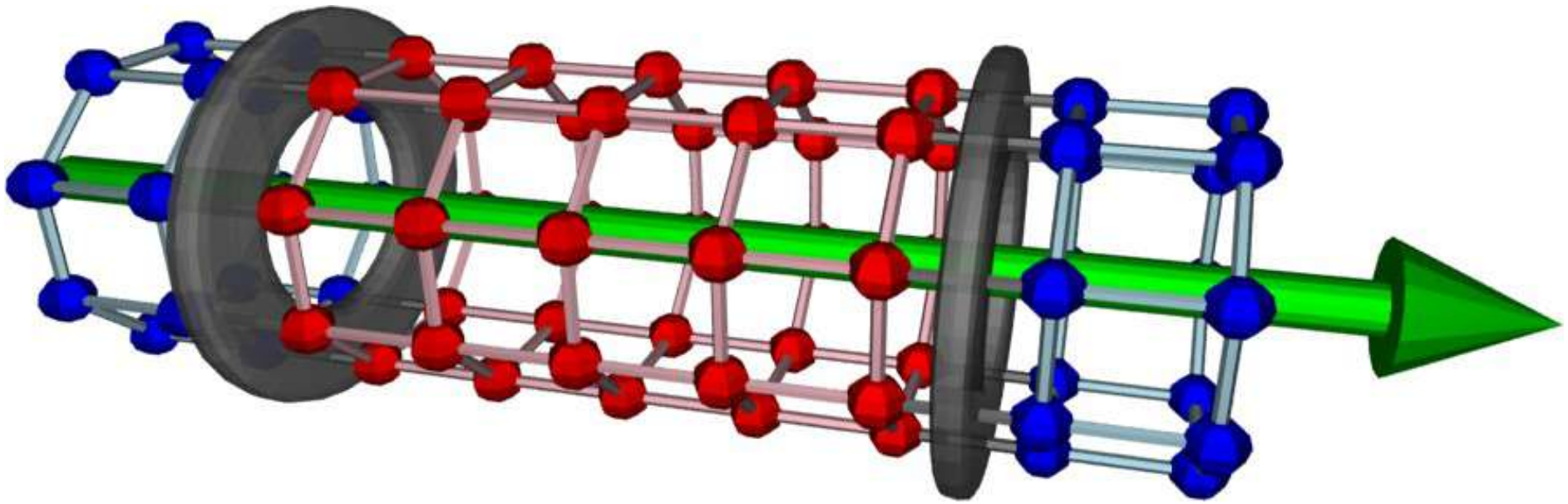
Modelling the magnetoresistance of disordered superconducting films

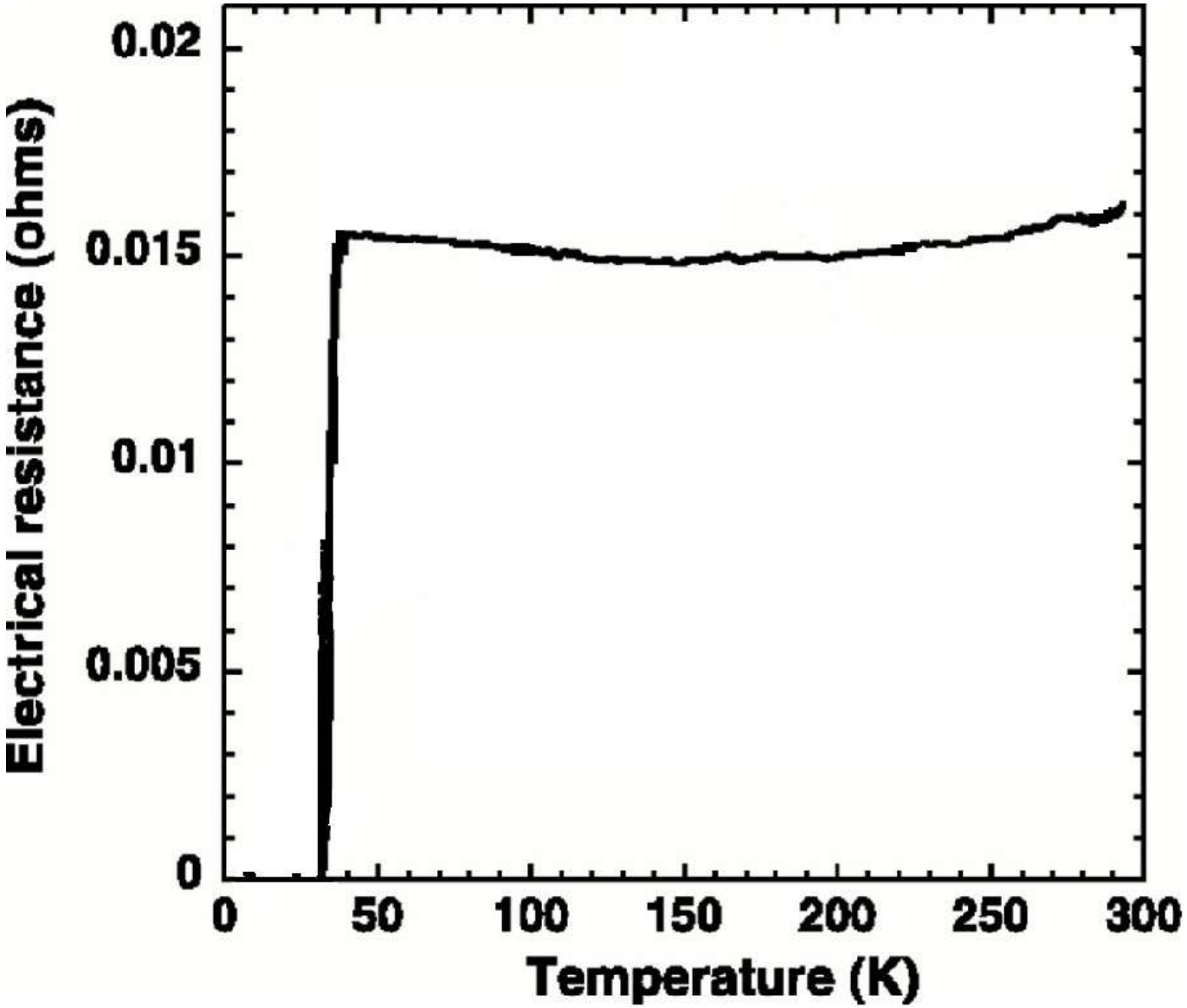


Gareth Conduit, Yigal Meir

University of Cambridge & Ben Gurion University

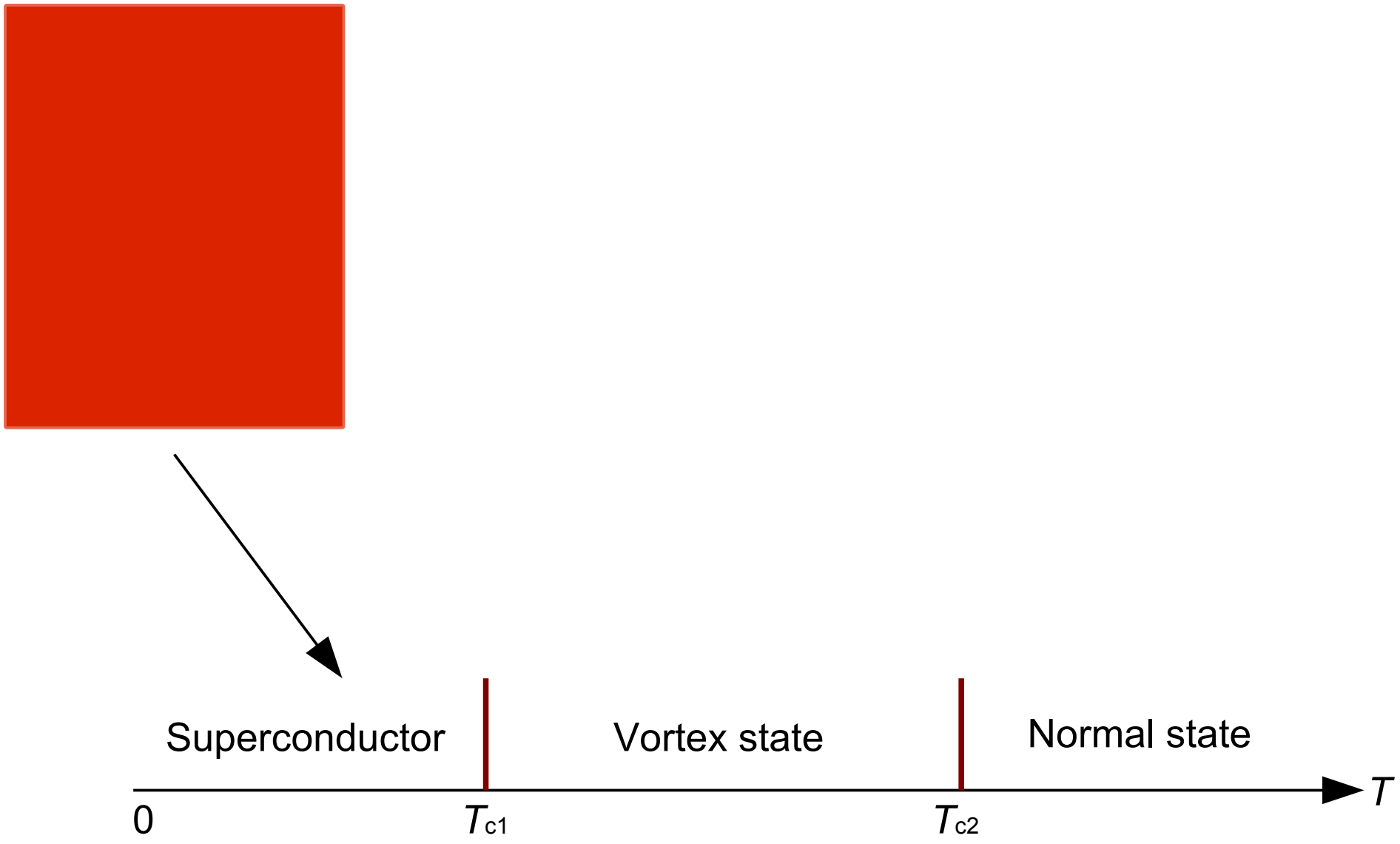
PRB **84**, 064513 (2011), arXiv:1107.1246, arXiv:1111.2941, arXiv:1112.1657

BCS superconductivity in MgB_2



Monteverde et al., Science **292**, 75 (2001)

Kosterlitz Thouless transition



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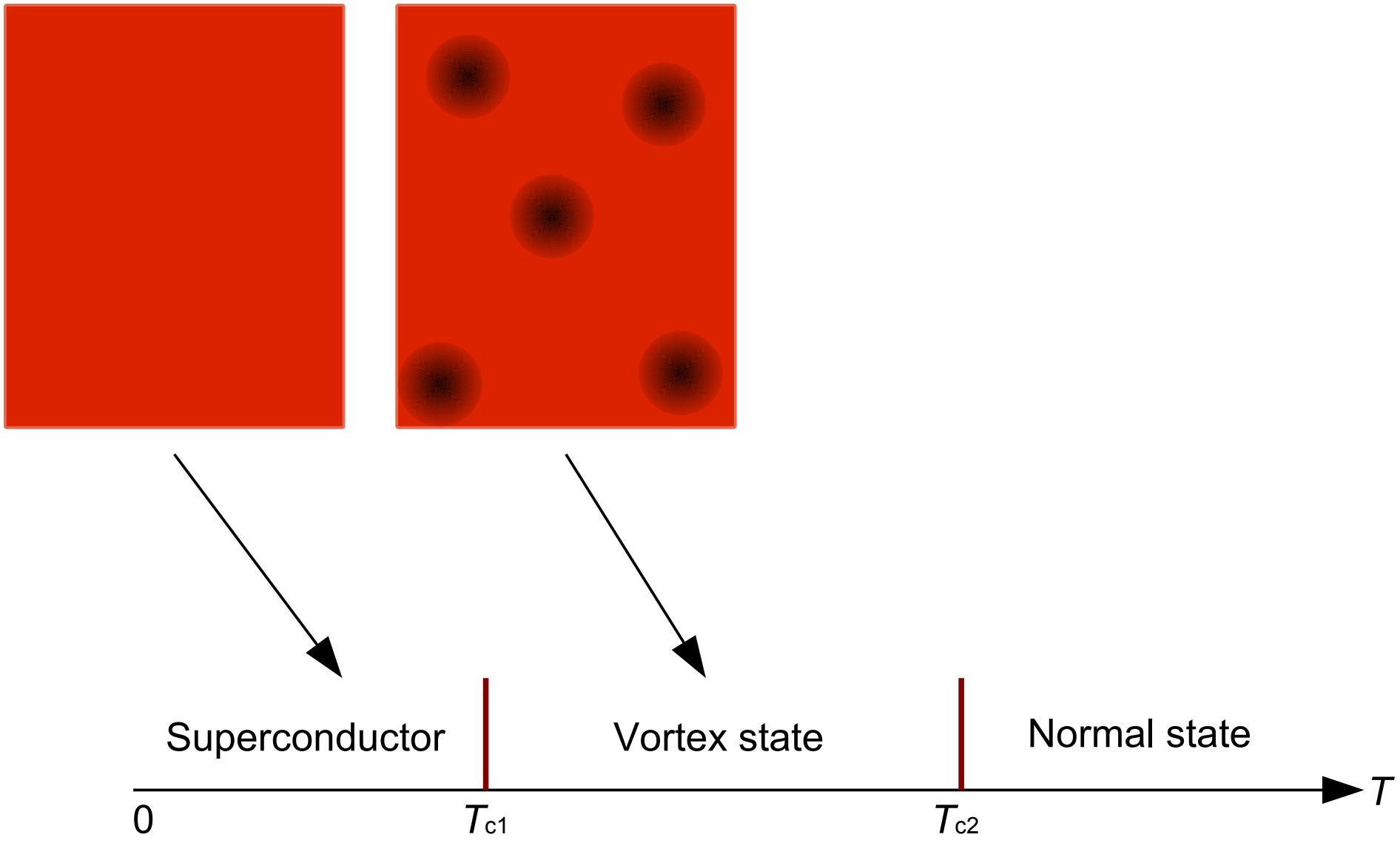

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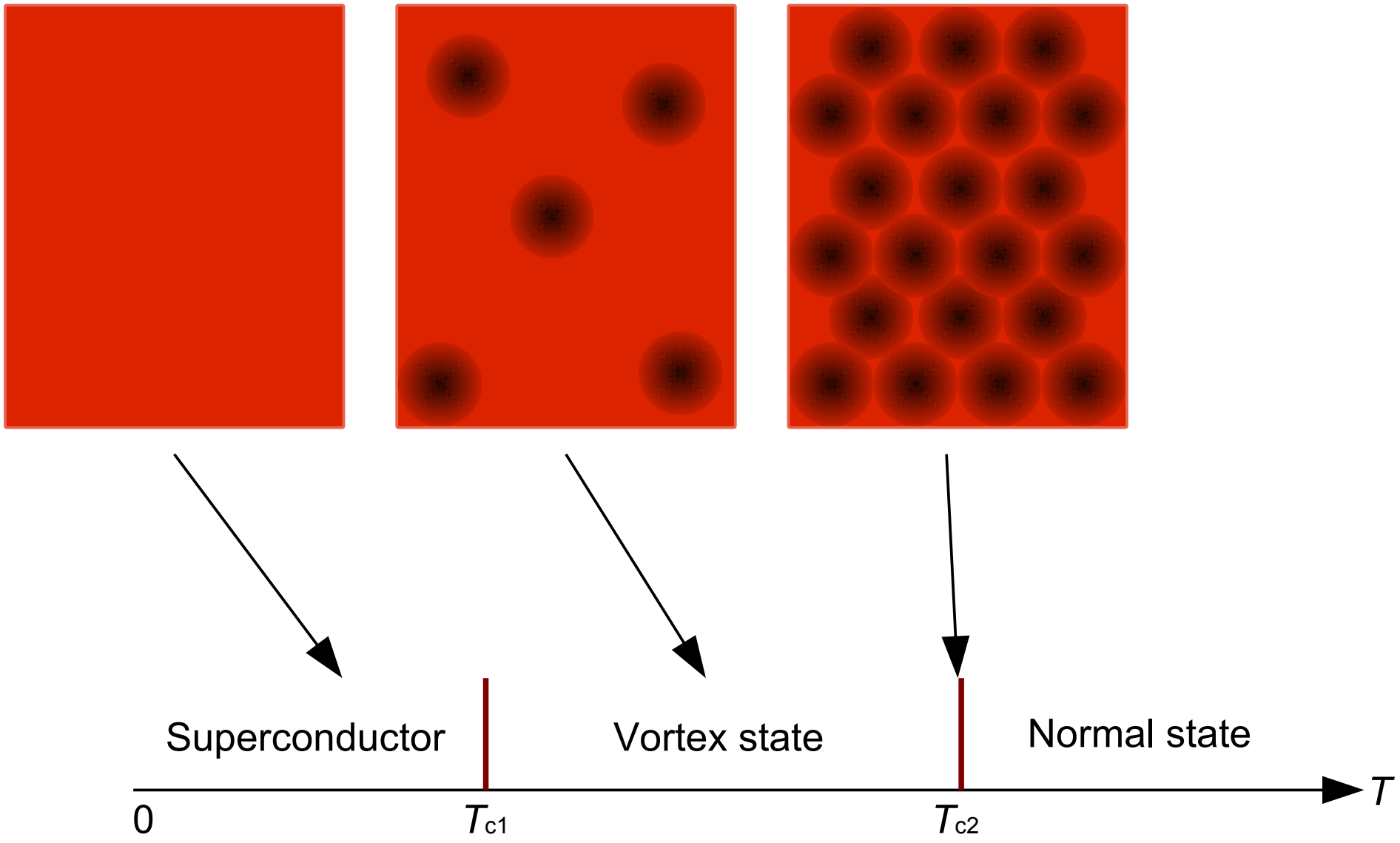
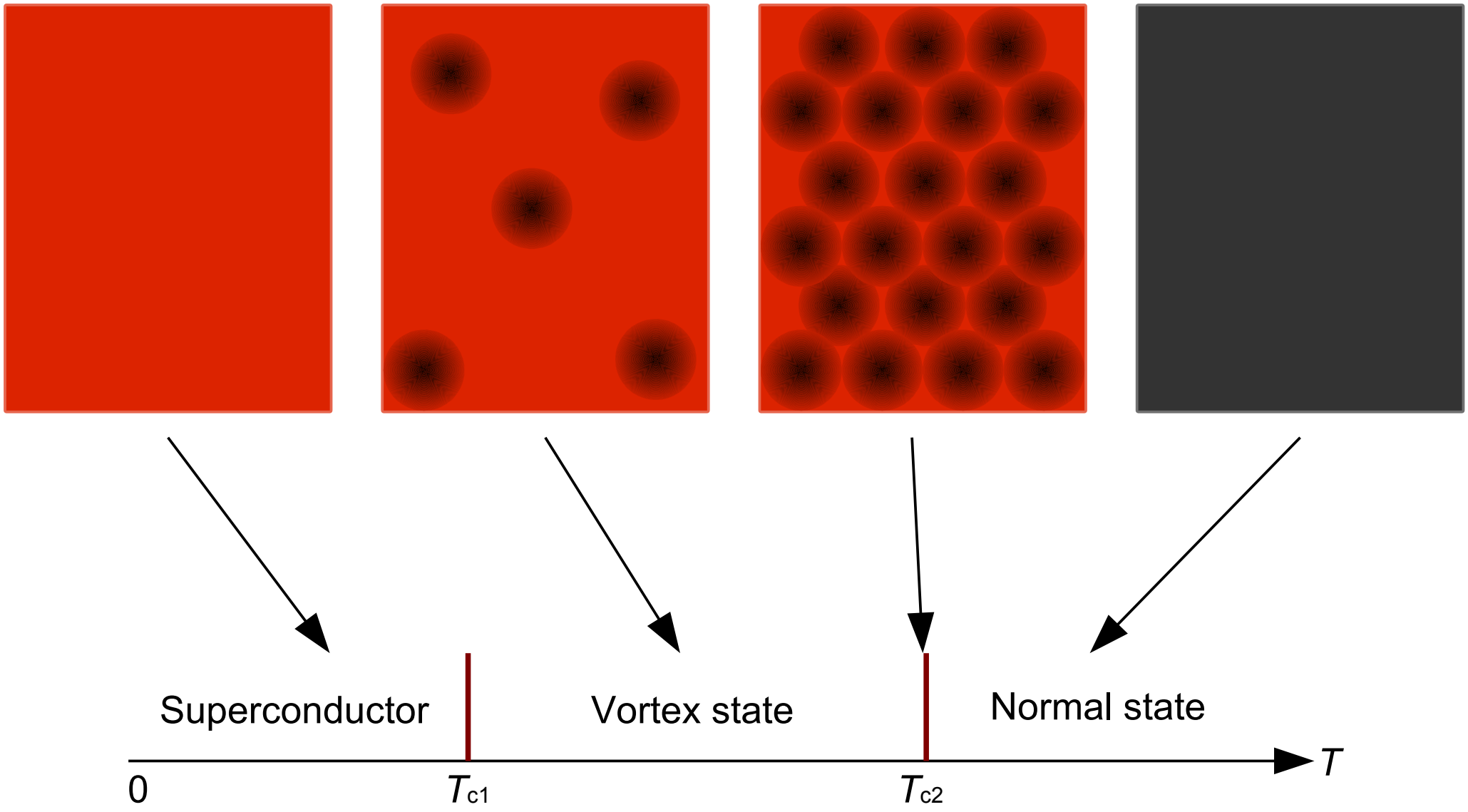
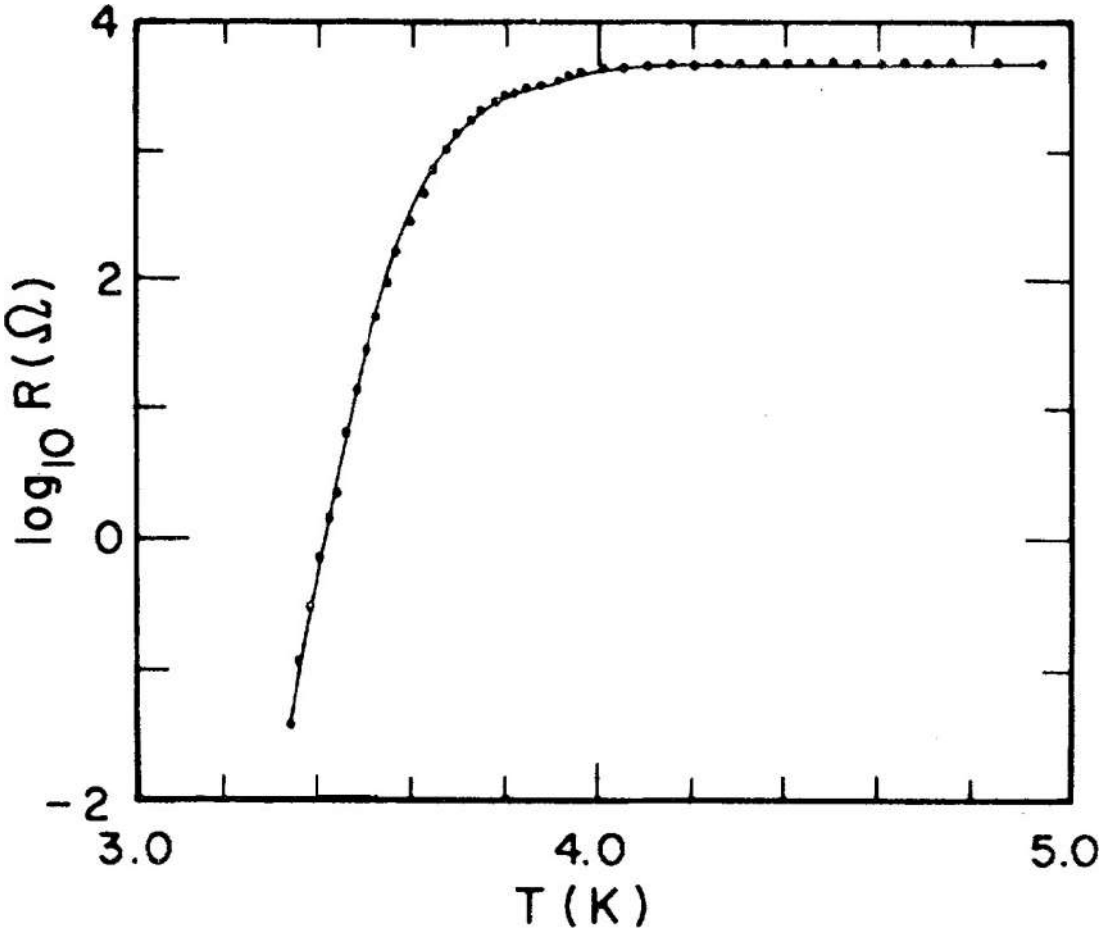
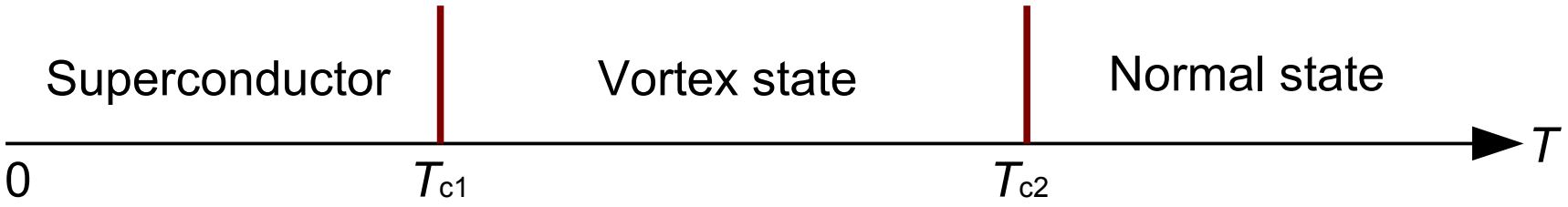


Kosterlitz Thouless transition

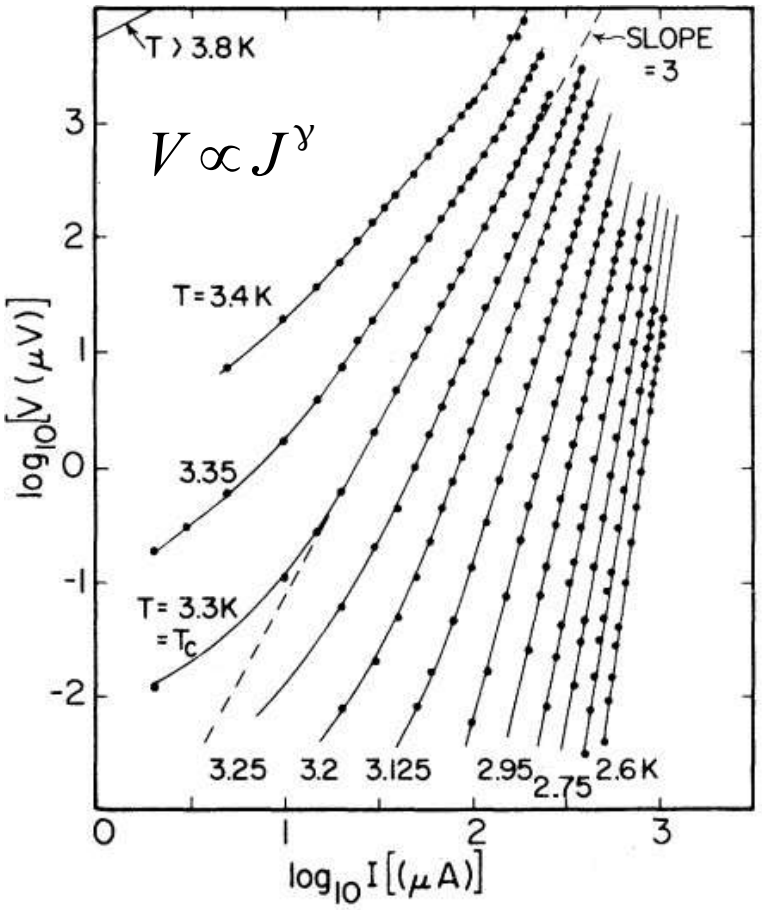


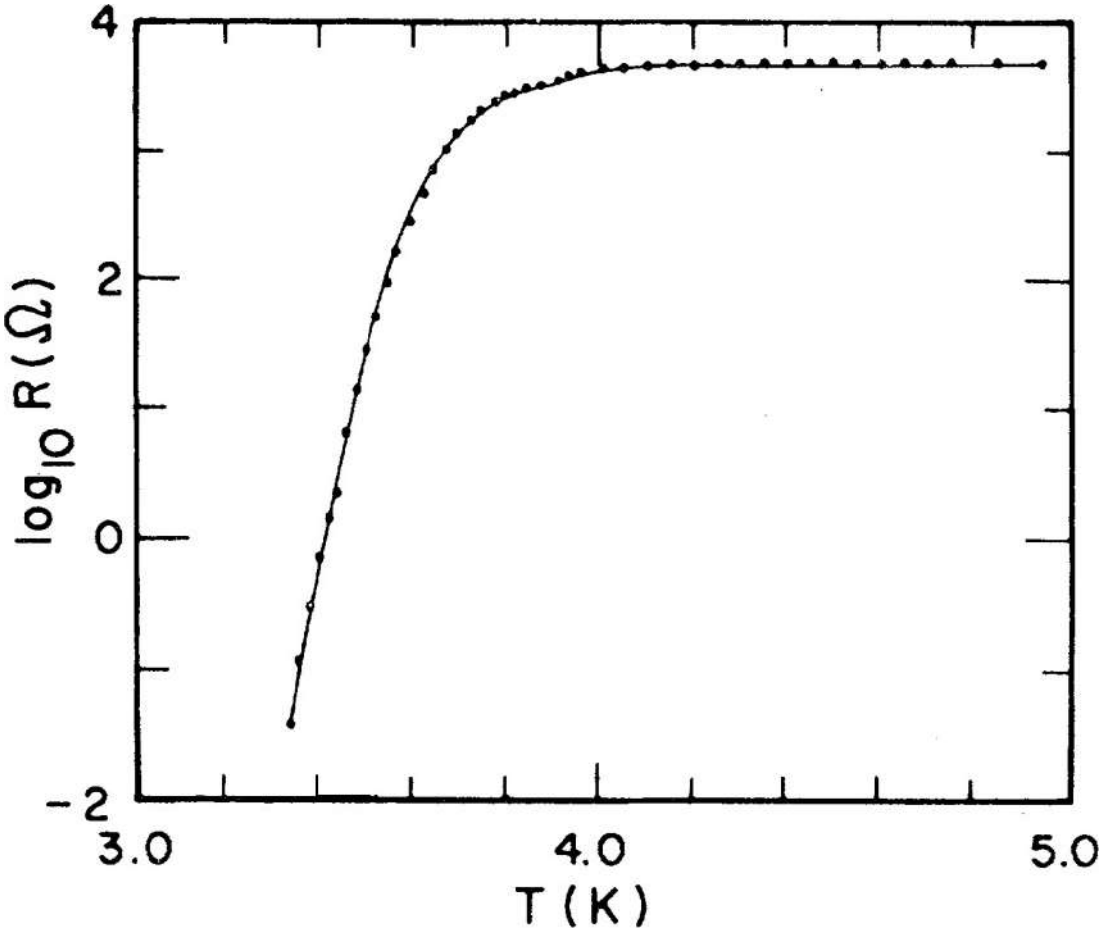
KT transition conductivity

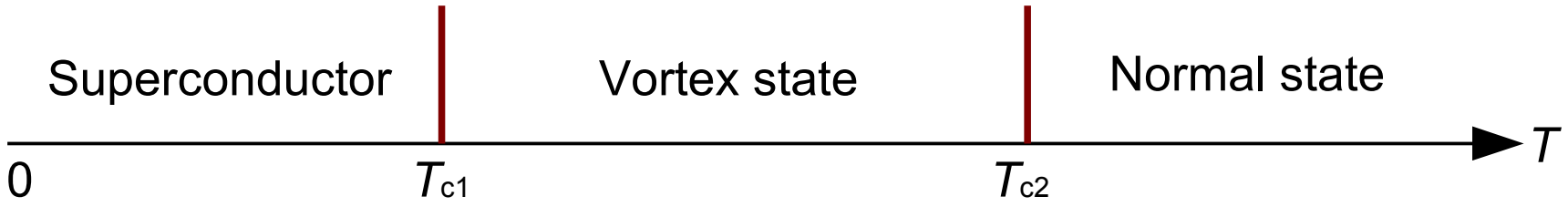




KT transition conductivity

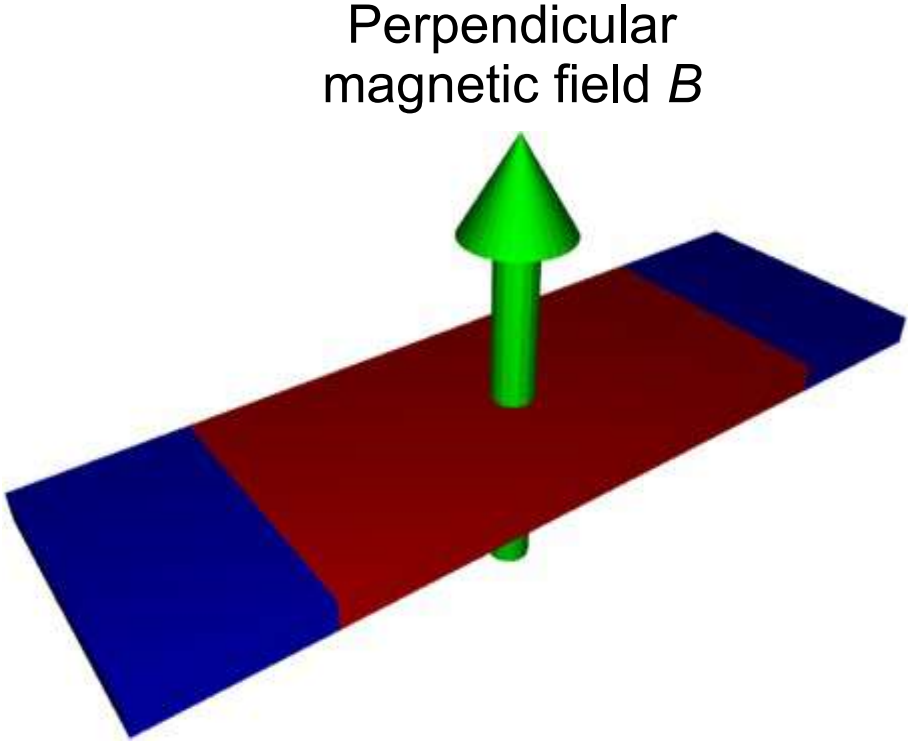


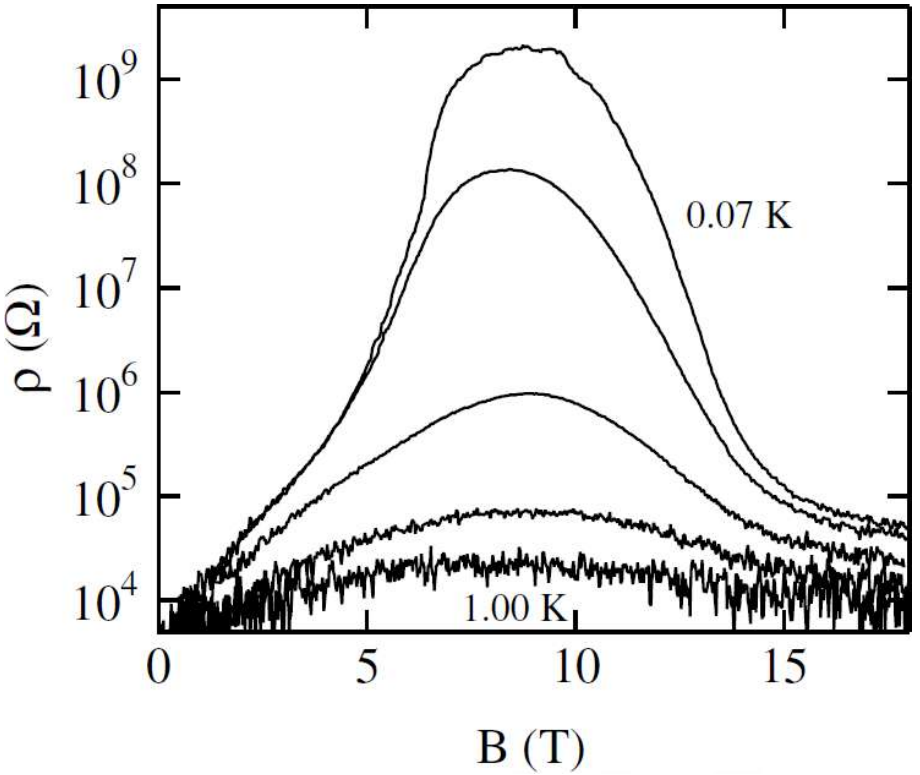




Transition in highly disordered systems

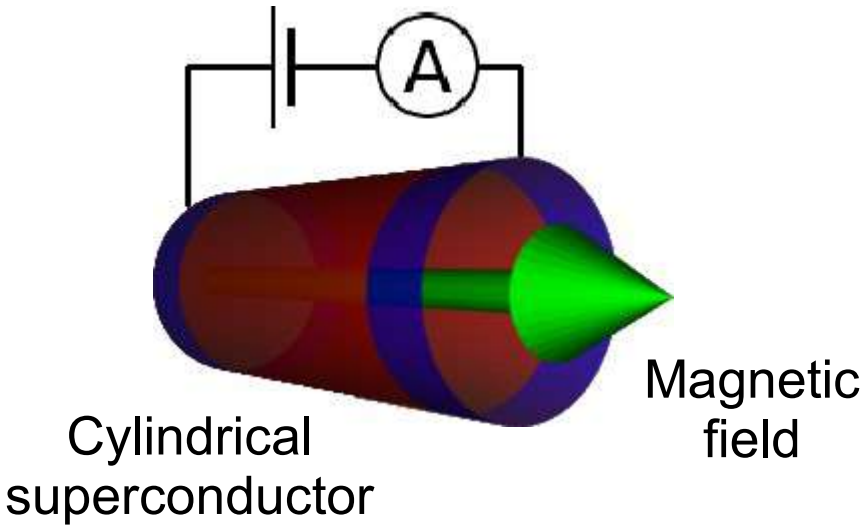
- Magnetoresistance peak [Sambandamurthy 04]

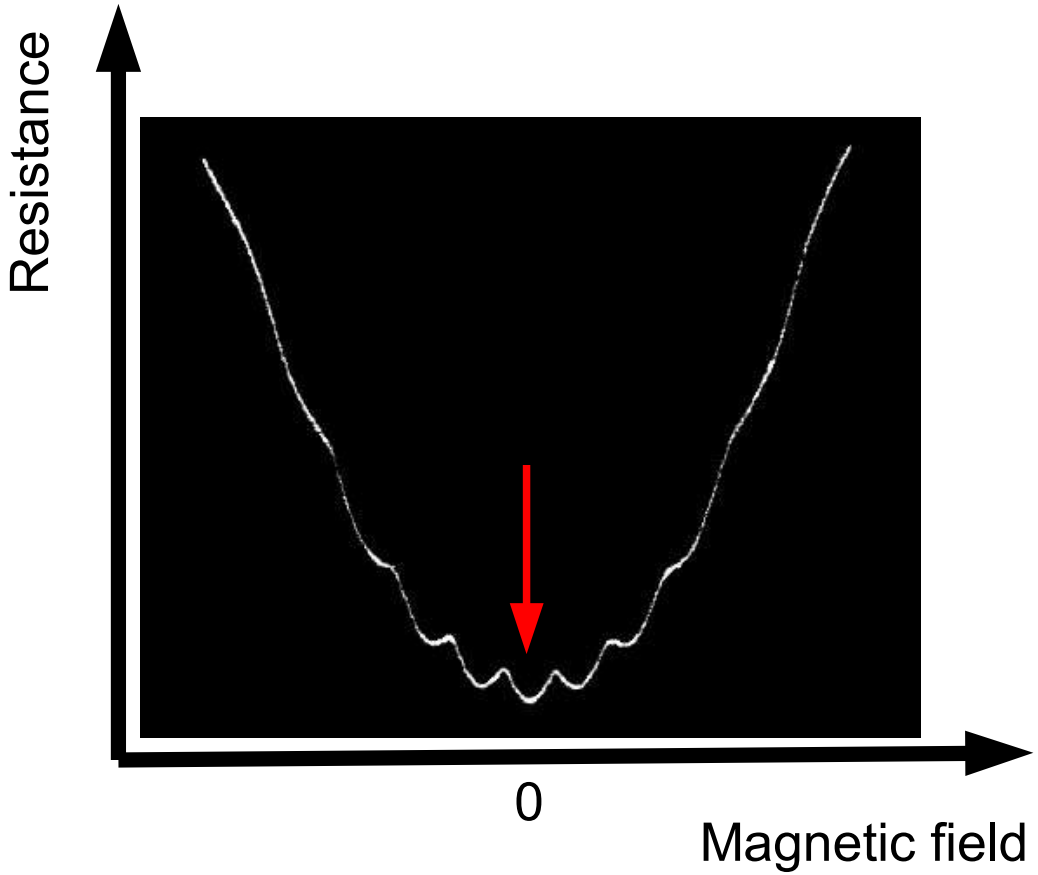




Little-Parks in a large diameter cylinder

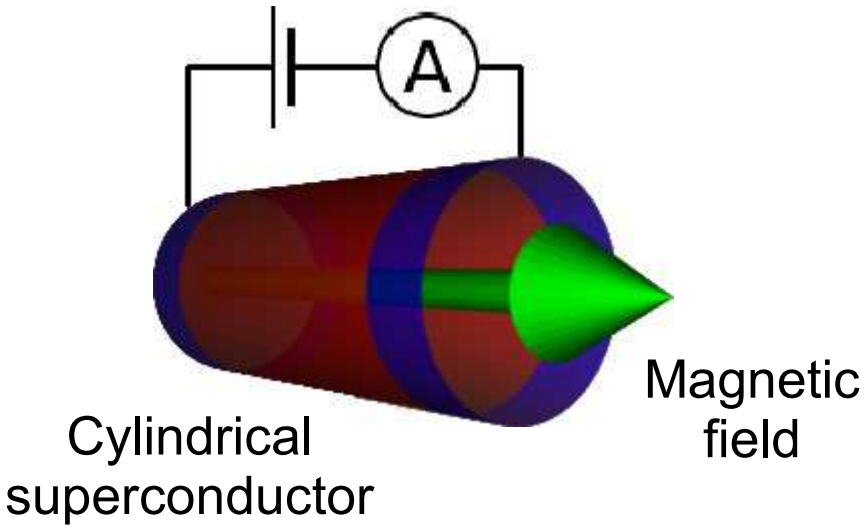
- Cylindrical superconductor held at transition temperature and zero threading flux [Little & Parks, PRL 1962]

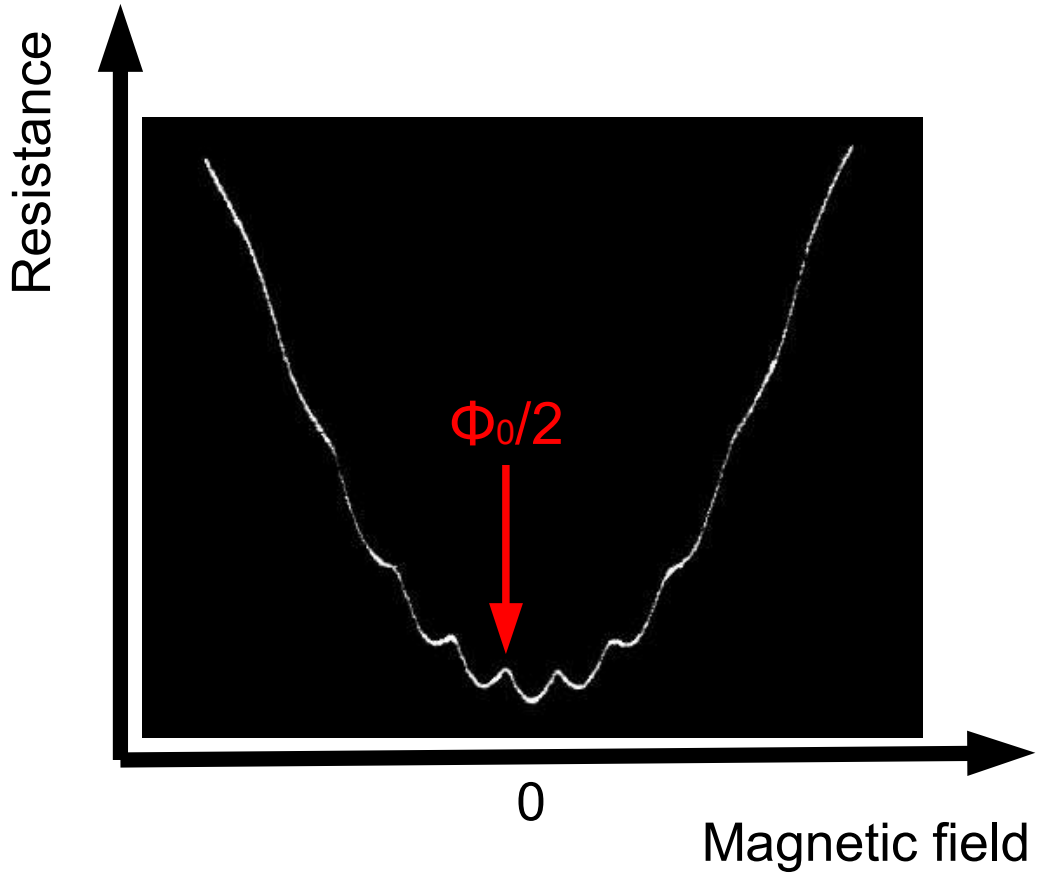




Little-Parks in a large diameter cylinder

- Cylindrical superconductor held at transition temperature and threading flux is increased [Little & Parks, PRL 1962]





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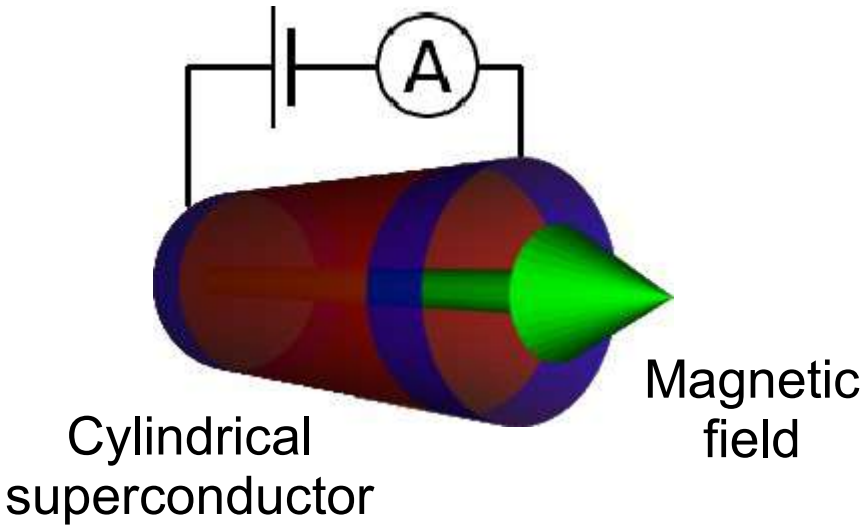
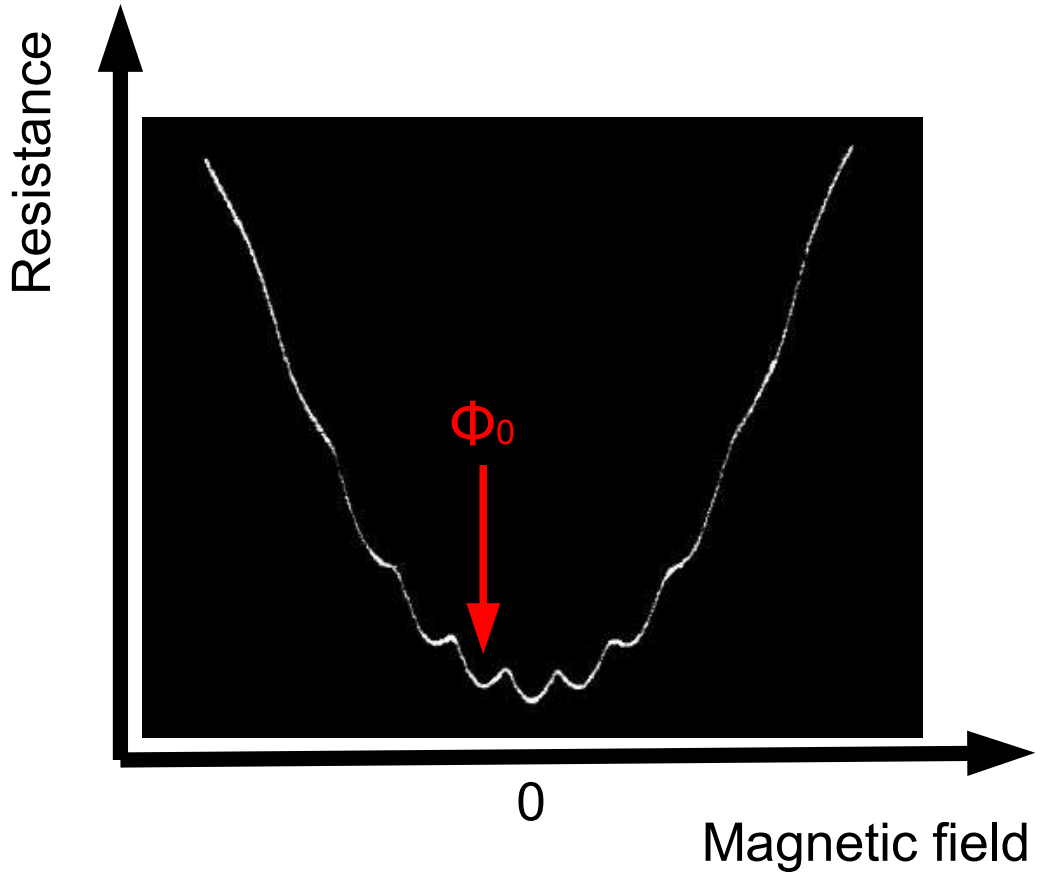
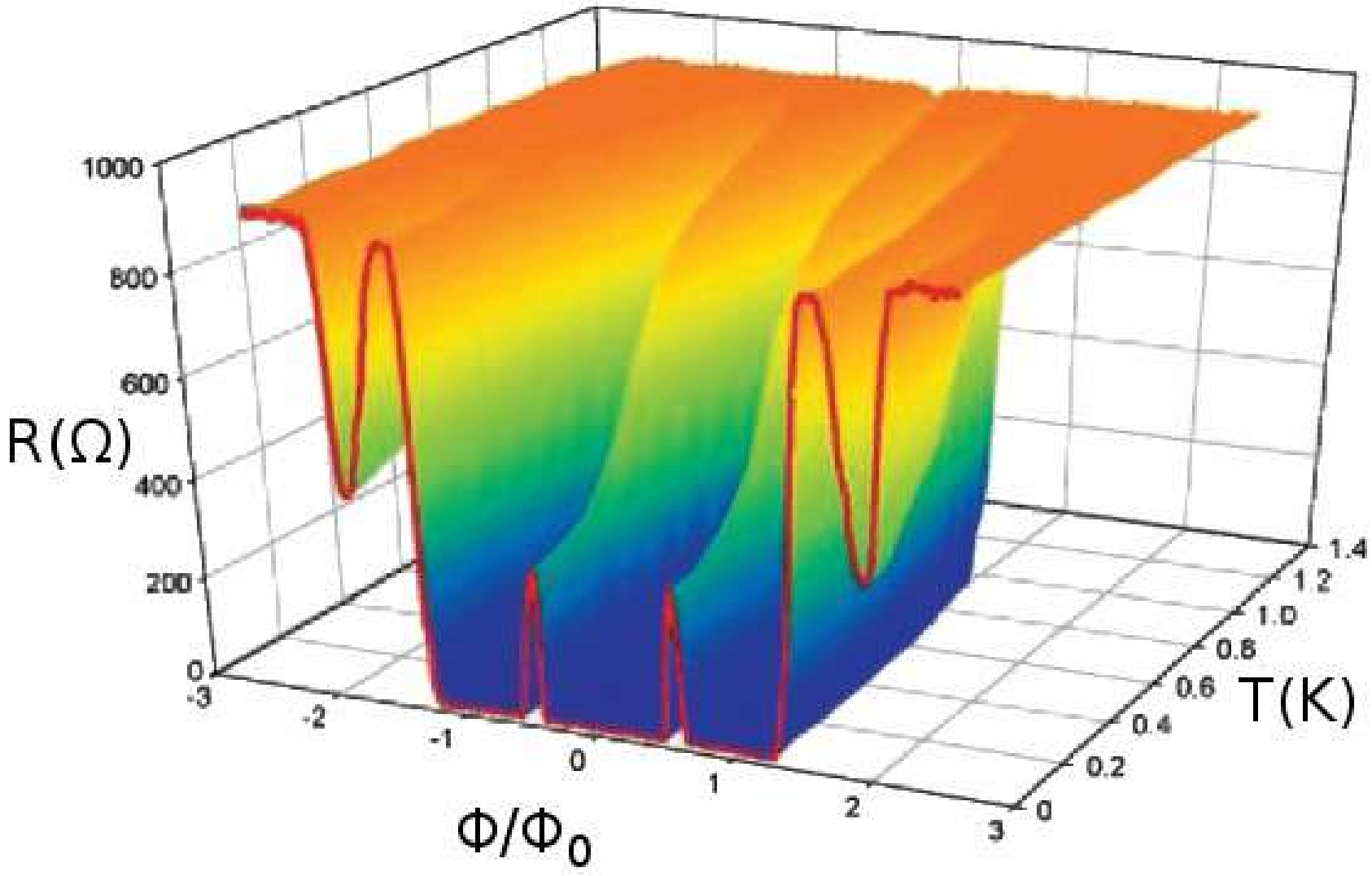




Little-Parks in a small diameter cylinder

- Reduce cylinder diameter to superconducting correlation length [Liu *et al*., Science 2001; Wang *et al*., PRL 2005]



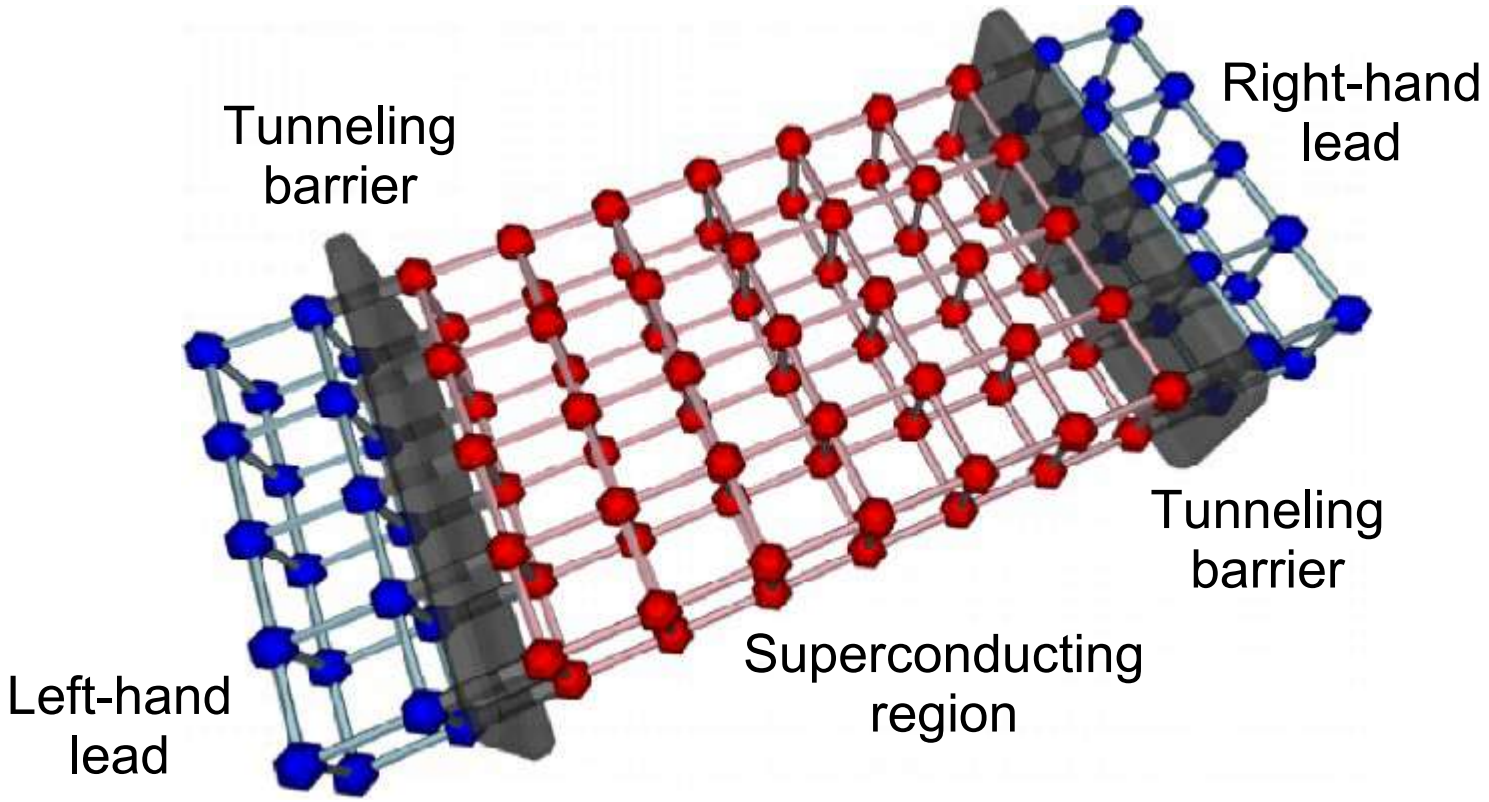
Strategy to study superconductors

- Develop new formalism to:
  - Calculate exact net current flow
  - Extract the microscopic current flow
  - Account for phase and amplitude fluctuations
  - Develop algorithm that permits access to large systems
- Test the formalism against a series of well-established results
- Study the Little Parks effect and magnetoresistance peak

How to calculate the current

- General expression for the current [Meir & Wingreen, PRL 1992]

$$J = \frac{\mathrm{i}e}{2h} \int \mathrm{d}\epsilon \Big[\mathrm{Tr} \left\{ \left(f_{\mathrm{L}}(\epsilon)\Gamma^{\mathrm{L}} - f_{\mathrm{R}}(\epsilon)\Gamma^{\mathrm{R}} \right) \left(G^{\mathrm{r}}_{\mathrm{e}\sigma} - G^{\mathrm{a}\sigma}_{\mathrm{e}} \right) \right\} + \mathrm{Tr} \left\{ (\Gamma^{\mathrm{L}} - \Gamma^{\mathrm{R}}) G^{<}_{\mathrm{e}\sigma} \right\} \Big]$$



Decoupling the interactions

- Negative *U* Hubbard model

$$\hat{H}_{\text{Hubbard}} = \sum_{i,\sigma} \epsilon_{i\sigma} c^{\dagger}_{i\sigma} c_{i\sigma} - \sum_{i} U_i c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow} c_{i\downarrow} c_{i\uparrow}$$

$$- \sum_{\langle i,j \rangle,\sigma} \left(t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + t^{*}_{ij} c^{\dagger}_{j\sigma} c_{i\sigma} \right)$$

- Decouple in density and Cooper pair channels

$$\rho_{i\sigma} = -|U_i| c^{\dagger}_{i\sigma} c_{i\sigma} \qquad \Delta_i = |U_i| \; c_{i\downarrow} c_{i\uparrow}$$

- Hamiltonian now contains single-body operators

$$\hat{\mathcal{H}}_{\text{BdG}} = \sum_{i,\sigma} (\epsilon_i + \rho_i) c^{\dagger}_{i\sigma} c_{i\sigma} - \sum_{\langle i,j \rangle,\sigma} \left(t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + t^{*}_{ij} c^{\dagger}_{j\sigma} c_{i\sigma} \right)$$

$$+ \sum_{i} \left(\Delta_i c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow} + \bar{\Delta}_i c_{i\downarrow} c_{i\uparrow} \right) + \sum_{i} \frac{|\Delta_i|^2 + \rho_i^2}{U_i}$$

Diagonalizing the Hamiltonian

- Hamiltonian now contains single-body operators

$$\hat{\mathcal{H}}_{\mathrm{BdG}} = \sum_{i,\sigma}(\epsilon_i + \rho_i)c^\dagger_{i\sigma}c_{i\sigma} - \sum_{\langle i,j\rangle,\sigma}\left(t_{ij}c^\dagger_{i\sigma}c_{j\sigma} + t^*_{ij}c^\dagger_{j\sigma}c_{i\sigma}\right)$$

$$+\sum_i\left(\Delta_i c^\dagger_{i\uparrow}c^\dagger_{i\downarrow} + \bar{\Delta}_i c_{i\downarrow}c_{i\uparrow}\right) + \sum_i \frac{|\Delta_i|^2 + \rho_i^2}{U_i}$$

- Energy eigenstates can be found from diagonalization of

$$\hat{\mathcal{H}}_{\mathrm{BgG}} = \frac{|\Delta|^2 + \rho^2}{U} + \begin{pmatrix} c^\dagger_\uparrow & c_\downarrow \end{pmatrix}\begin{pmatrix} \epsilon+\rho & \Delta \\ \bar{\Delta} & -(\epsilon+\rho)\end{pmatrix}\begin{pmatrix} c_\uparrow \\ c^\dagger_\downarrow \end{pmatrix} + \epsilon + \rho$$

Accelerated Metropolis sampling

- To perform thermal sum calculate

$$\langle J \rangle = \sum_{\Delta,\rho} J[\Delta,\rho] e^{-\beta(E[\Delta,\rho]-E_0)}$$

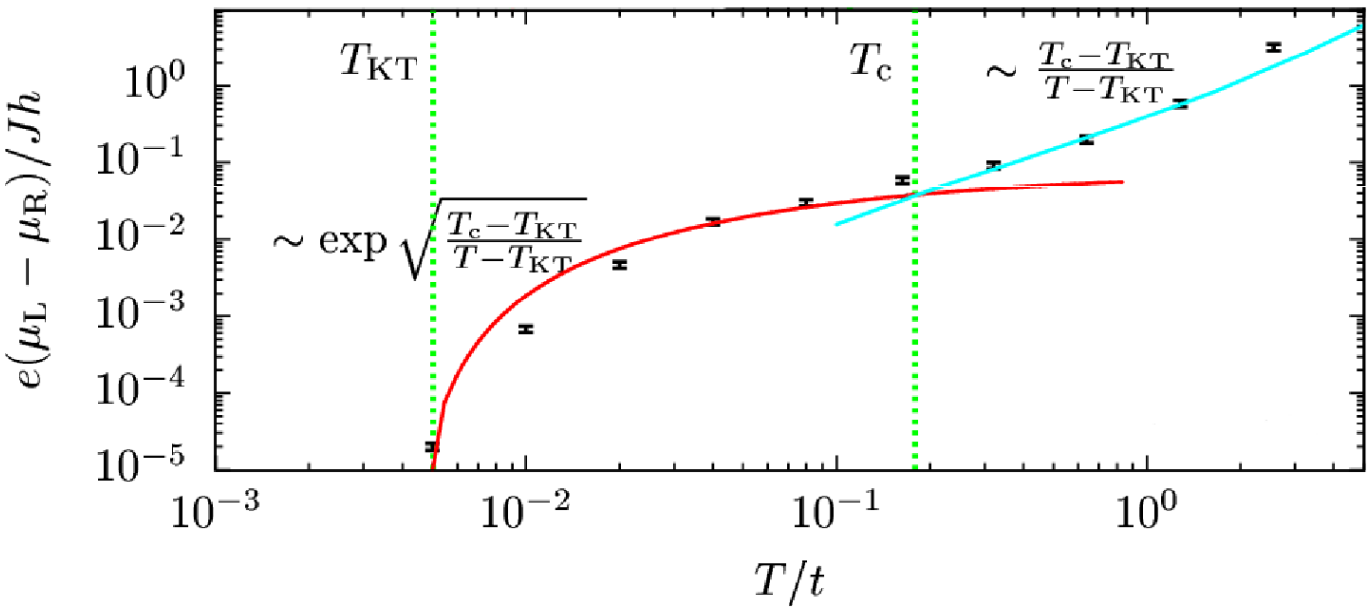
- Propose new configuration of Δ and ρ, accept with probability

$$\exp\left(\beta E[\Delta_{\text{old}},\rho_{\text{old}}]-\beta E[\Delta_{\text{new}},\rho_{\text{new}}]\right)$$

- Calculating $E[\Delta,\rho]$ costs $O(N^3)$, where N is the number of sites
- New method calculates $E[\Delta,\rho]-E[\Delta+\delta\Delta,\rho+\delta\rho]$ using a order M Chebyshev expansion [Weisse 09] in $O(N^{0.9}M^{2/3})$ time

Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear *IV* characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



Halperin & Nelson, J. Low Temp. Phys 1979

Ambegaokar *et al*., PRB 1980

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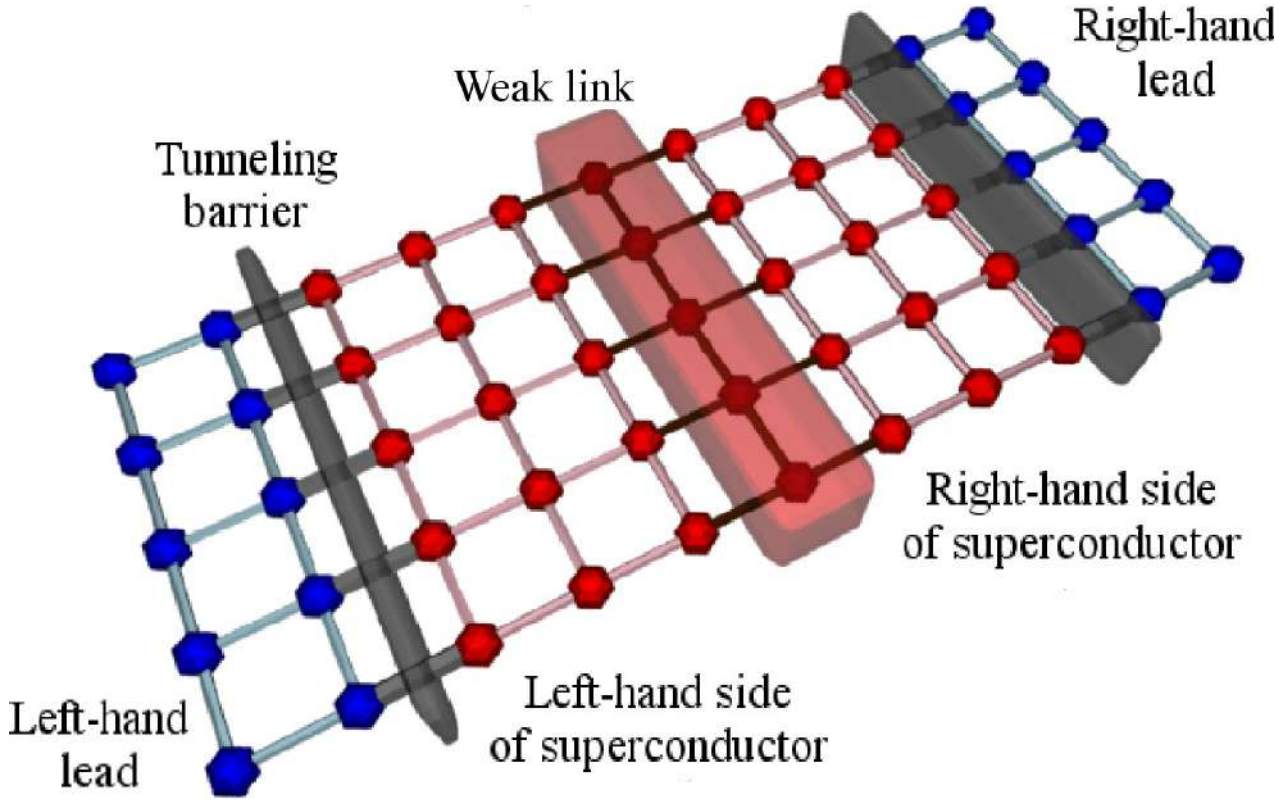

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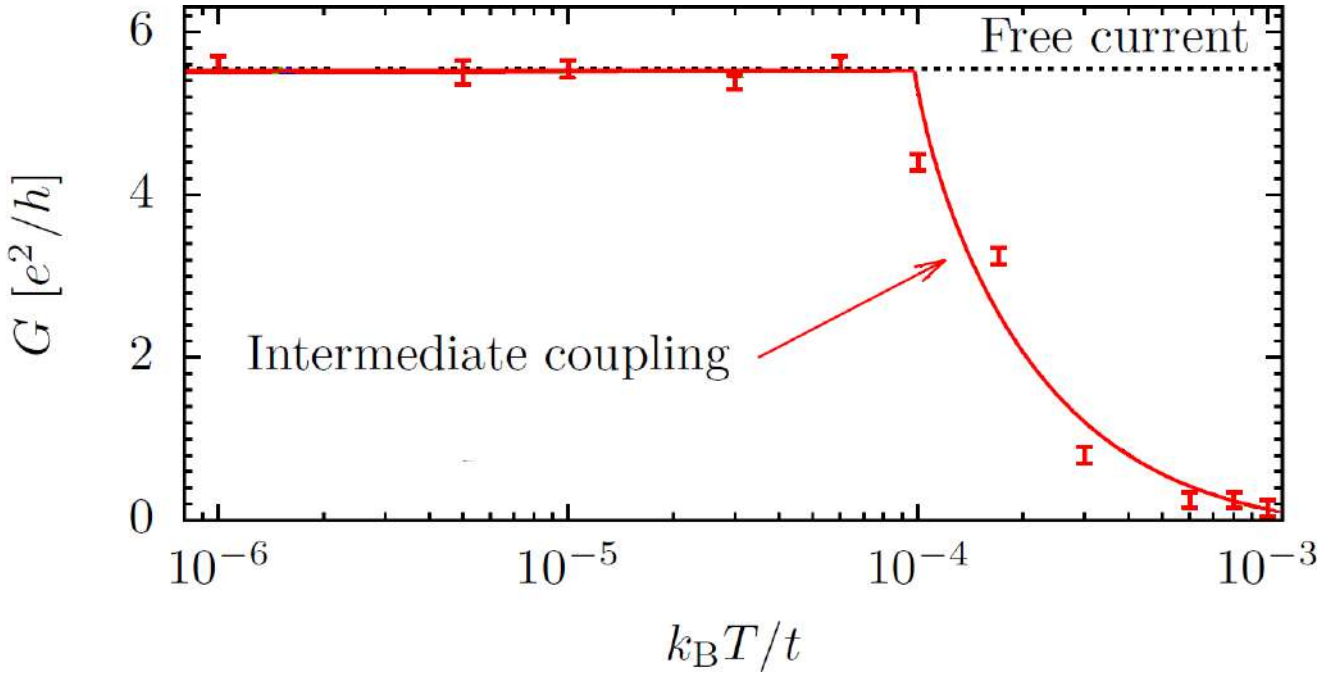

Ambegaokar & Baratoff, PRL **10**, 486 (1963)

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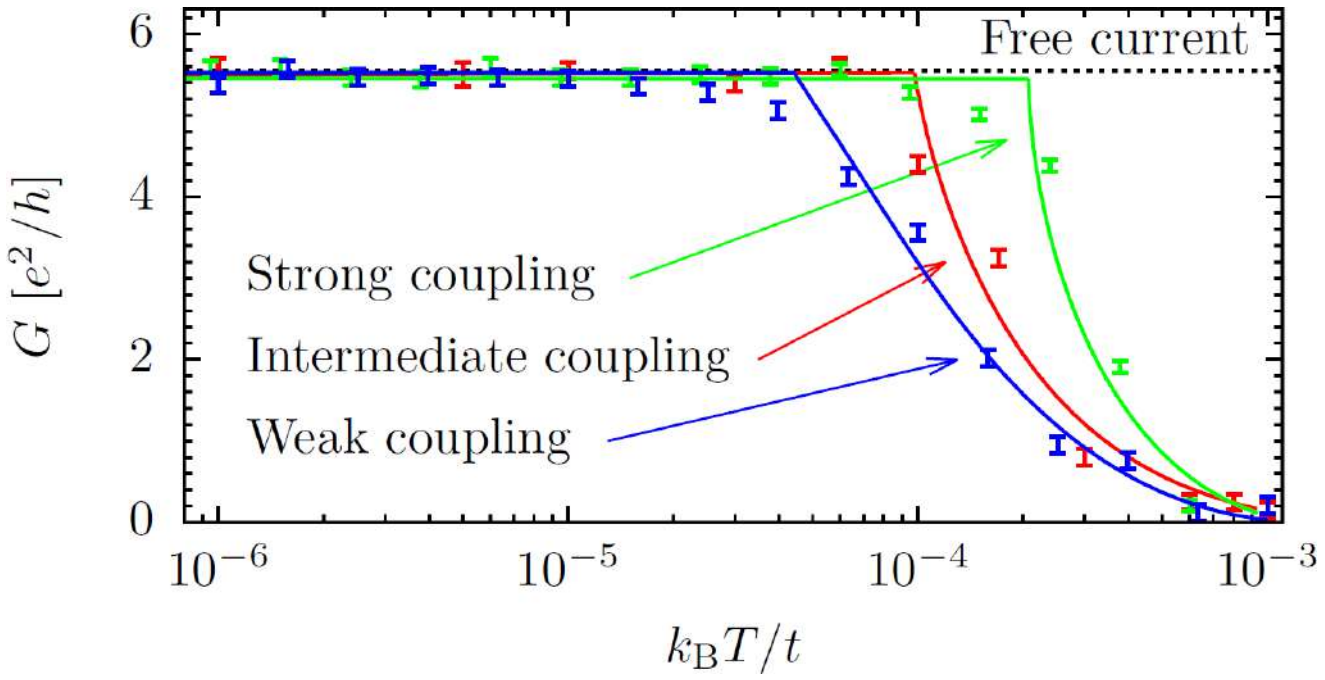

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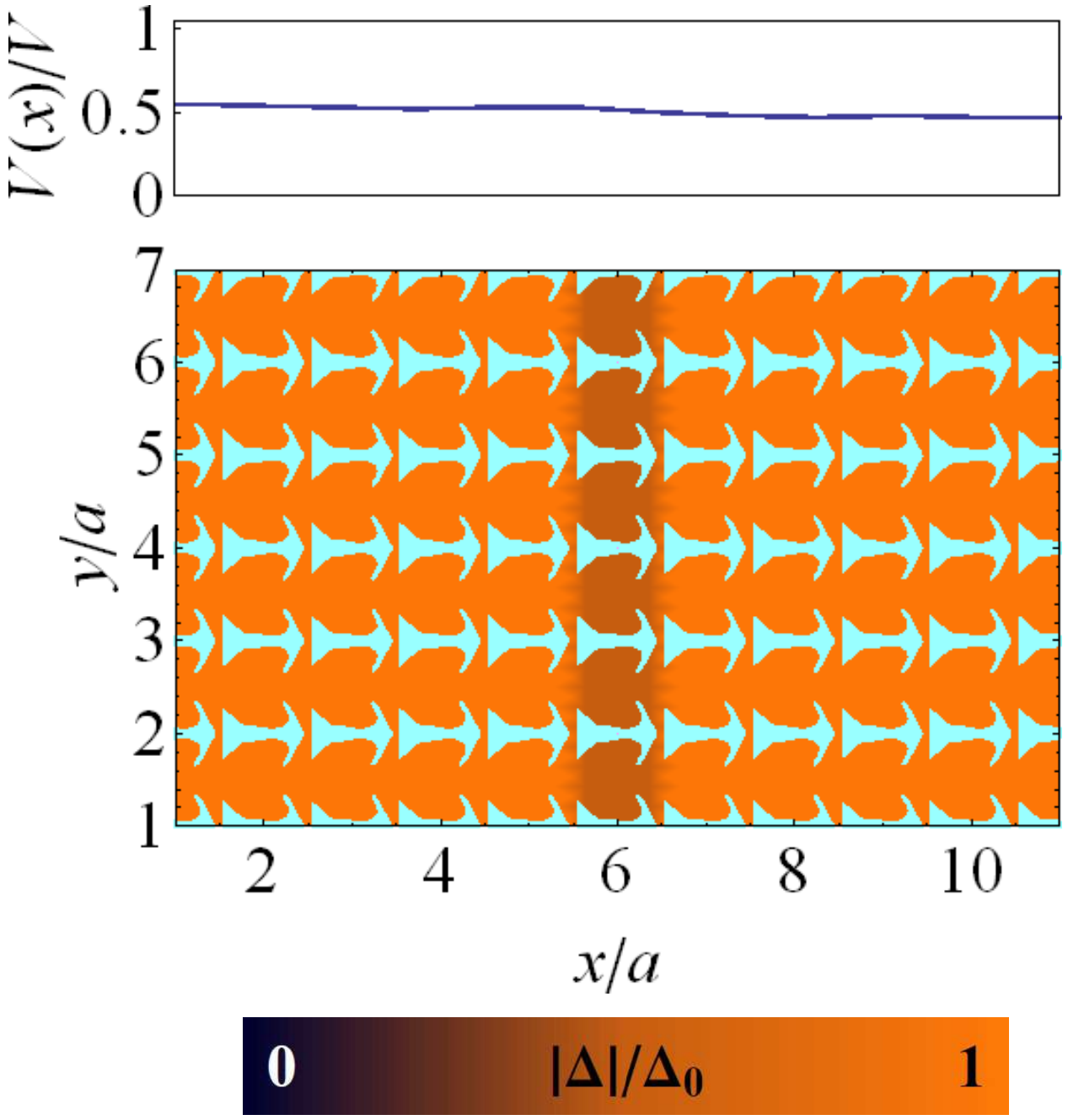

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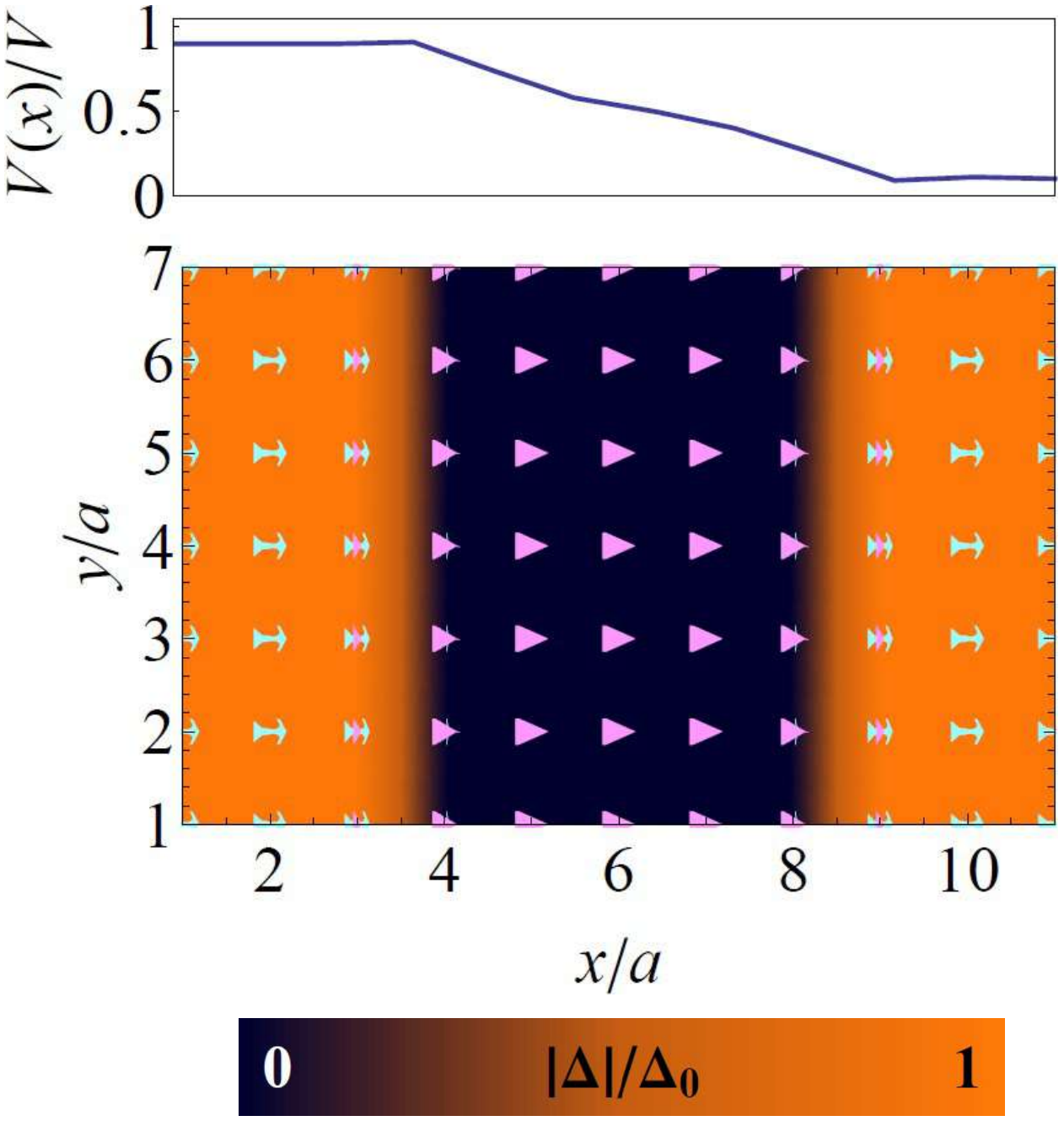

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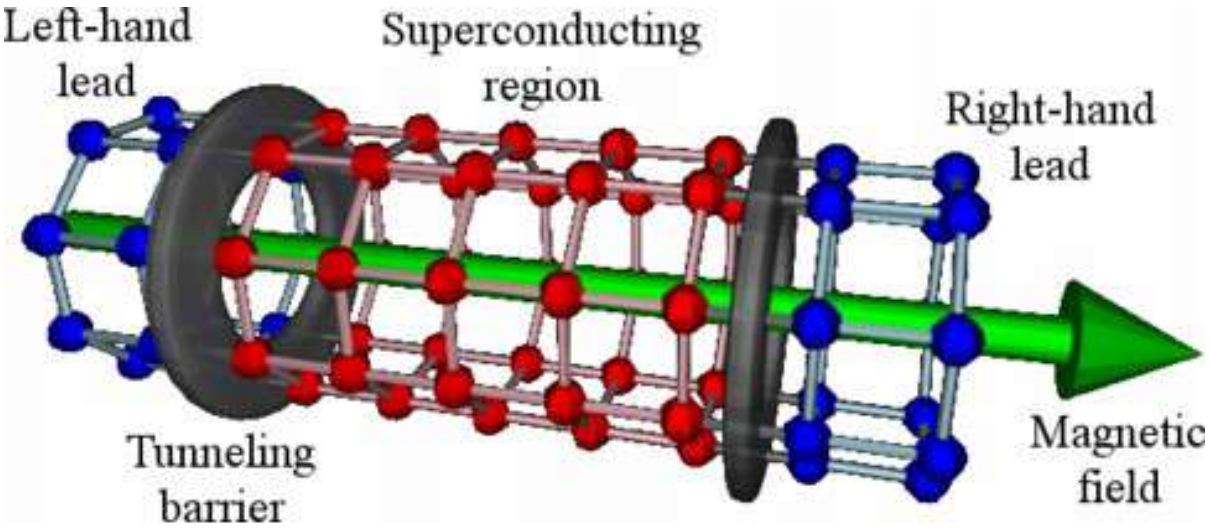
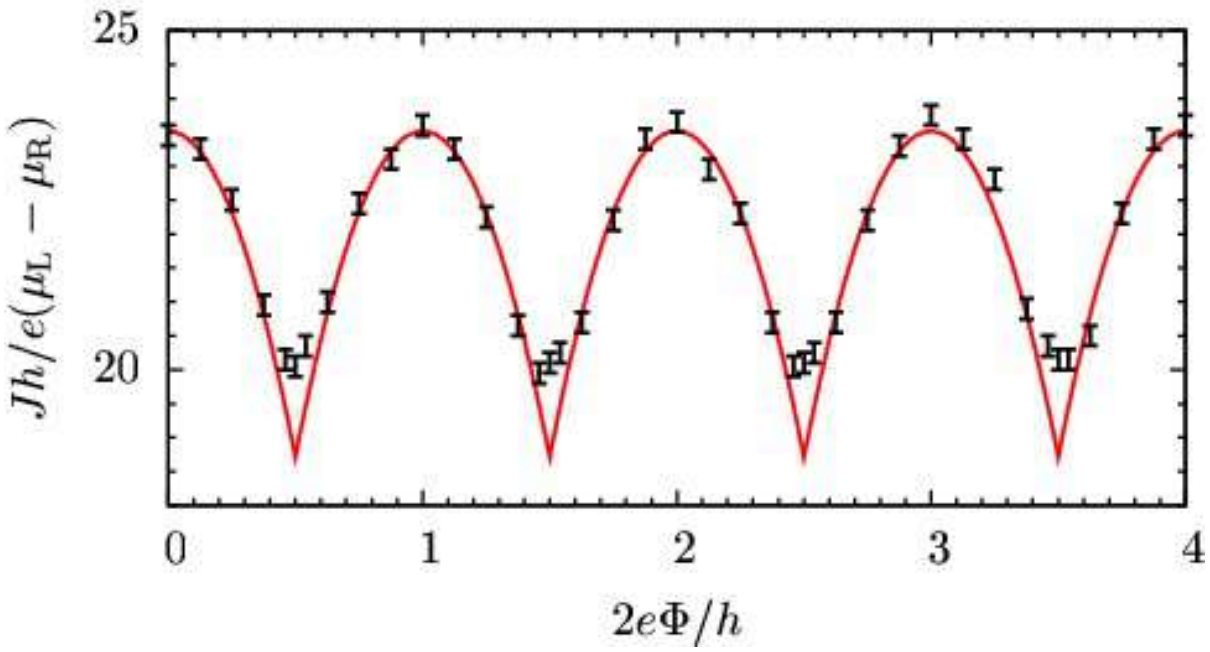
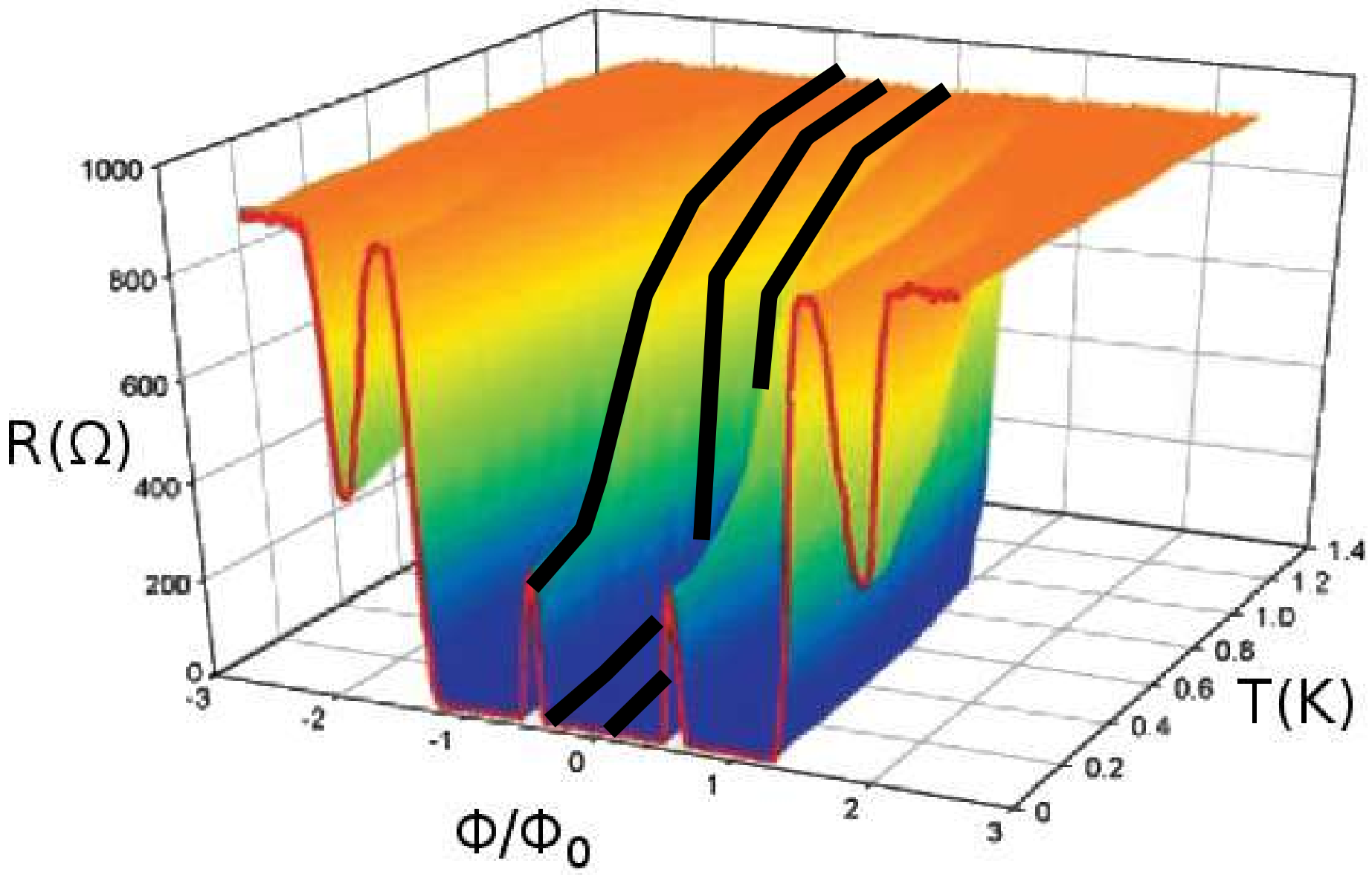
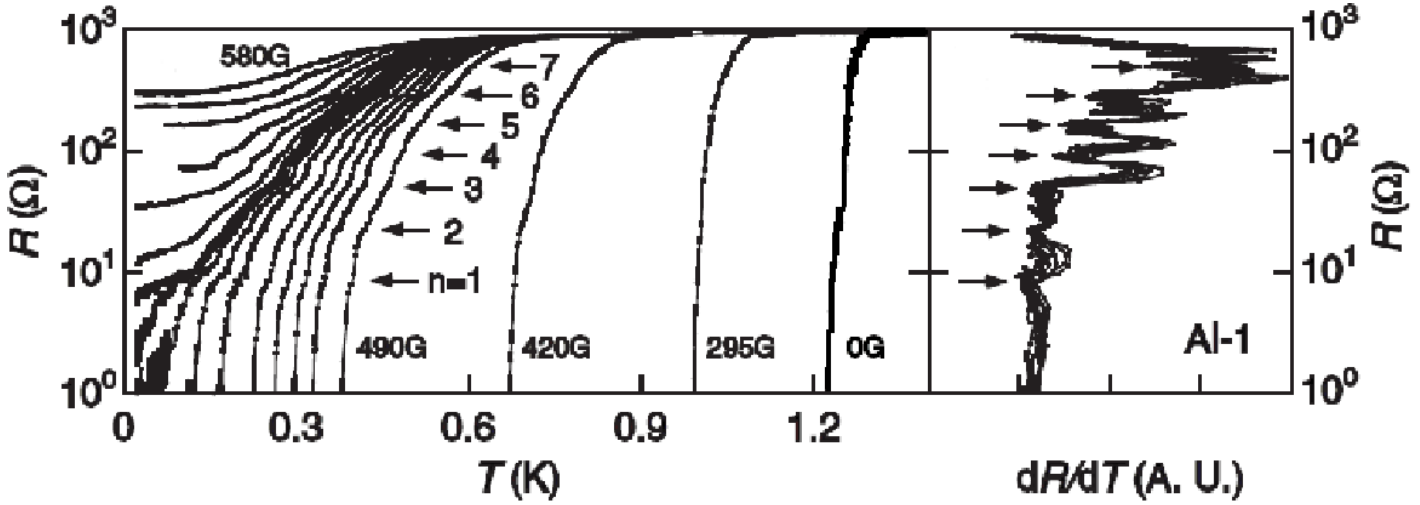


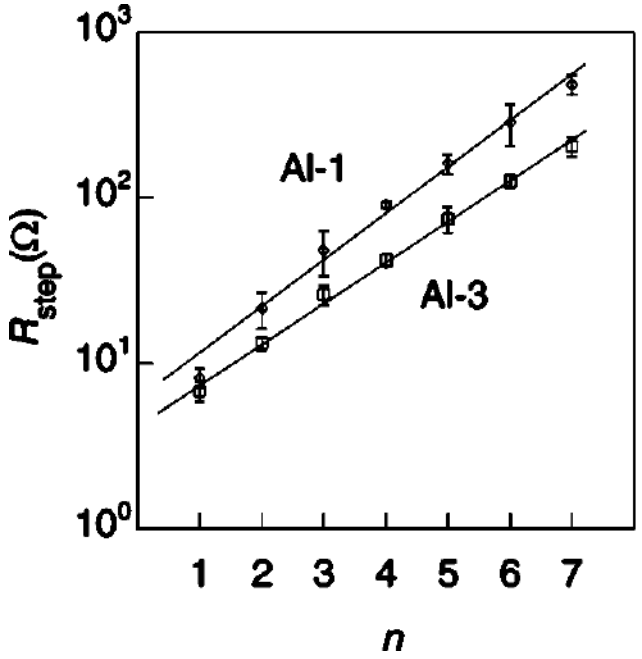


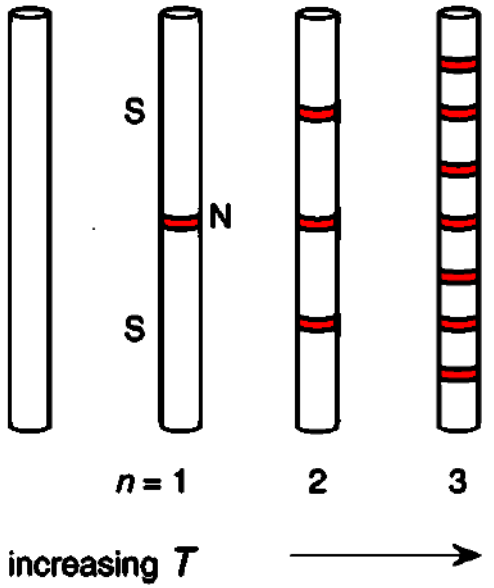
Little-Parks in a small diameter cylinder



Quantum phase transition hypothesis

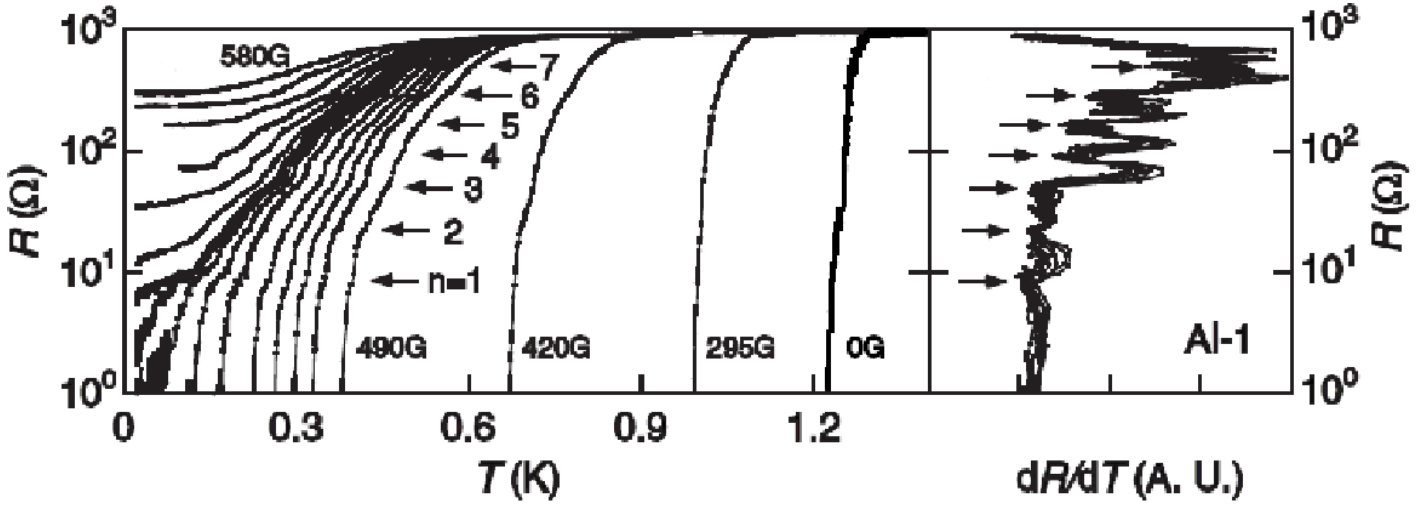


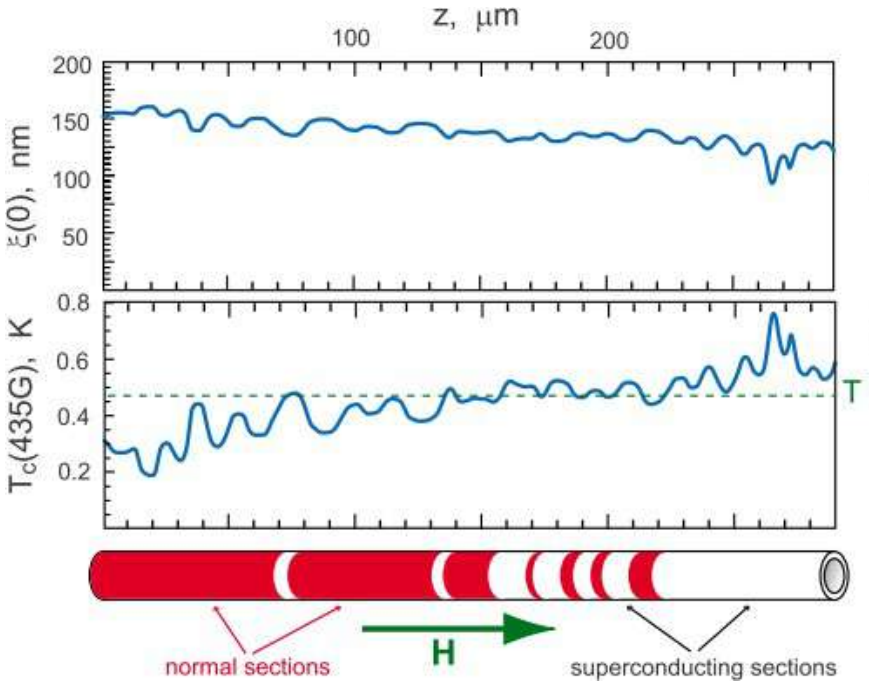


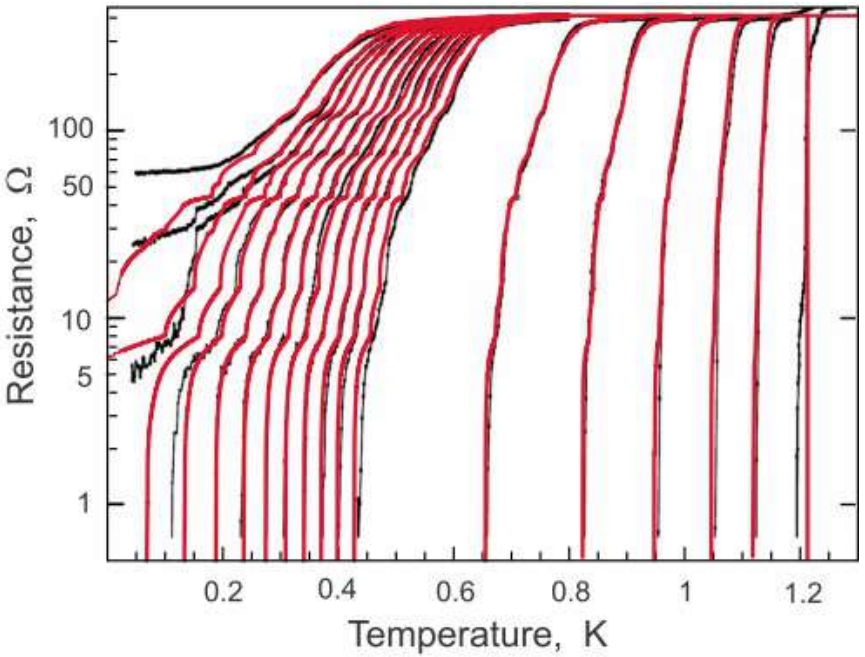


Wang et al. PRL (2005)

Mean-field BCS transition hypothesis



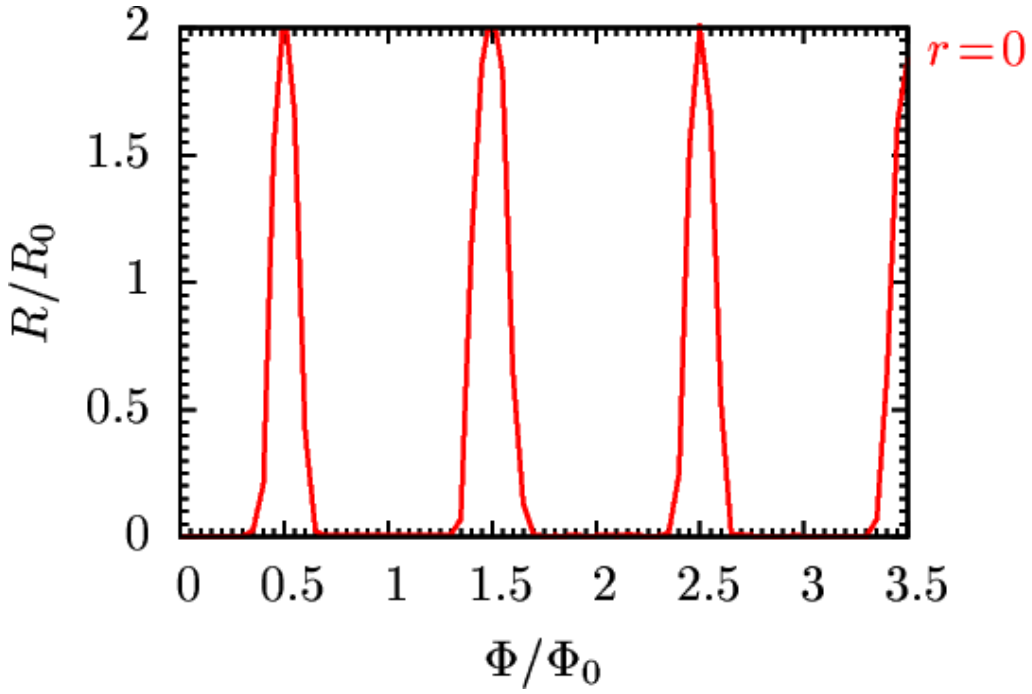


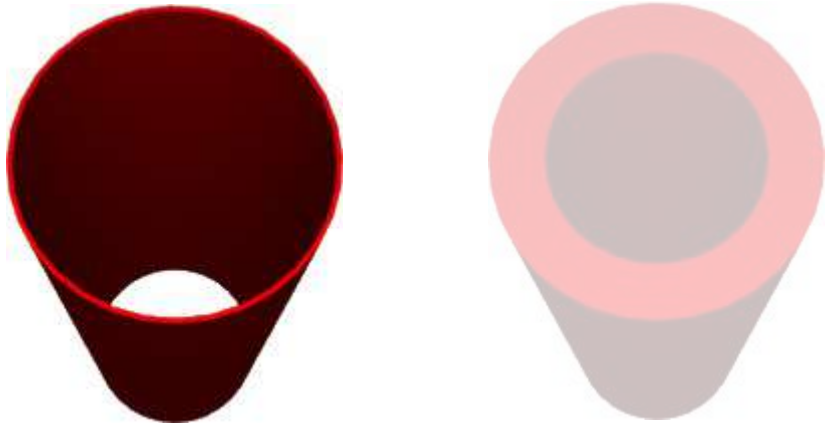


Dao & Chibotaru, PRB (2009)

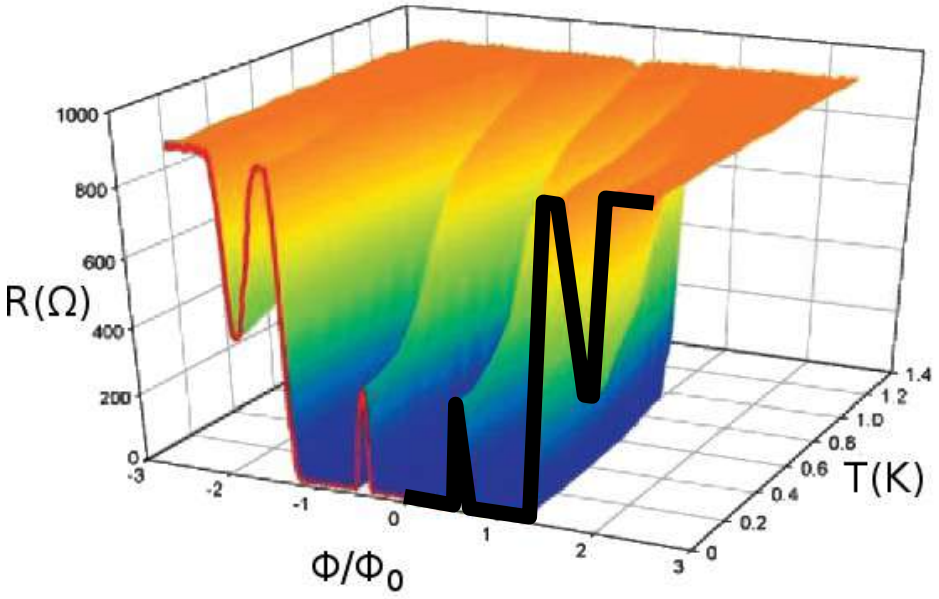
Little-Parks in a small diameter cylinder

Theory:





Experiment:



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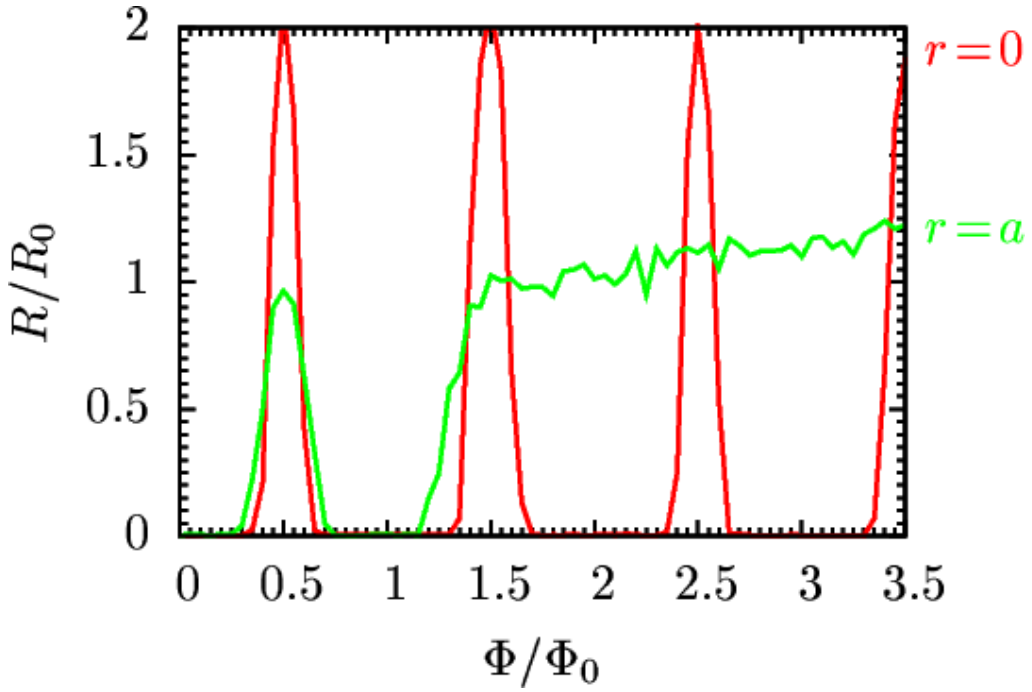

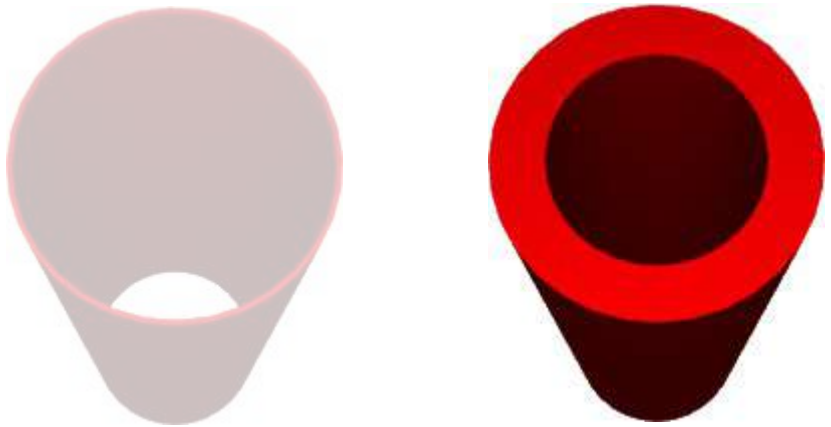

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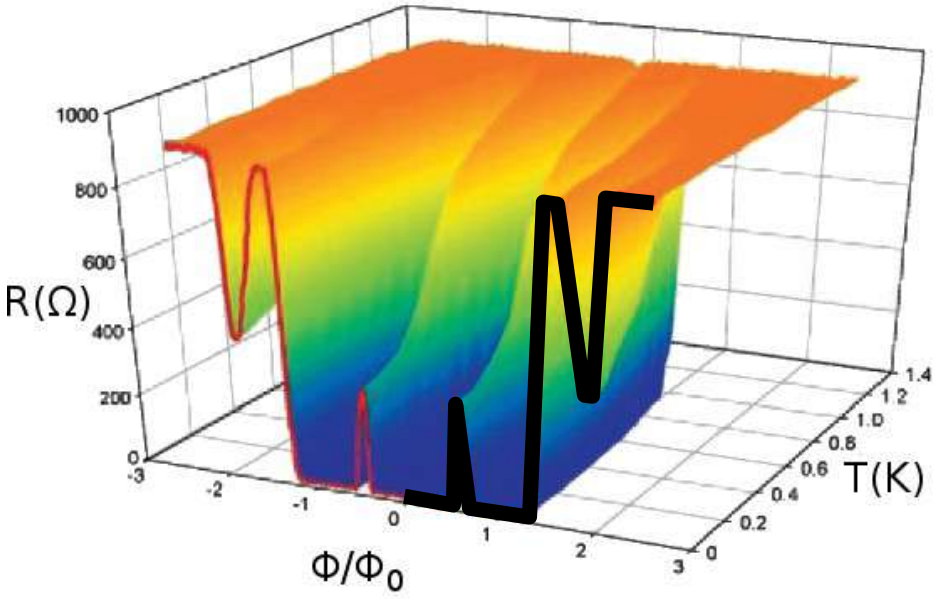

Little-Parks in a small diameter cylinder

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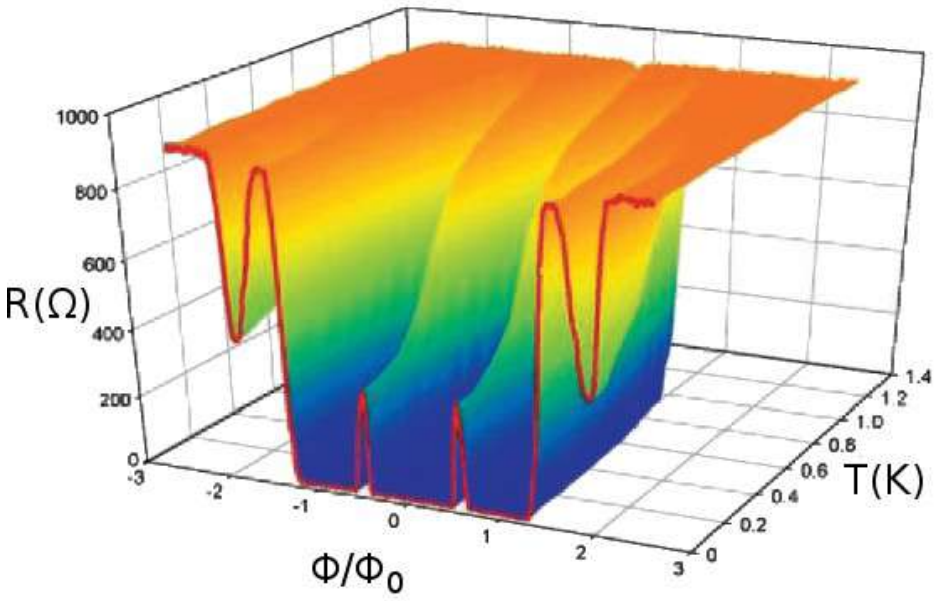

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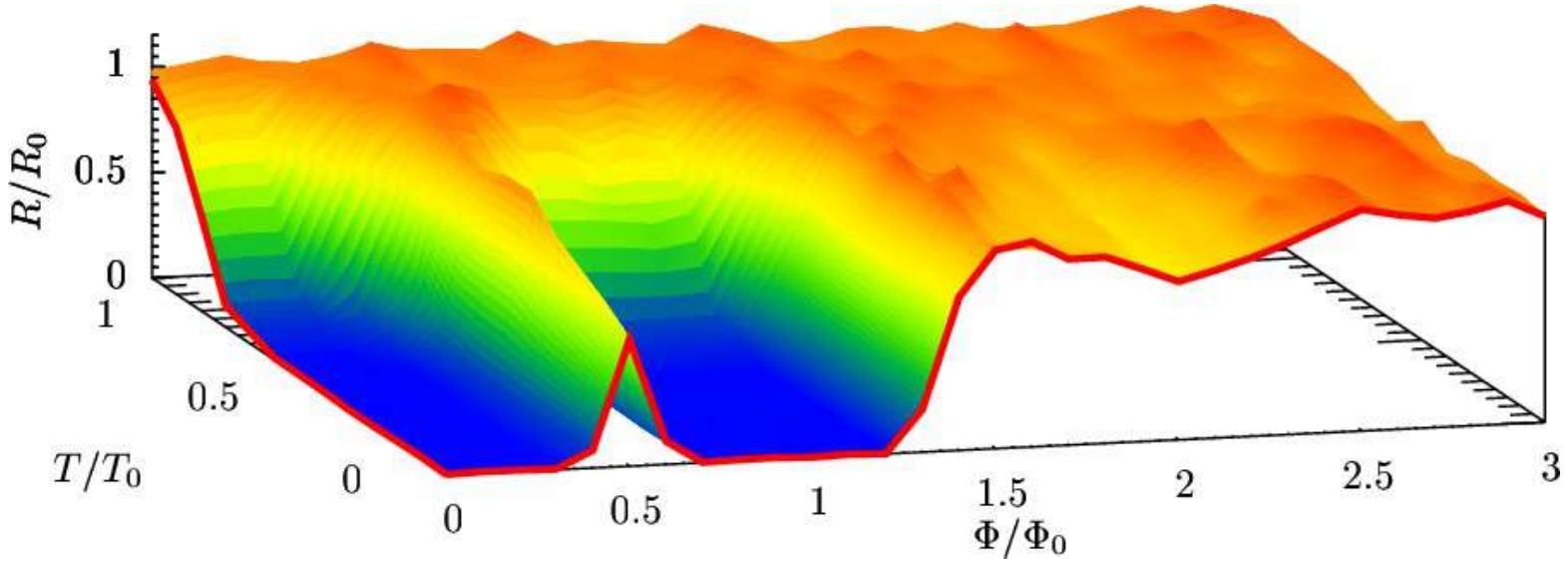

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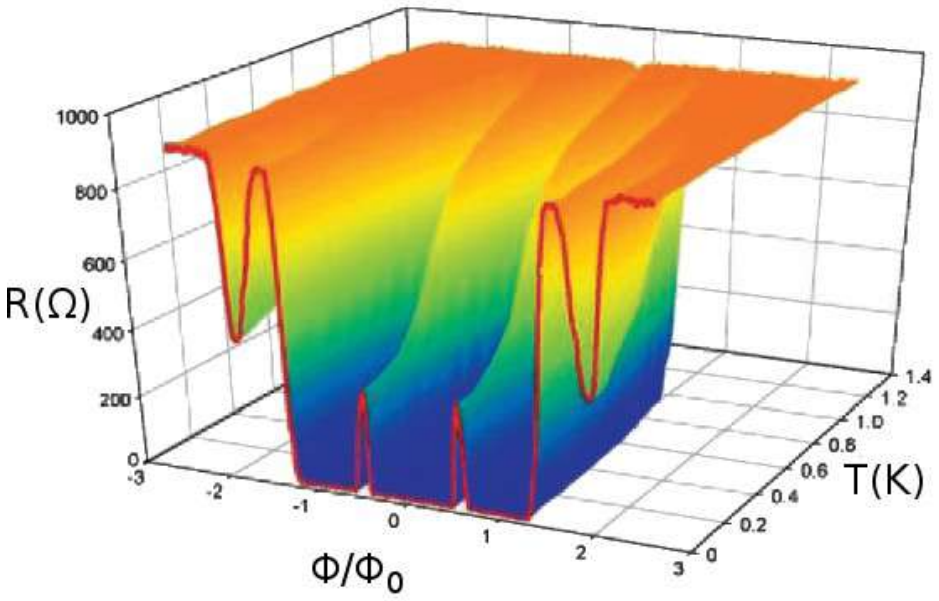

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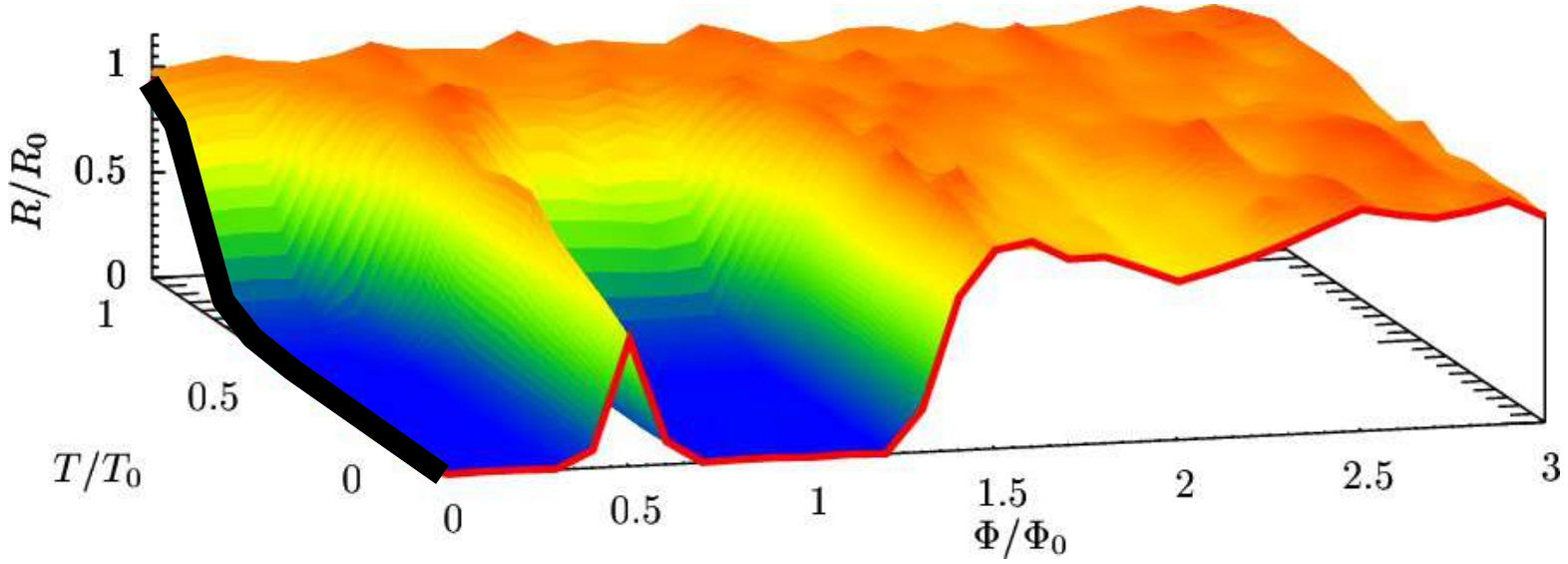
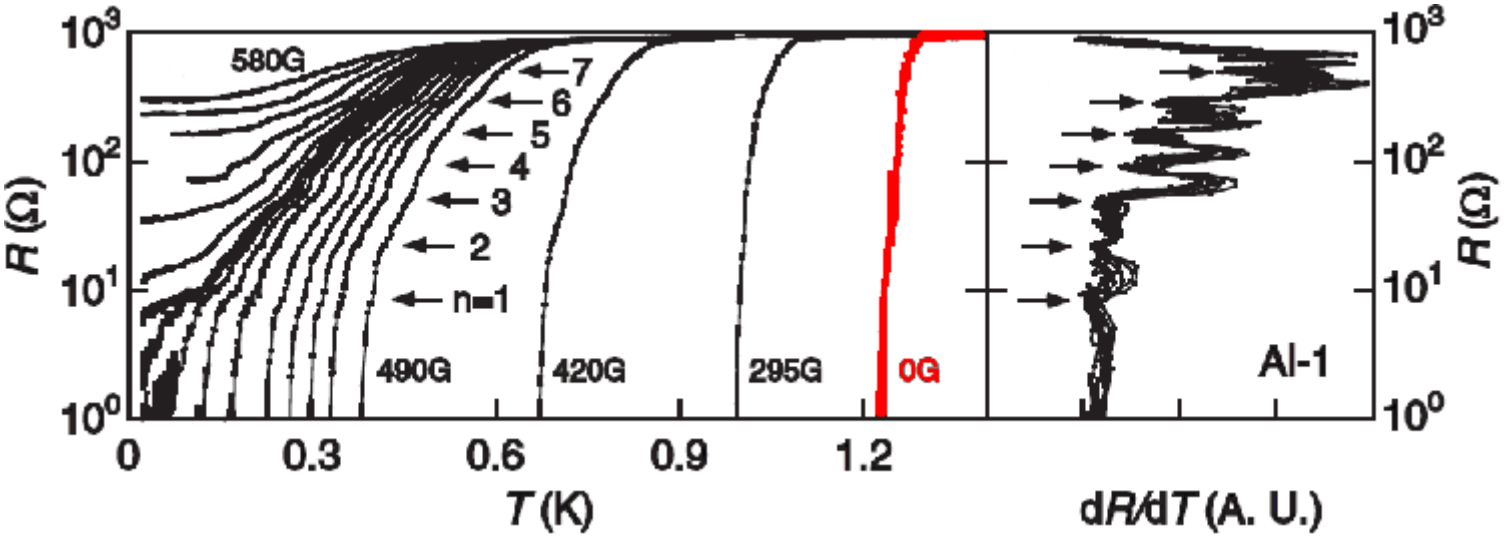
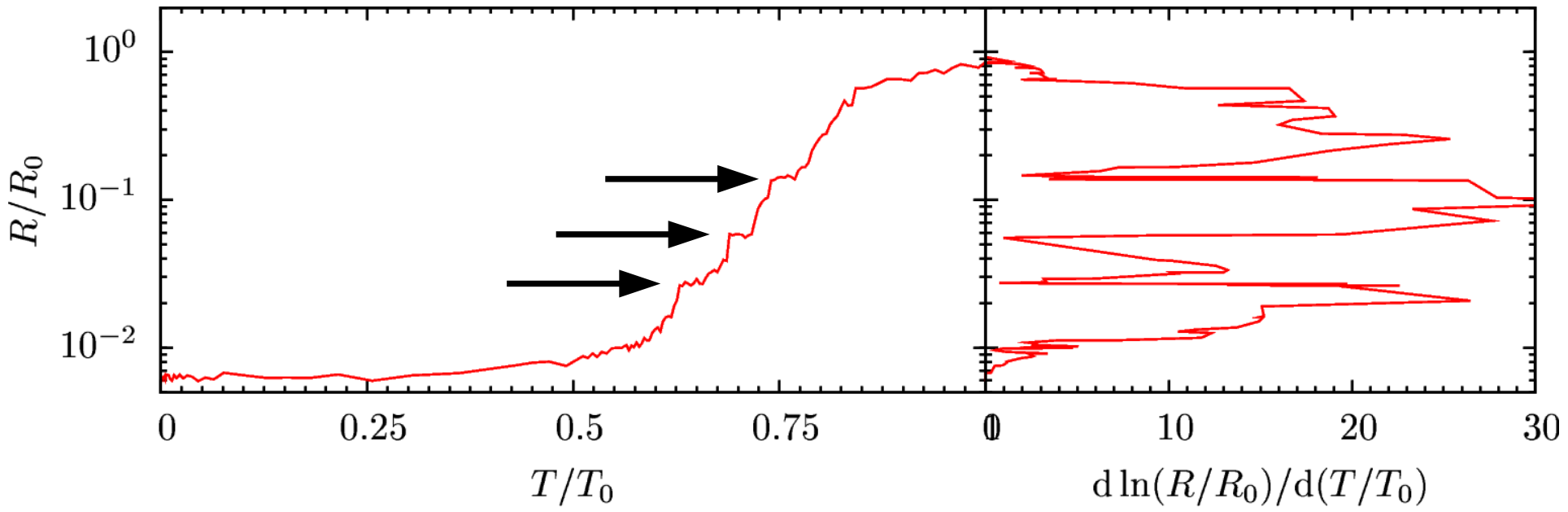


Evidence of phase reconstruction

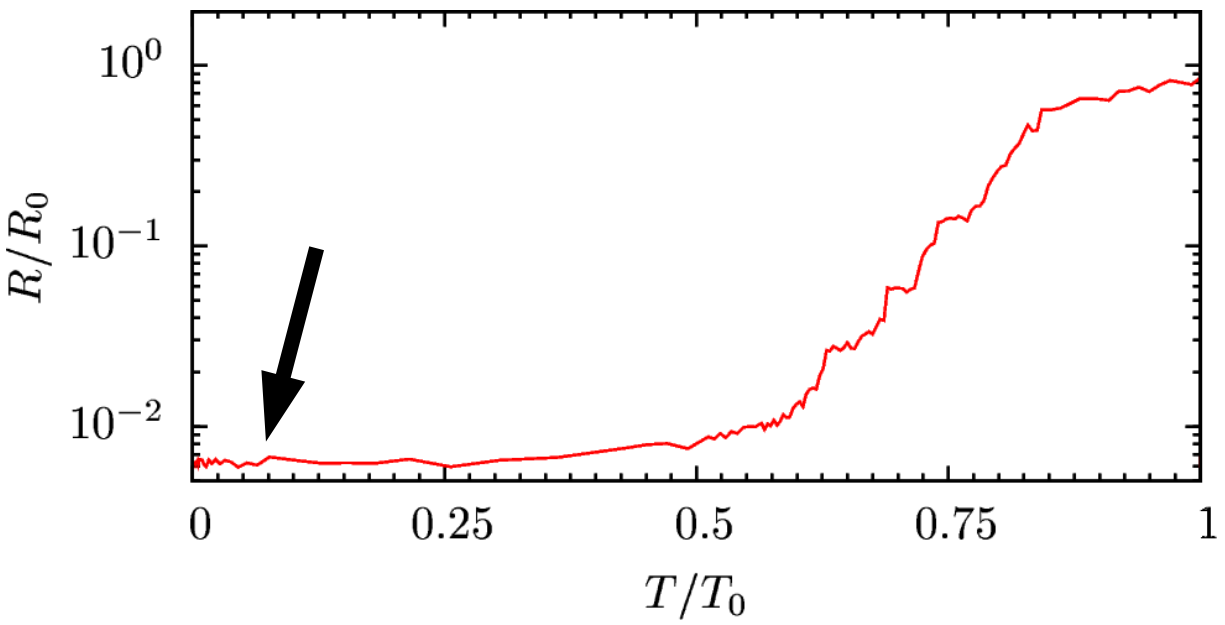
- Experiment:

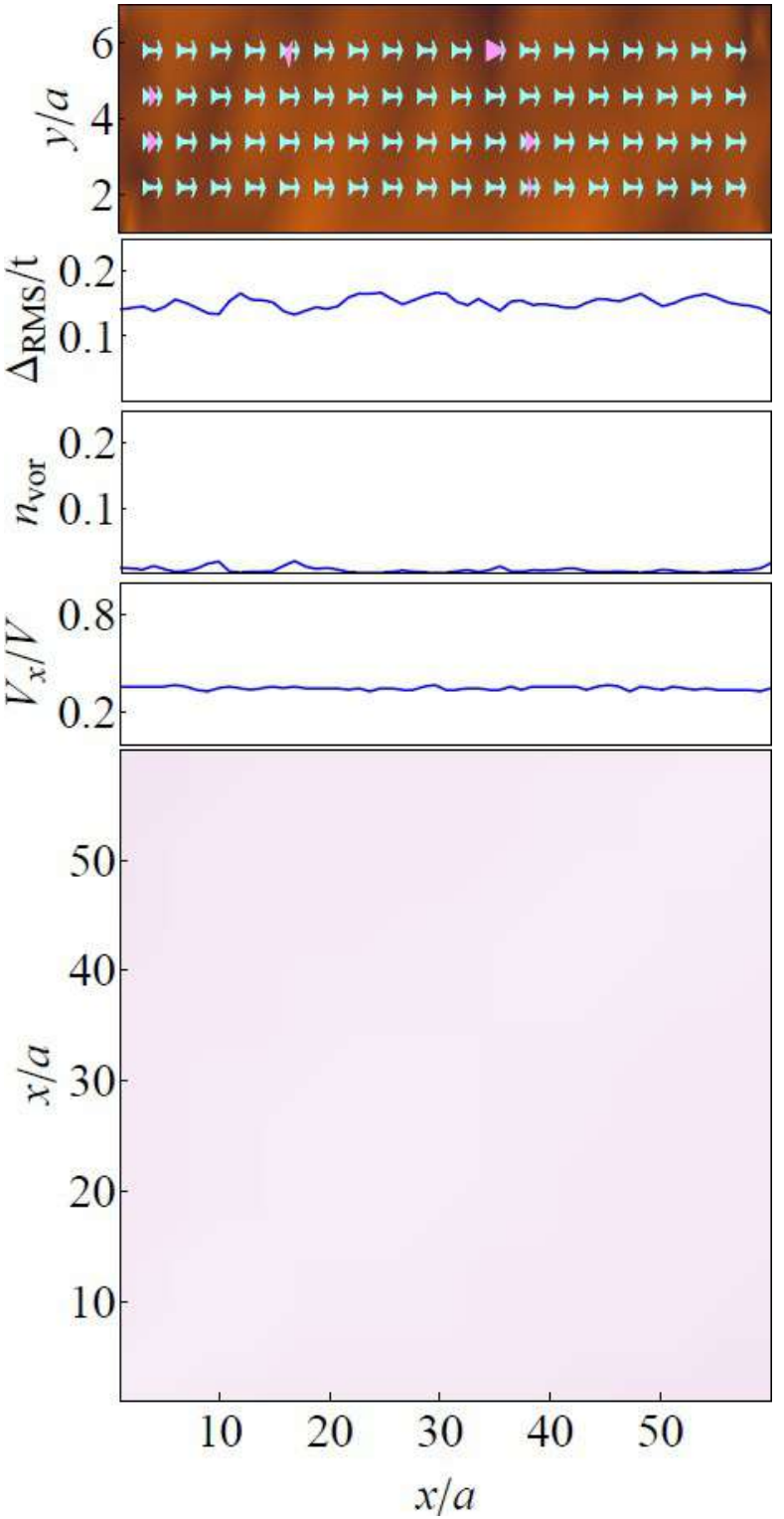


- Theory:



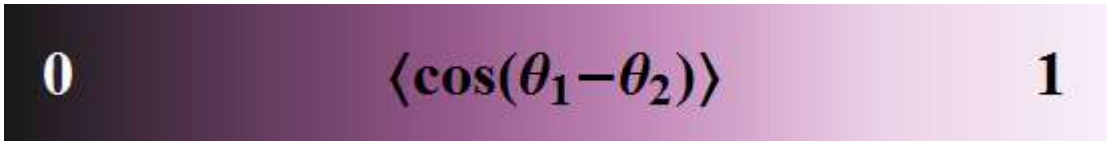
Completely superconducting



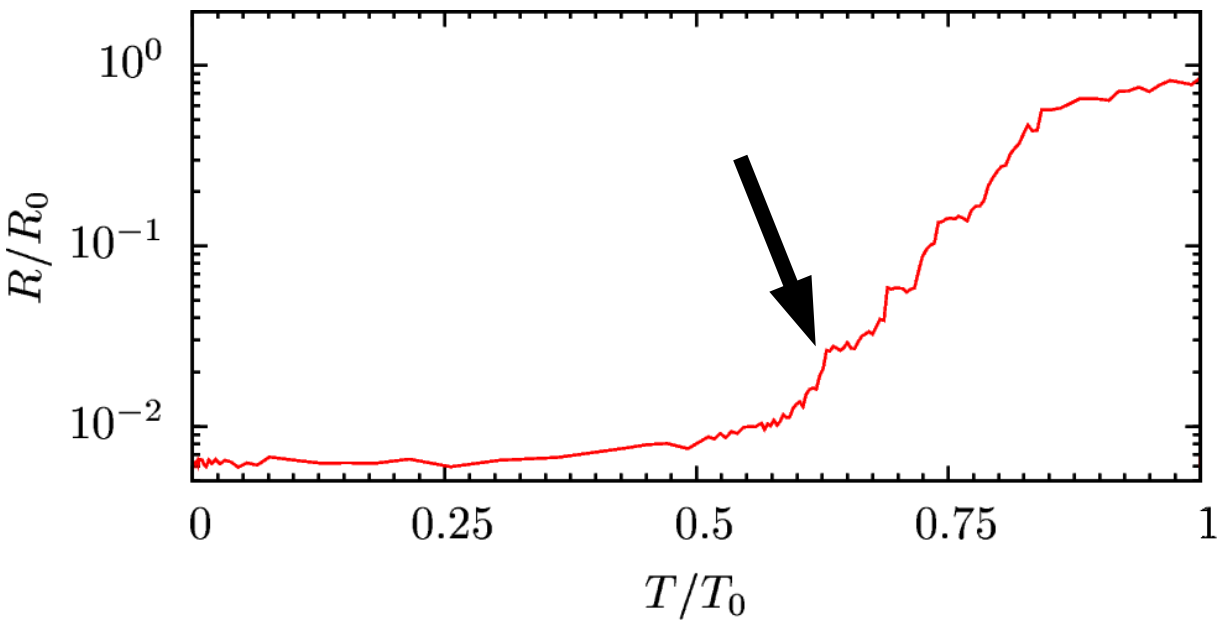


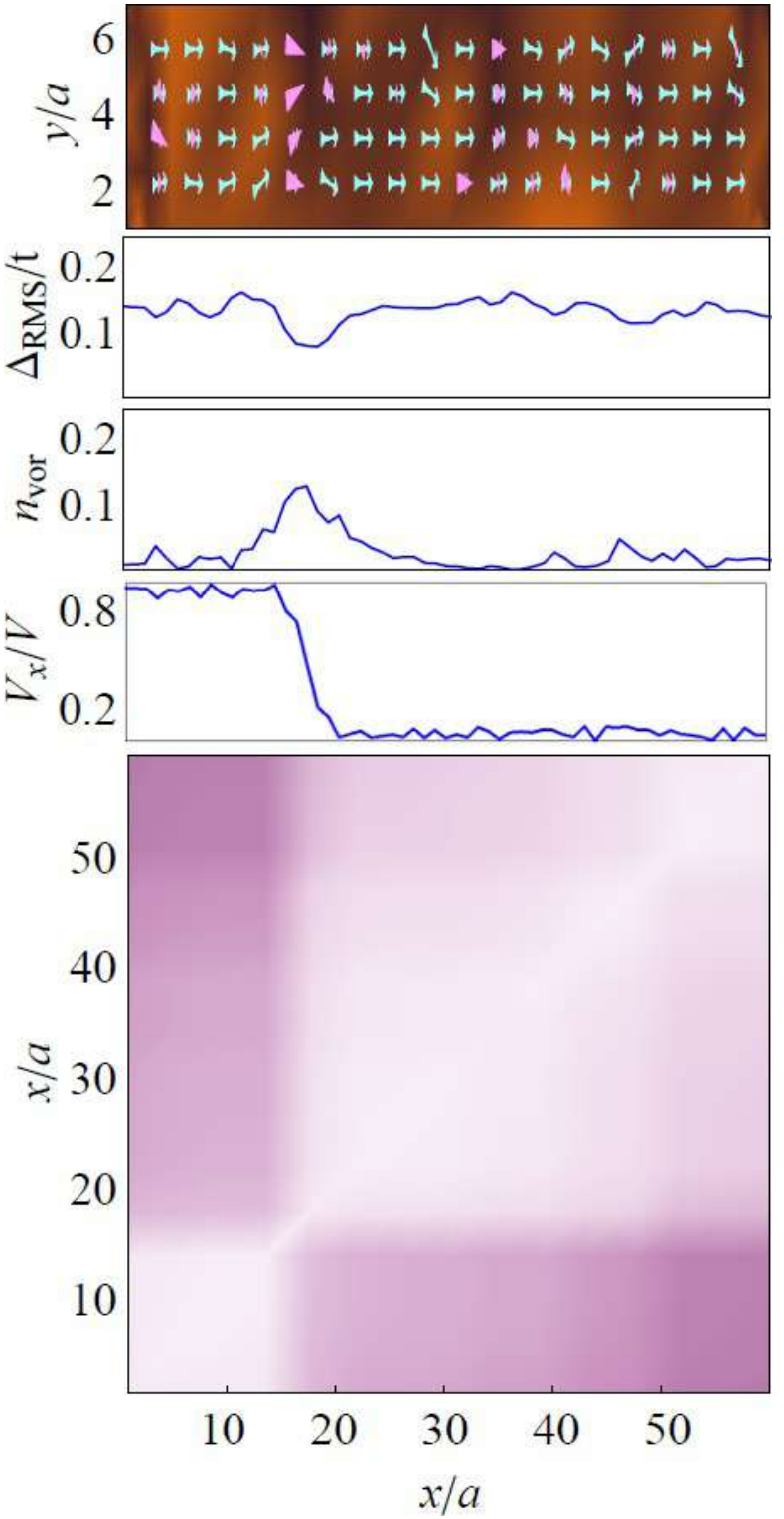
Superconducting current

Normal current



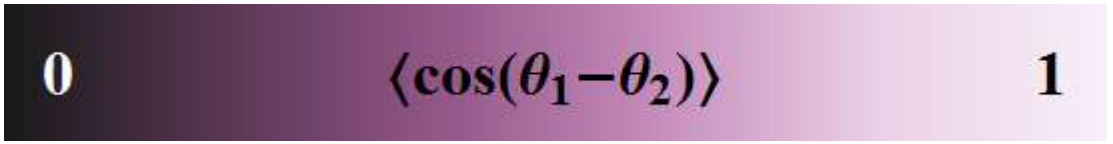
Two superconducting regions



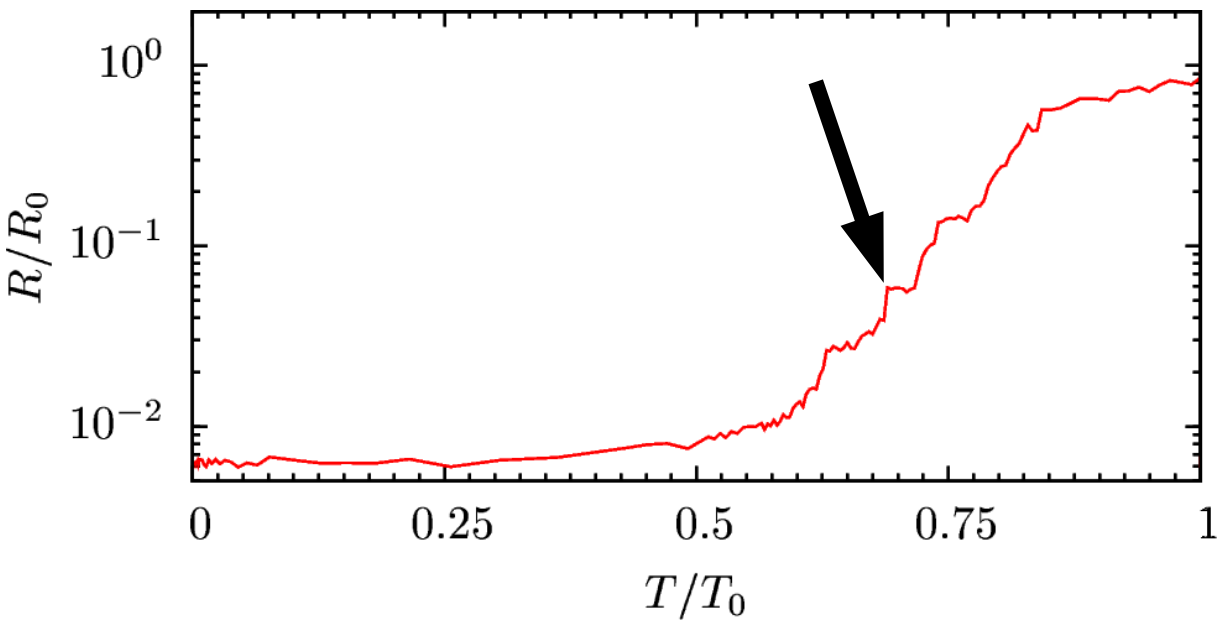


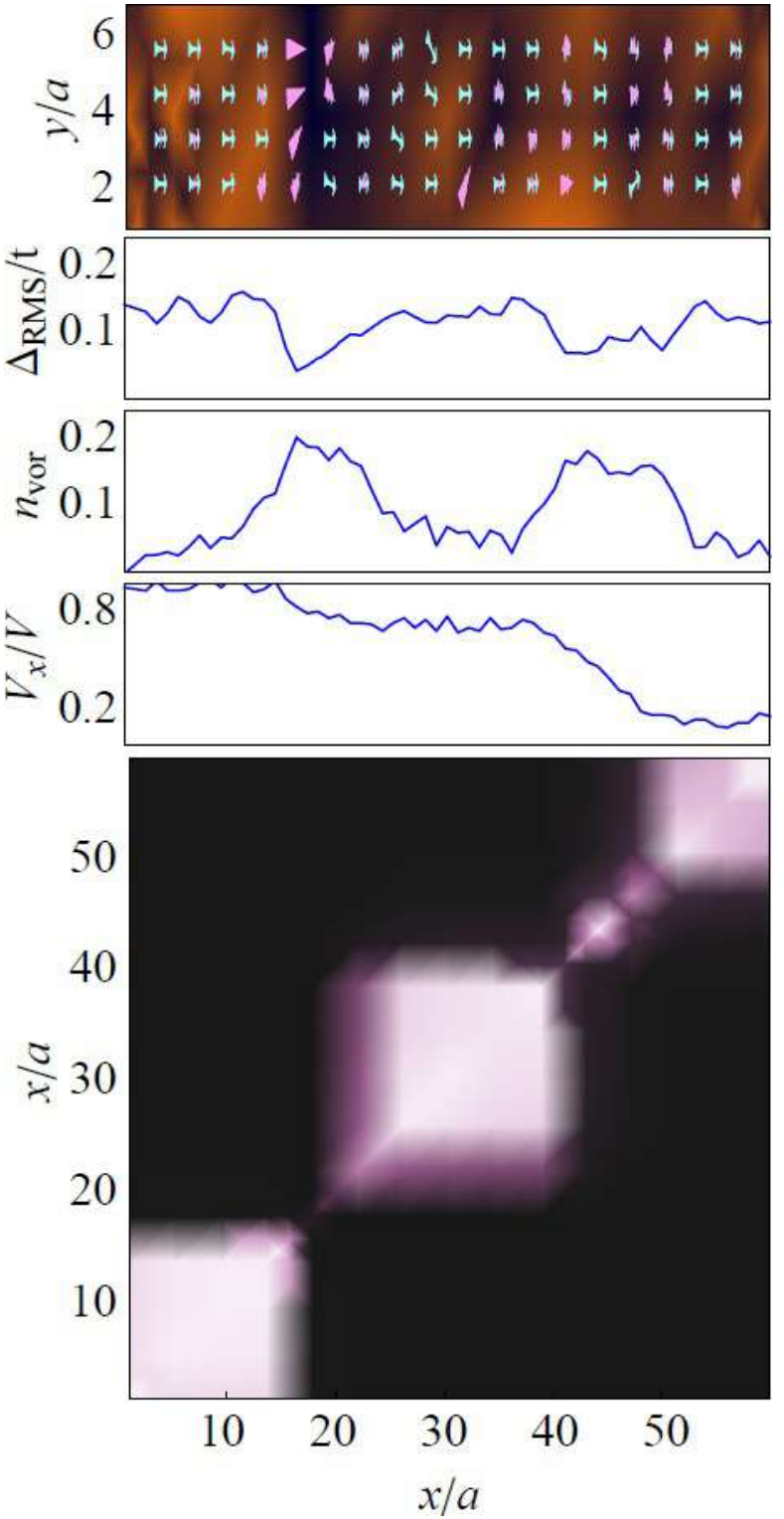
Superconducting current

Normal current



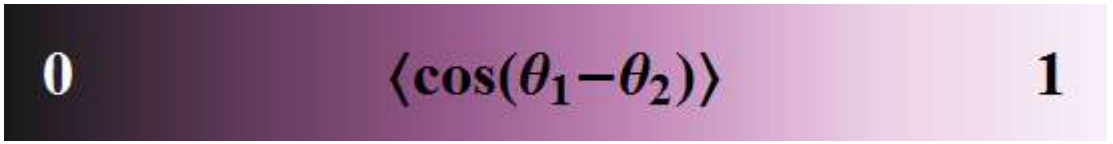
Three superconducting regions



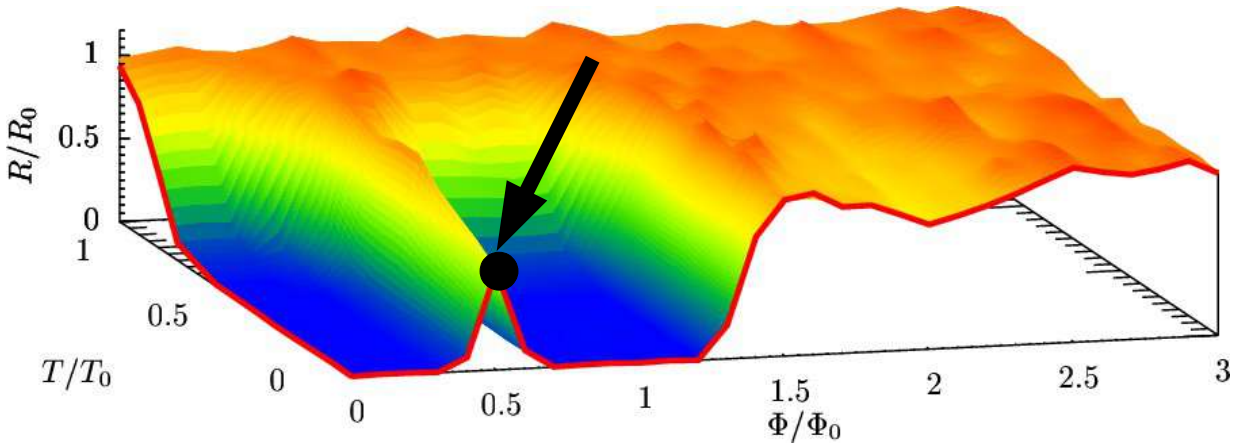


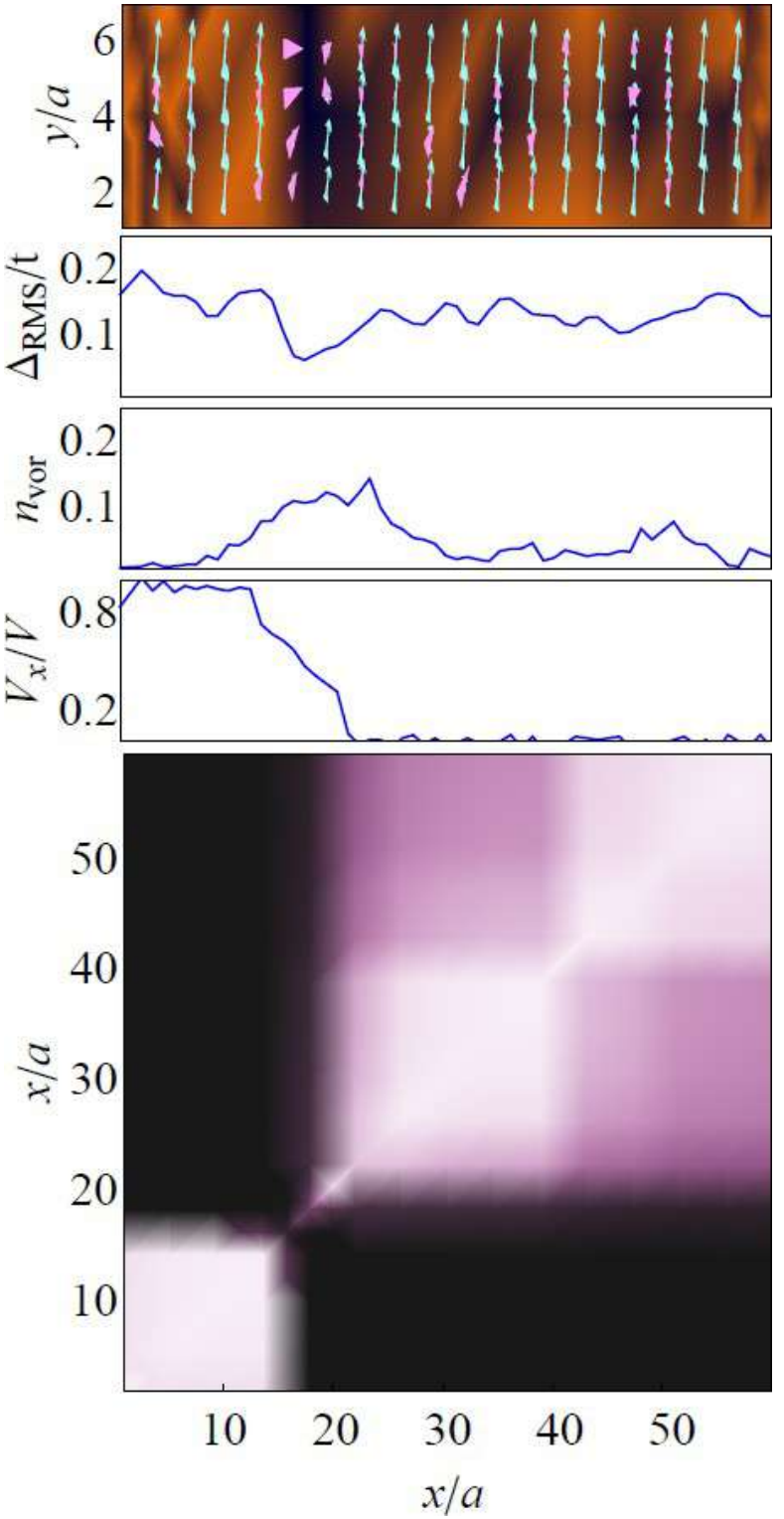
Superconducting current

Normal current



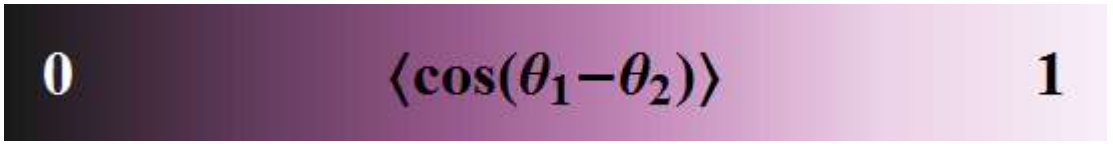
Half flux quantum normal state





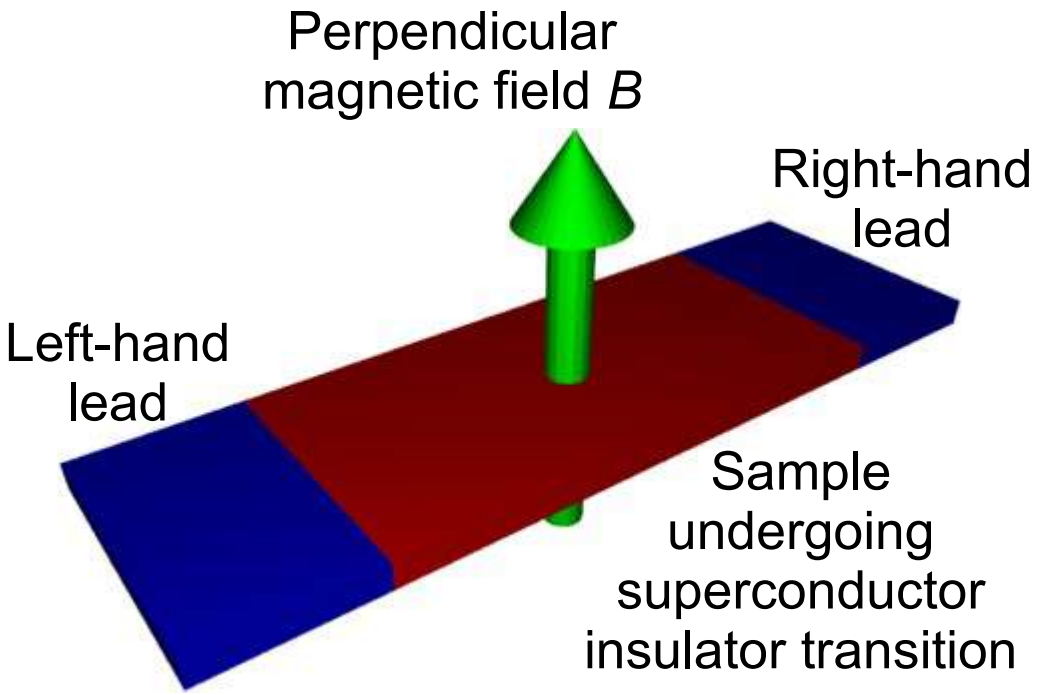
Superconducting current

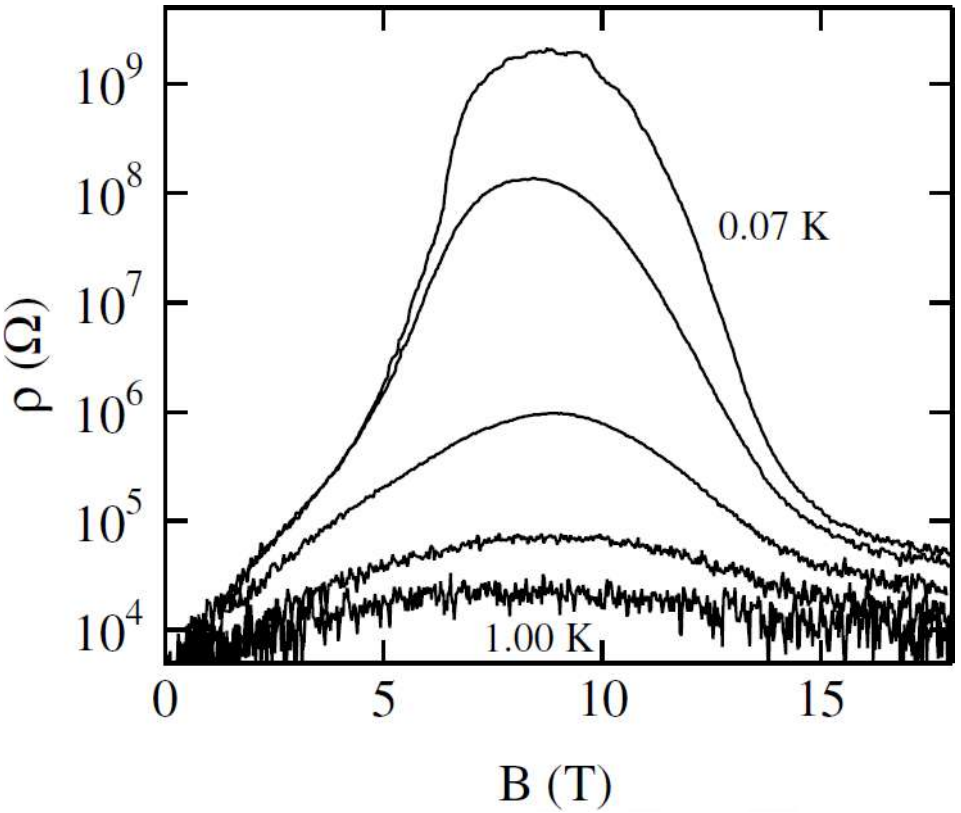
Normal current



Magnetoresistance peak

- Study superconductor-insulator transition in dirty sample with perpendicular magnetic field





Magnetoresistance peak

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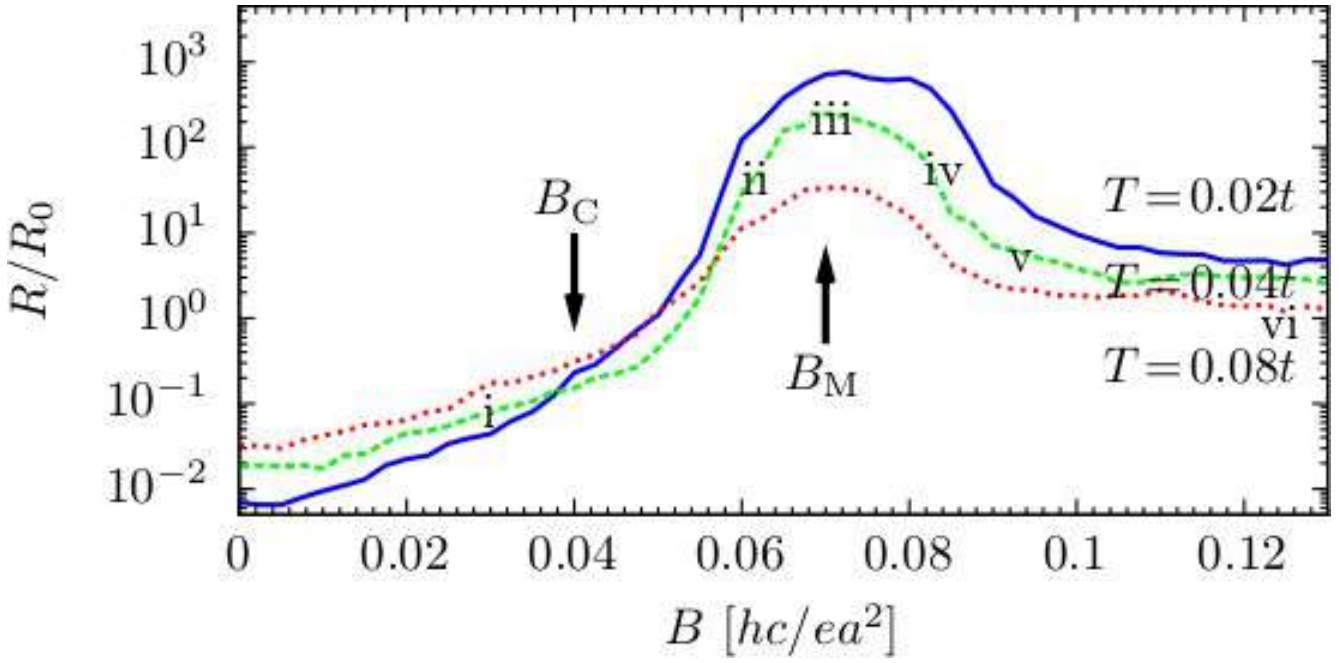
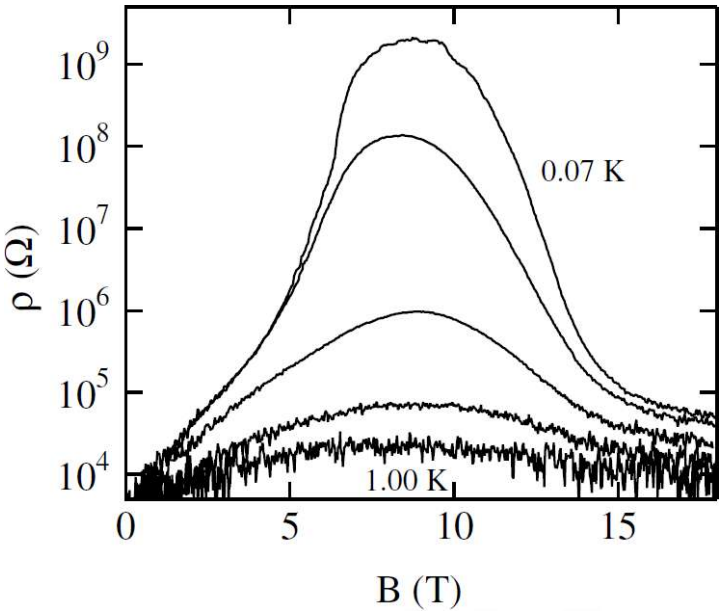
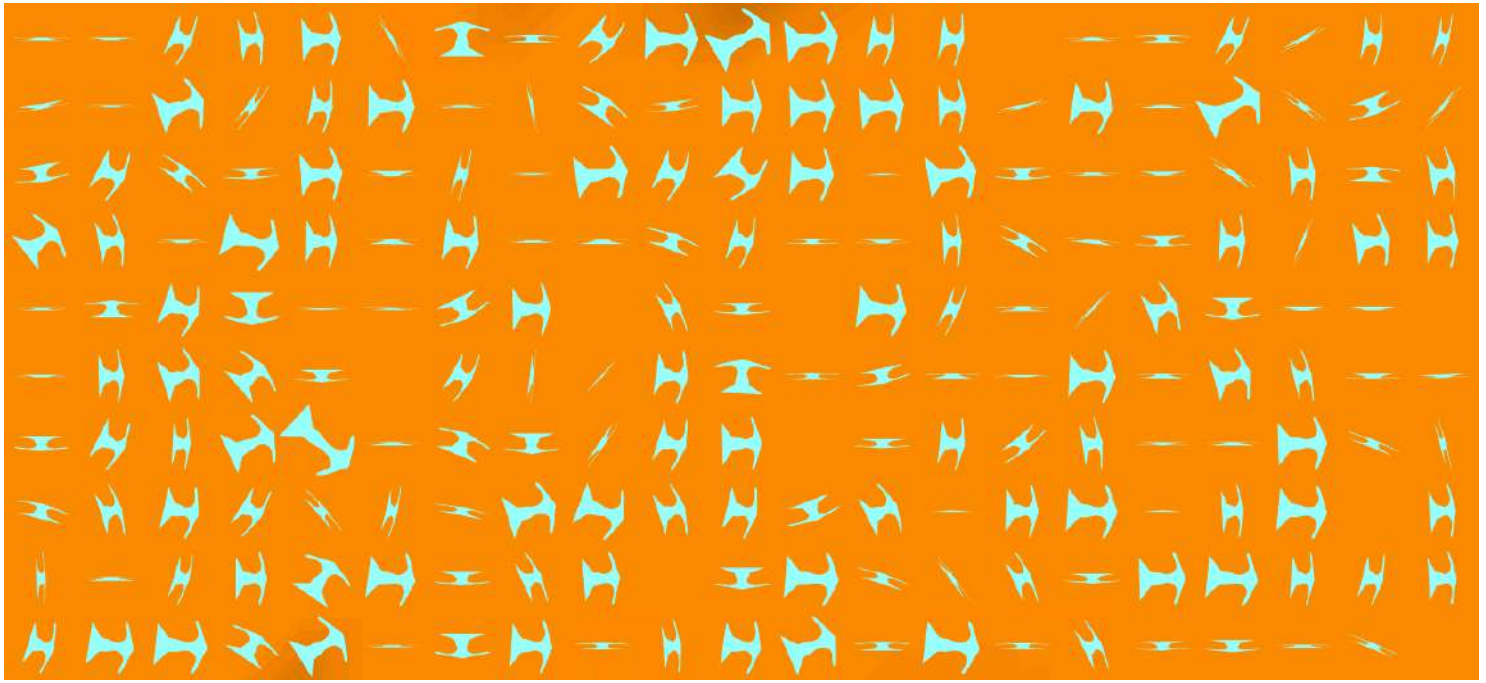
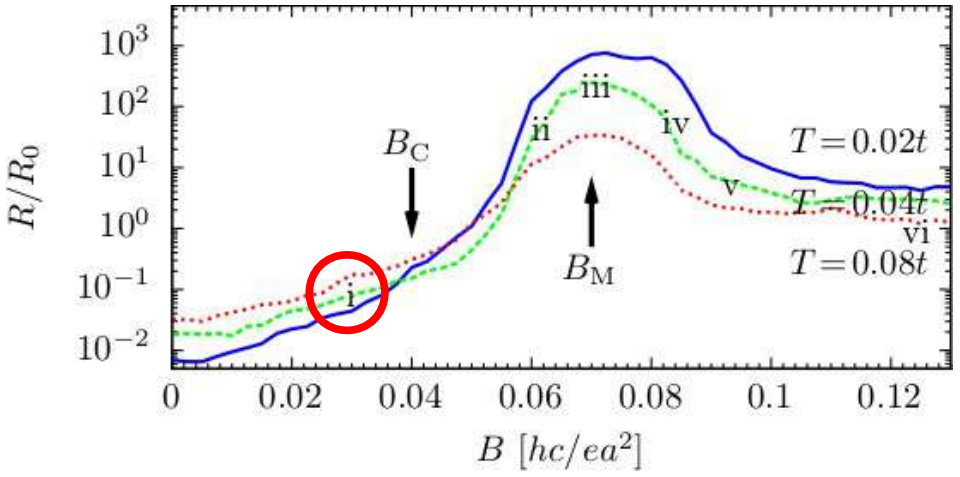




Clues: current maps

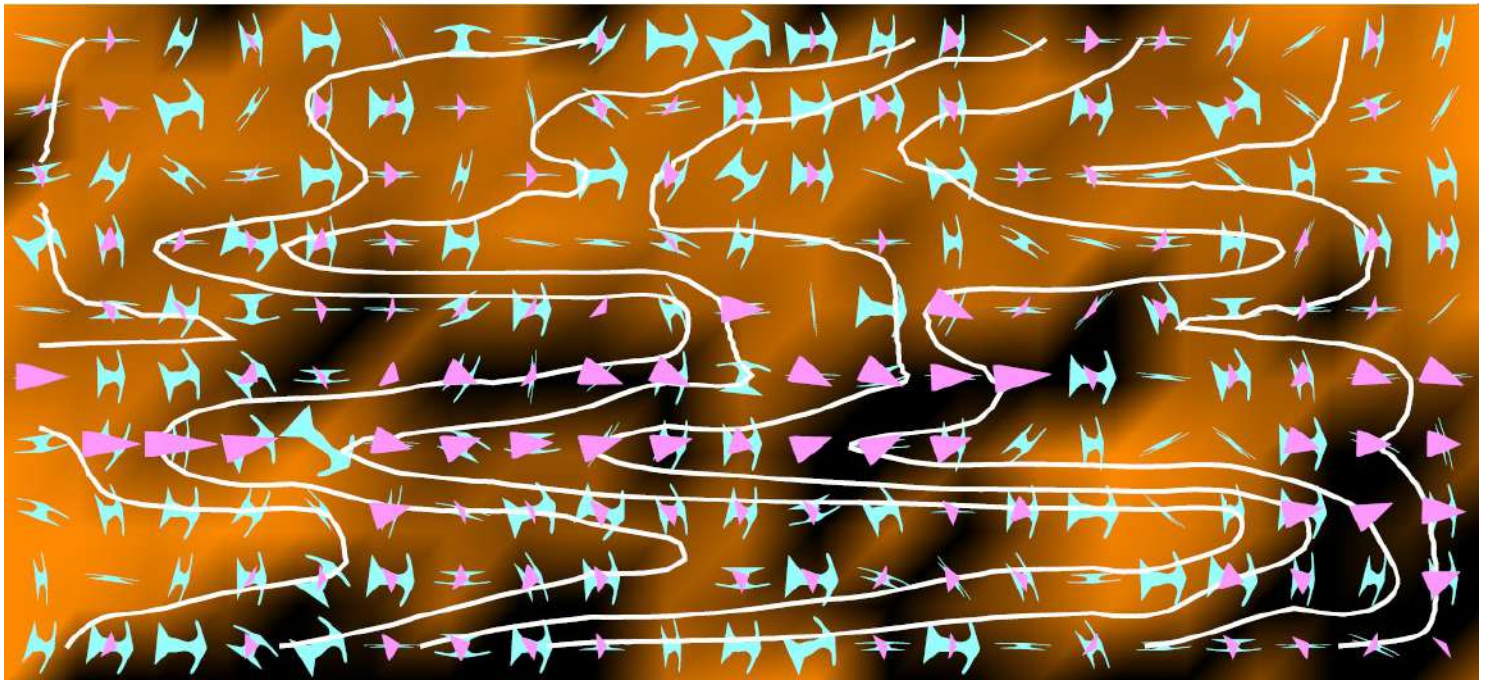


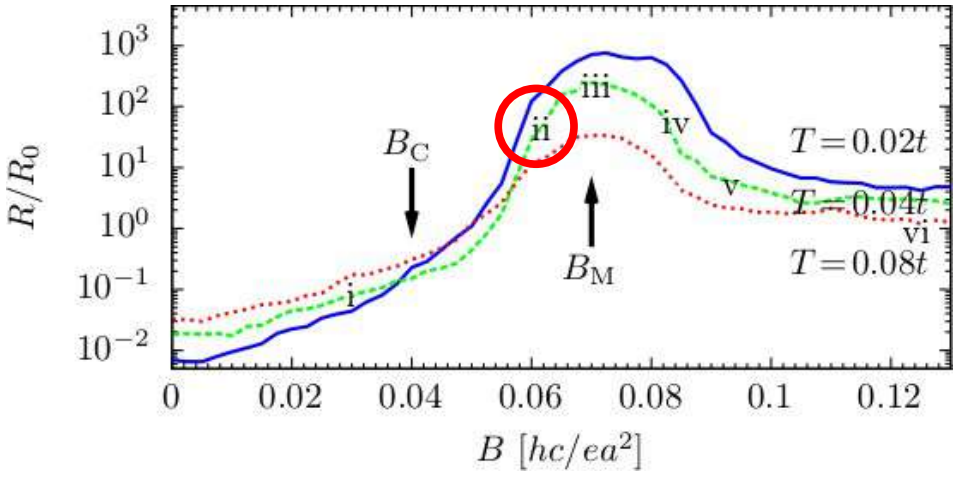


Superconducting current

Normal current

Clues: current maps

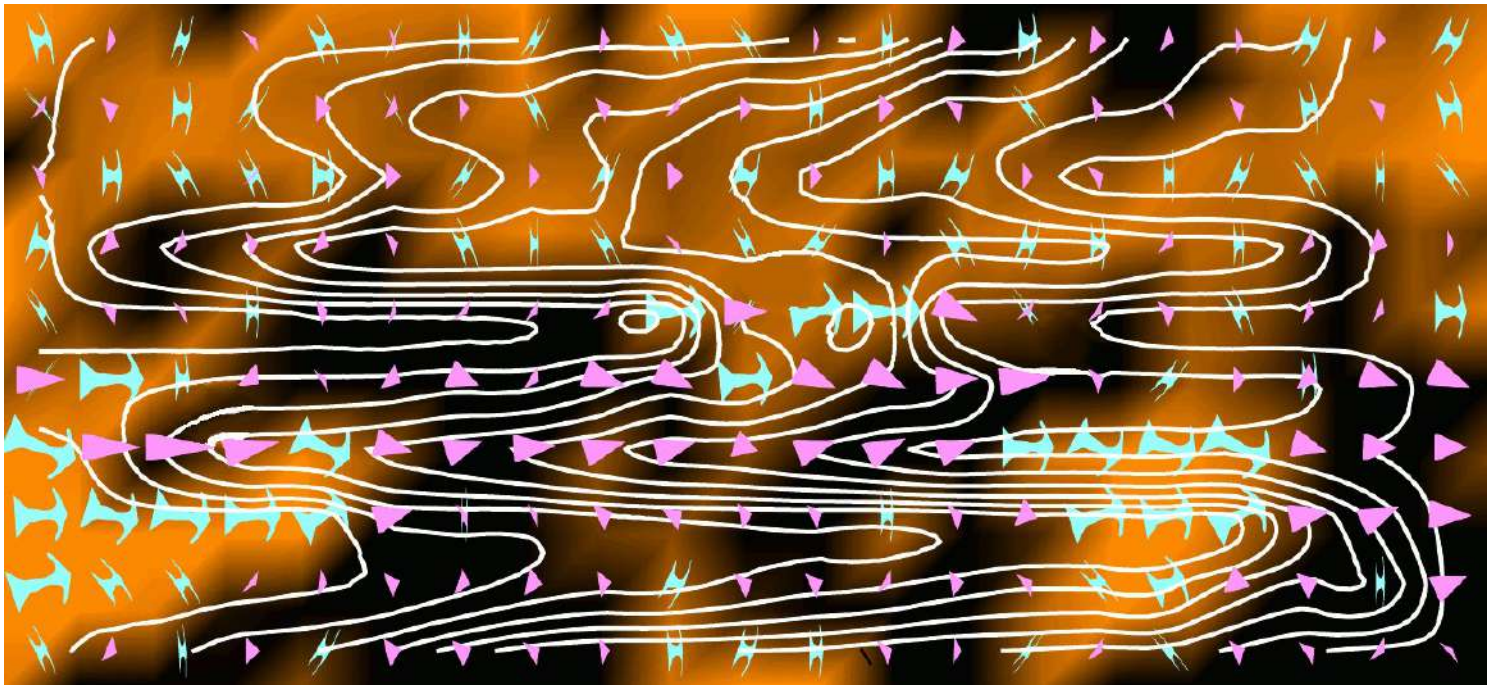


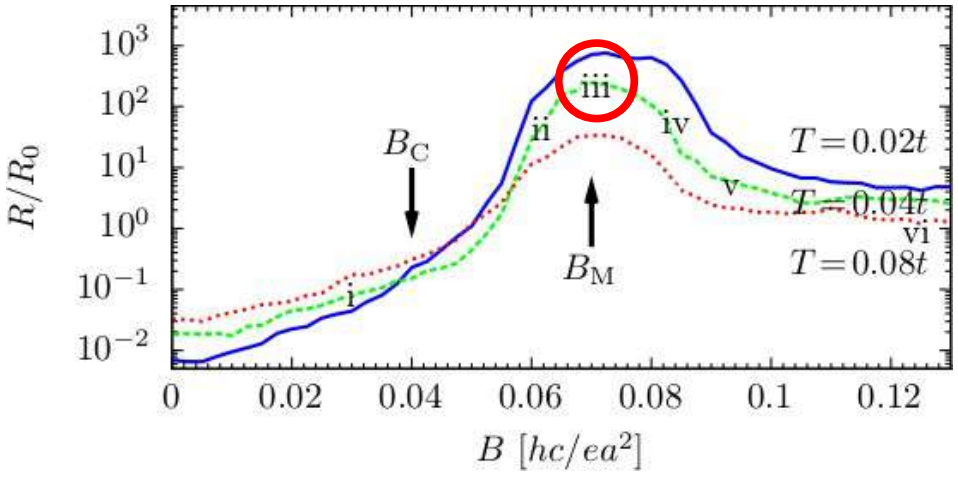


Superconducting current

Normal current

Clues: current maps

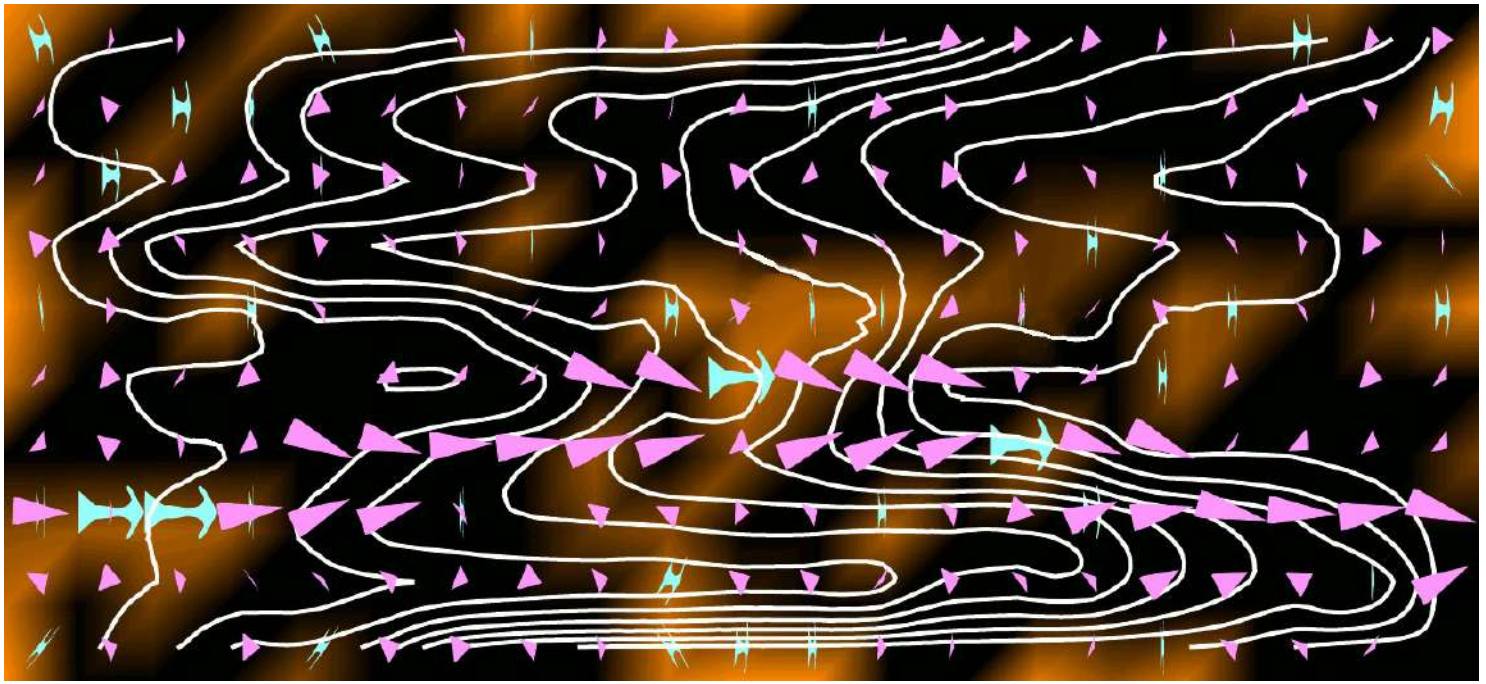


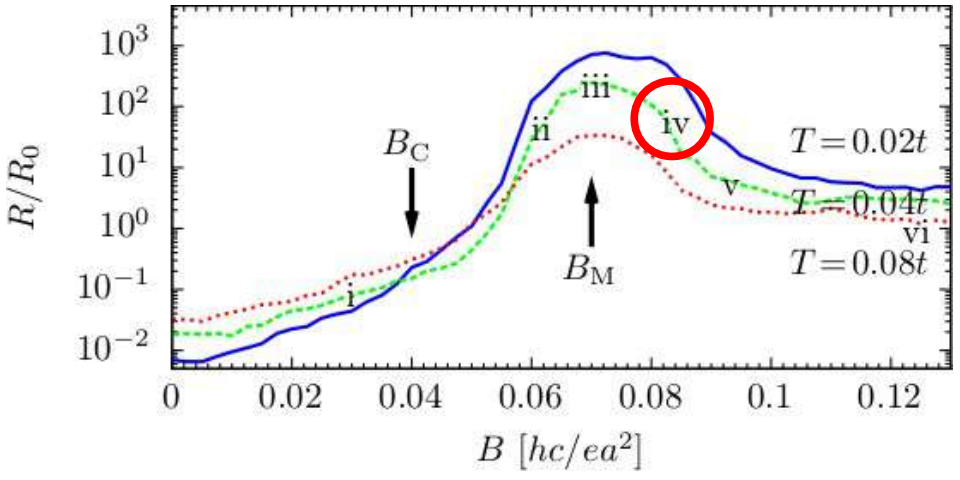


Superconducting current

Normal current

Clues: current maps





Superconducting current

Normal current

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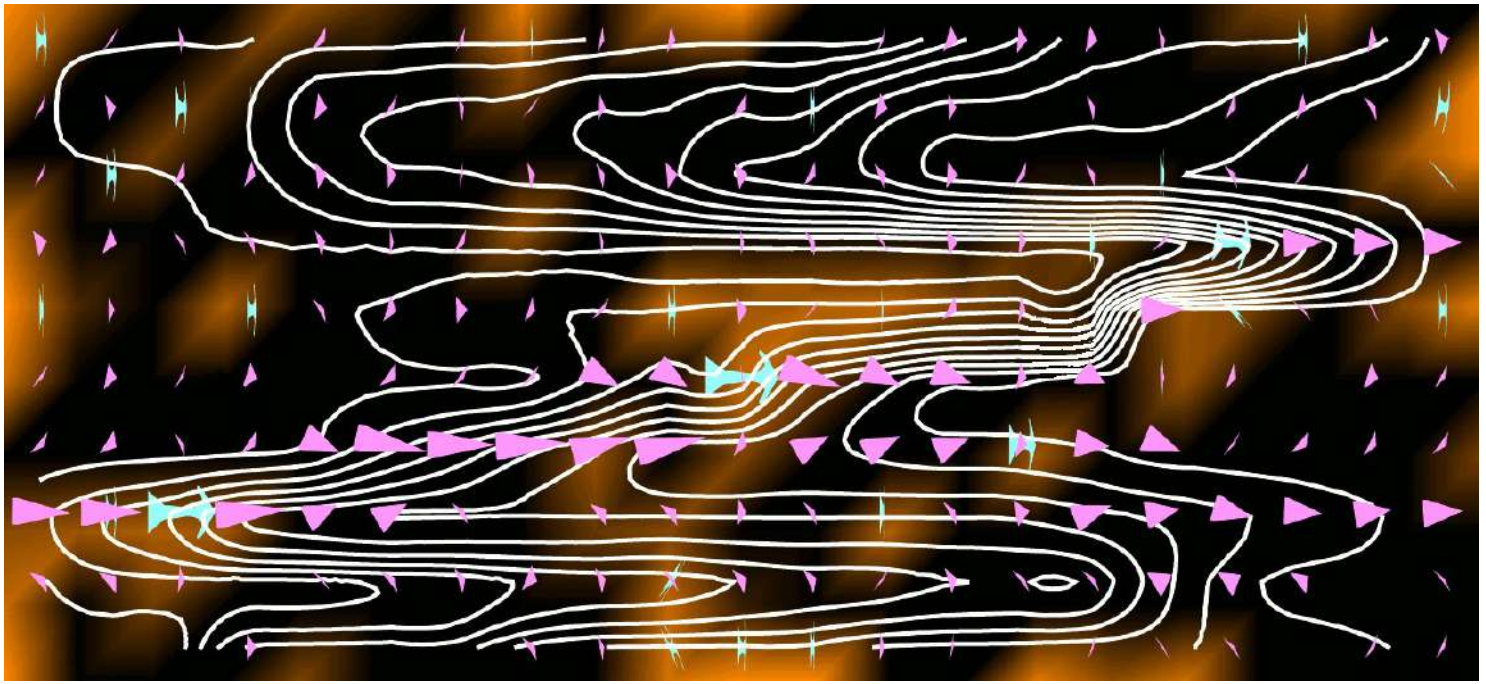
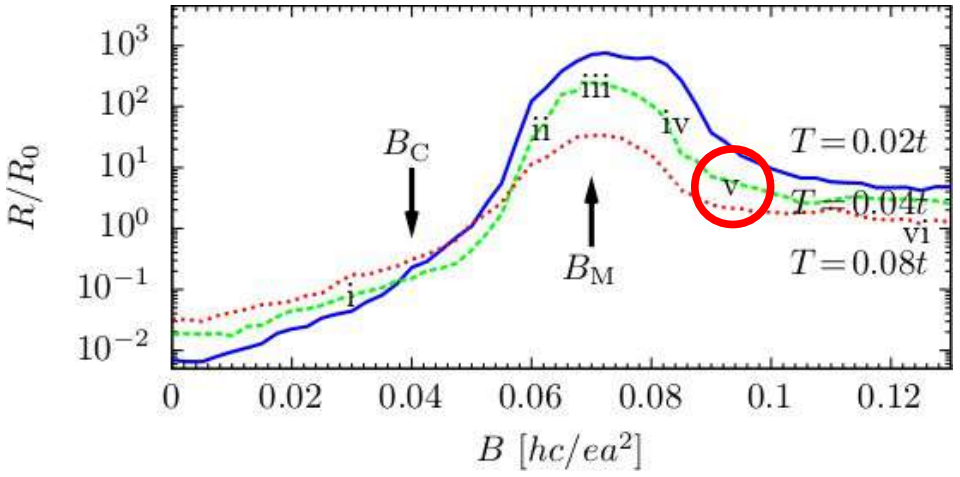
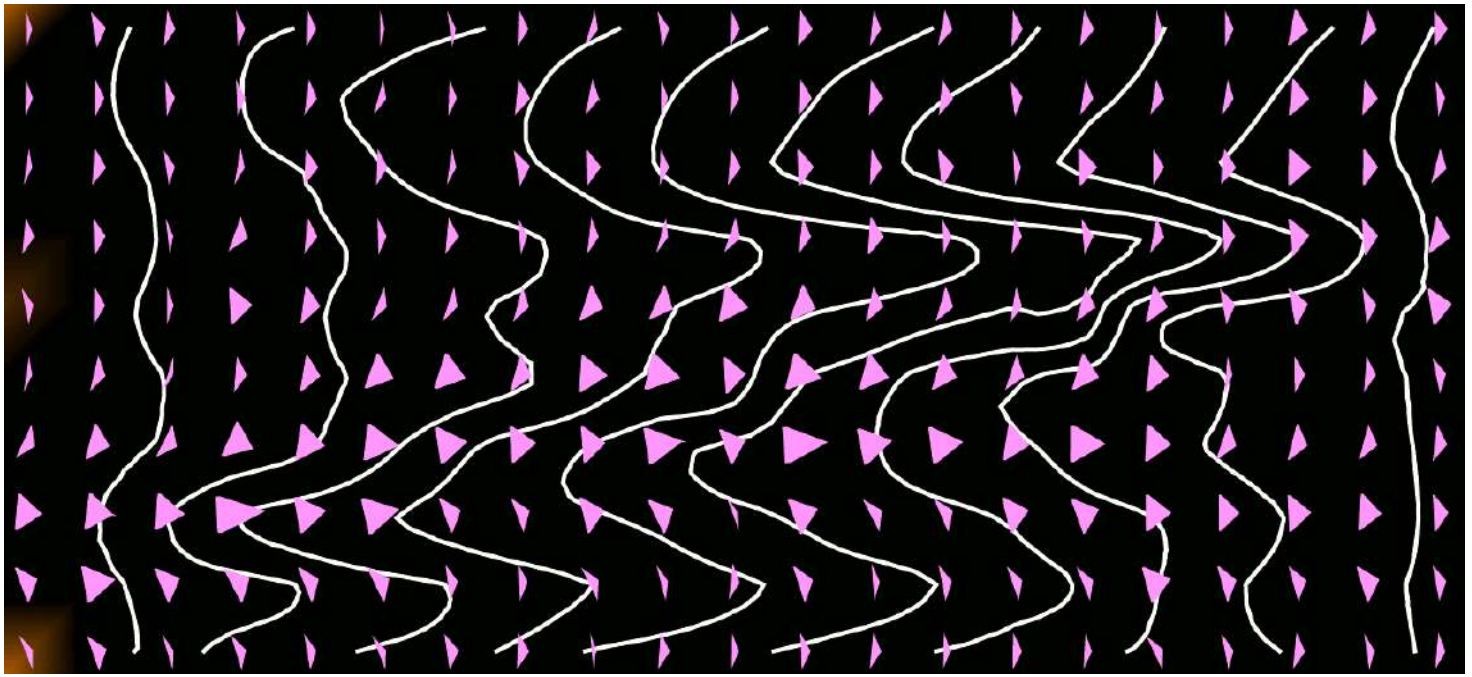


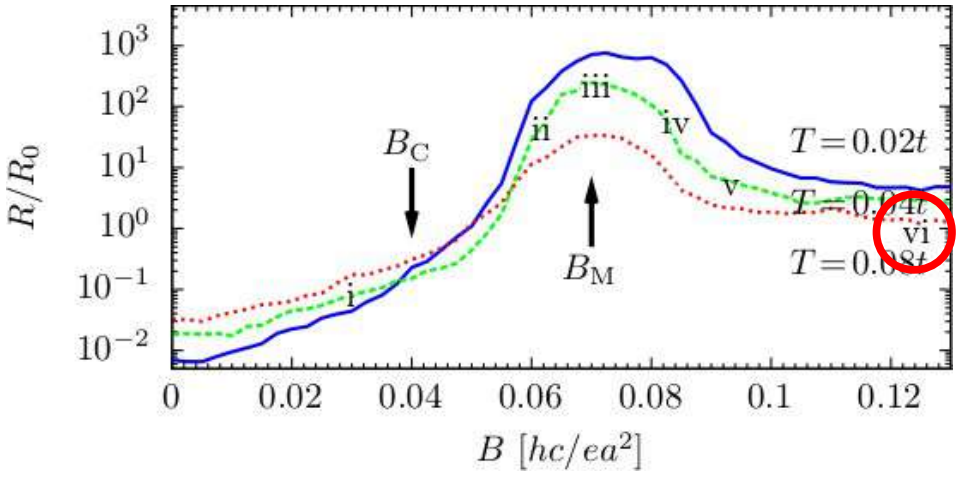


Superconducting current

Normal current

Clues: current maps



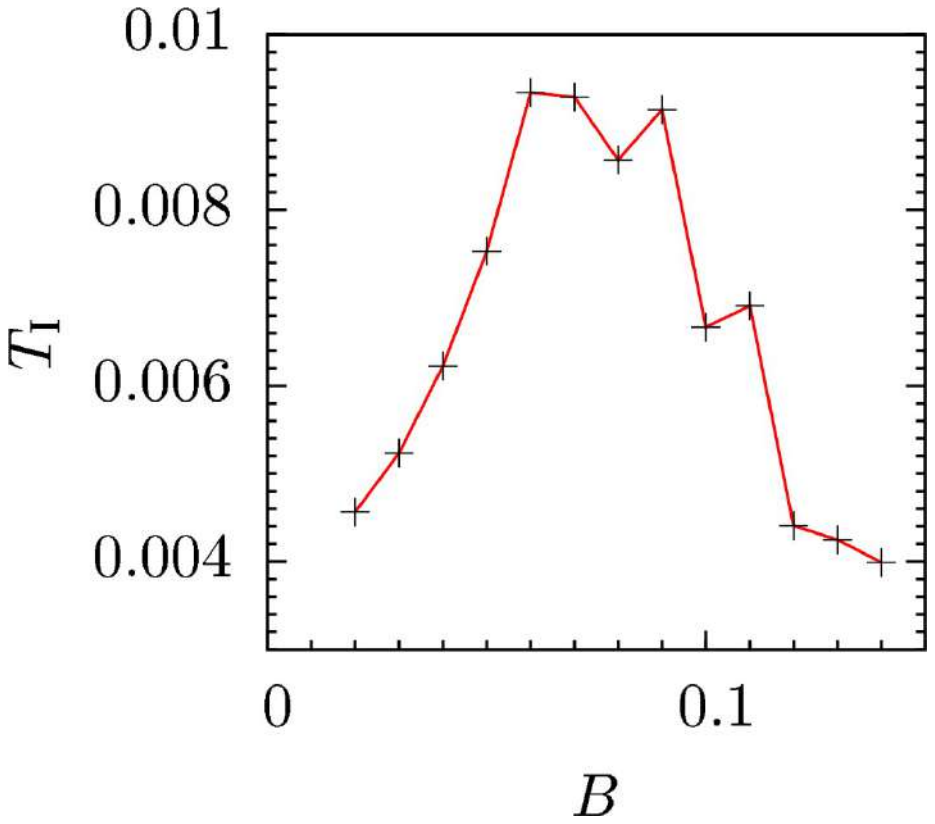


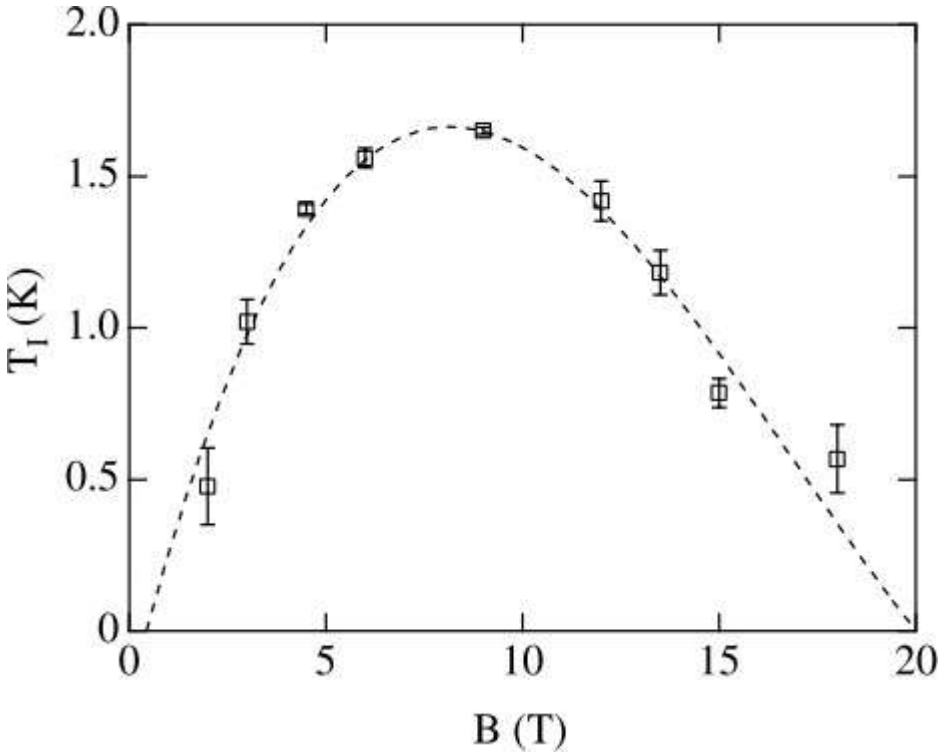
Superconducting current

Normal current

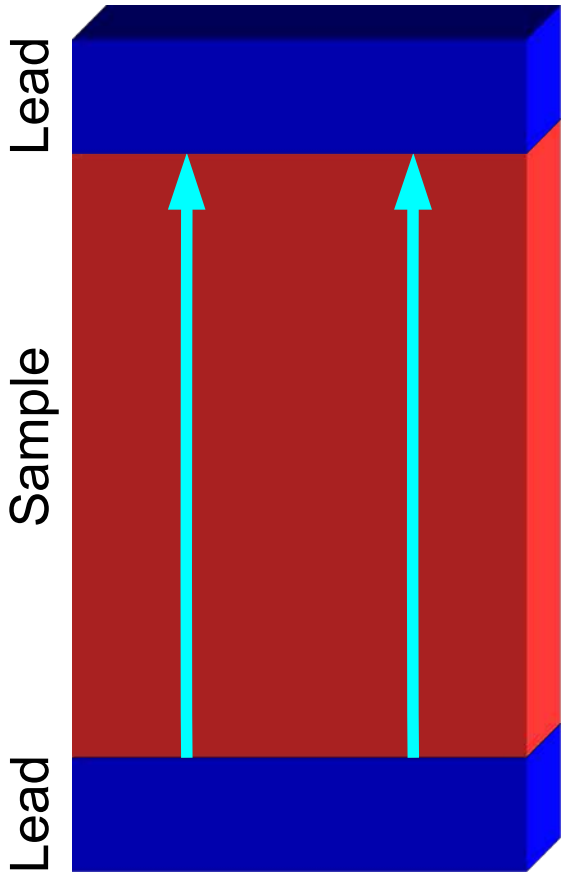
Clues: activated transport

- Activated transport $\rho = \rho_0 e^{T_1/T}$

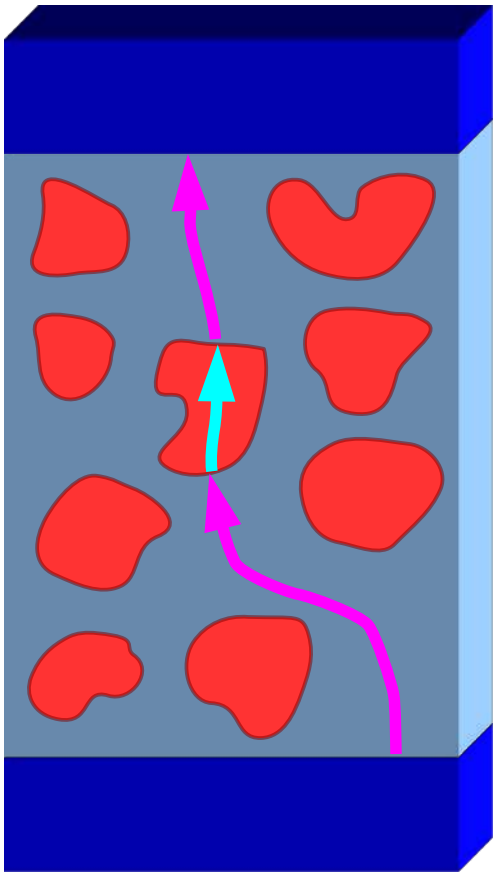




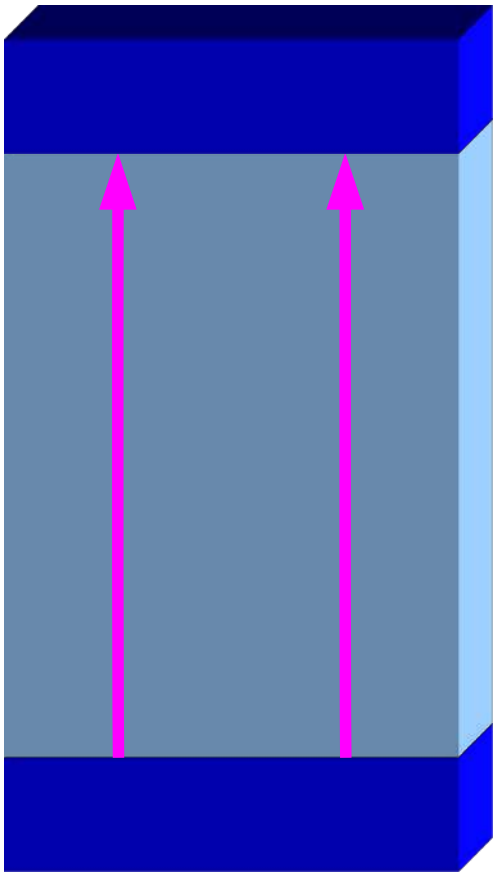
Proposed mechanism



Sample entirely superconducting

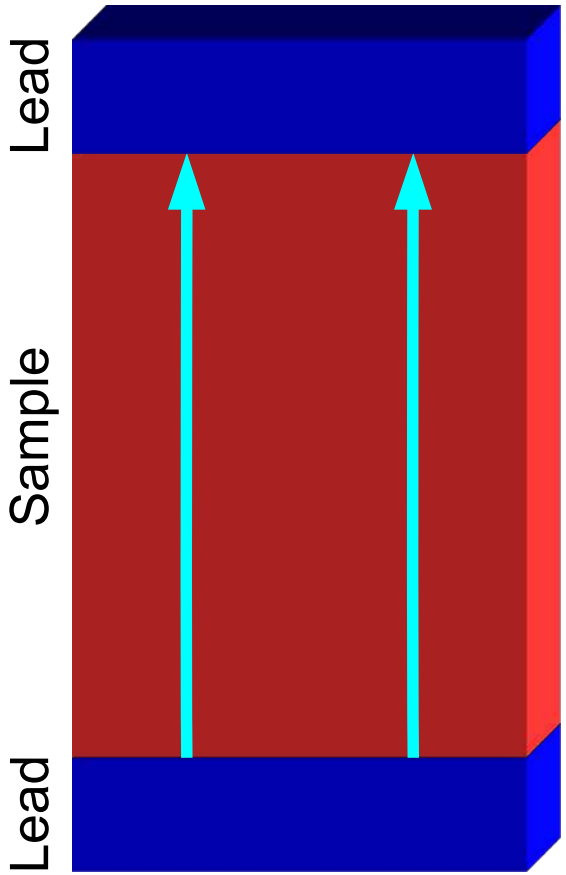


Superconducting puddles have a charging energy and a tunneling barrier

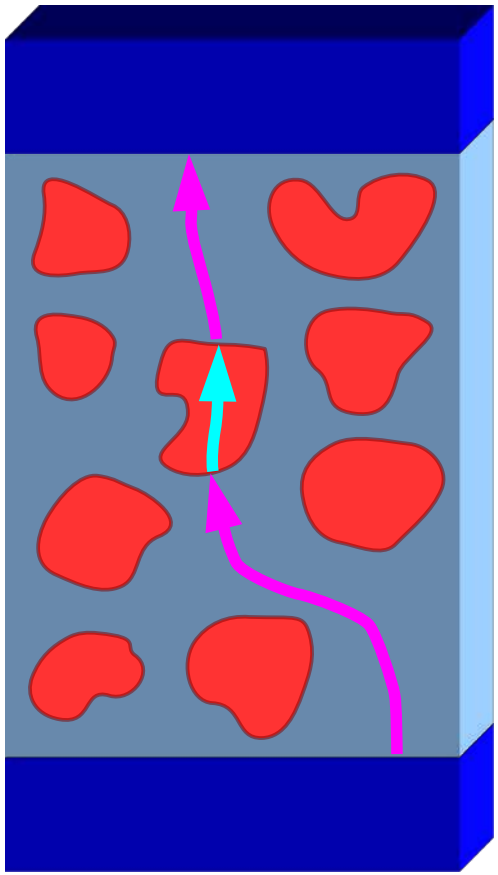


Sample entirely normal

Proposed mechanism



Sample entirely superconducting



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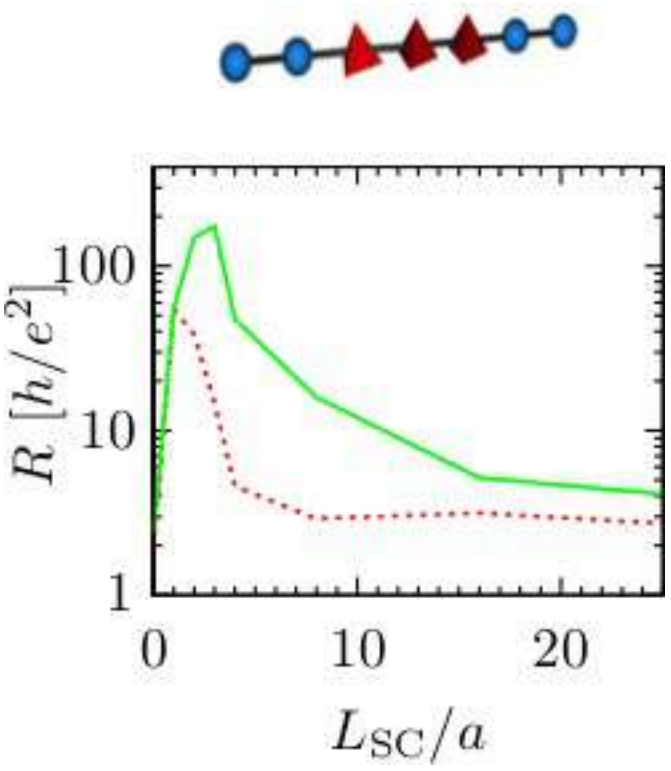
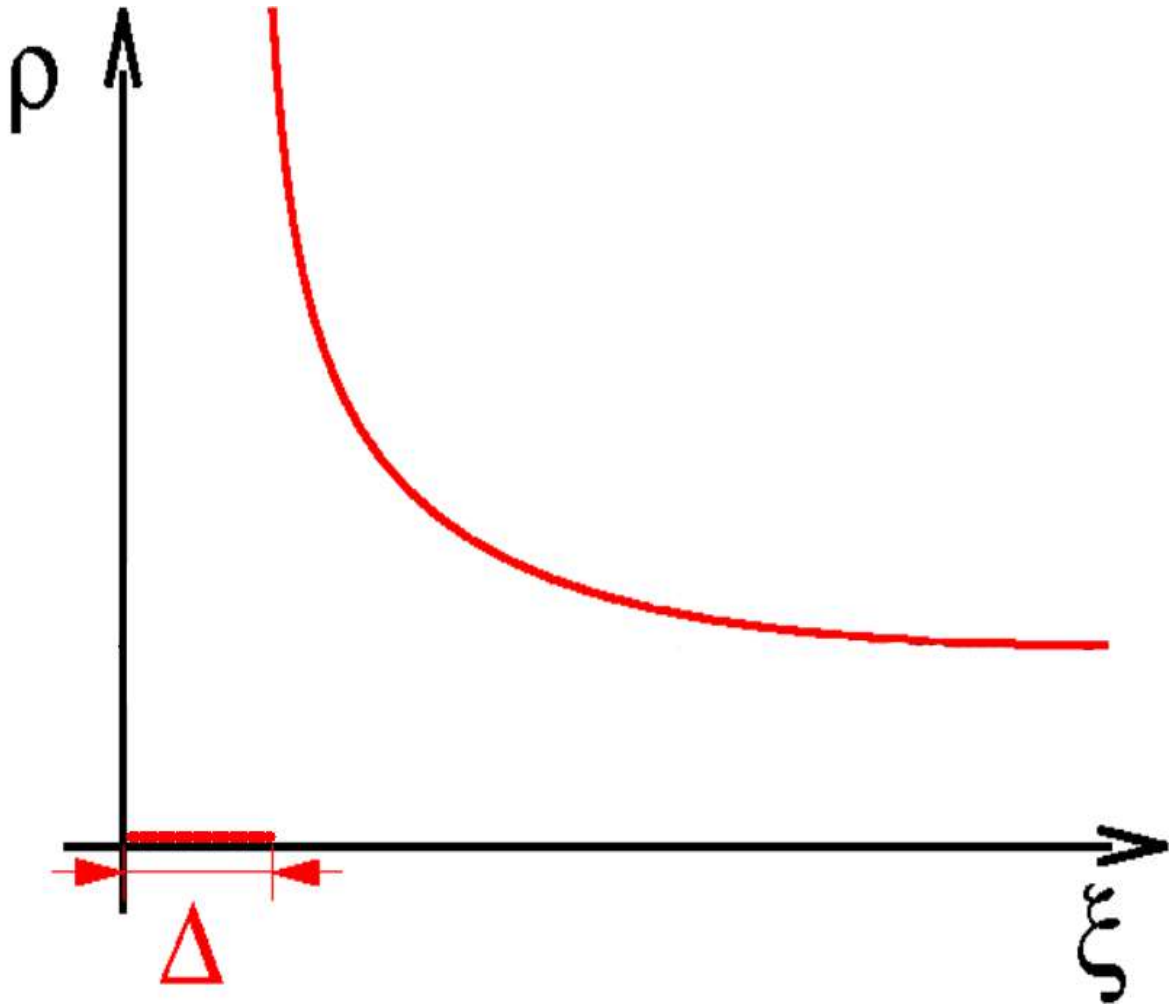


Summary & future prospects

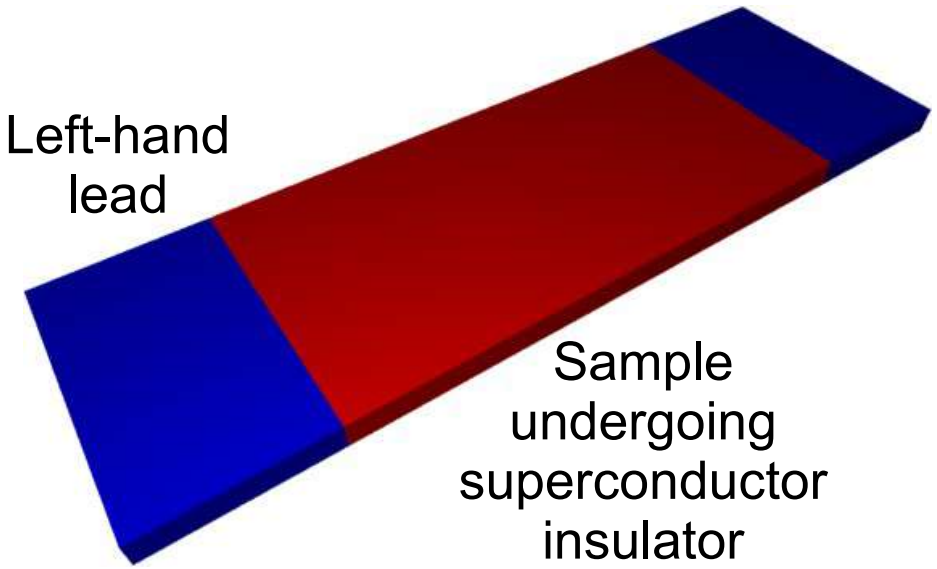
- Developed new formalism that includes thermal phase fluctuations to calculate and probe transport in superconductors
- New numerical techniques permit access to large systems
- Tested formalism against a series of well established results
- Shown that superconductor-insulator transition in small diameter cylinders is driven by phase fluctuations
- Shown that magnetoresistance peak could be driven by condensation of superconducting puddles
- Flexibility allows us to study wide range of unexplained effects

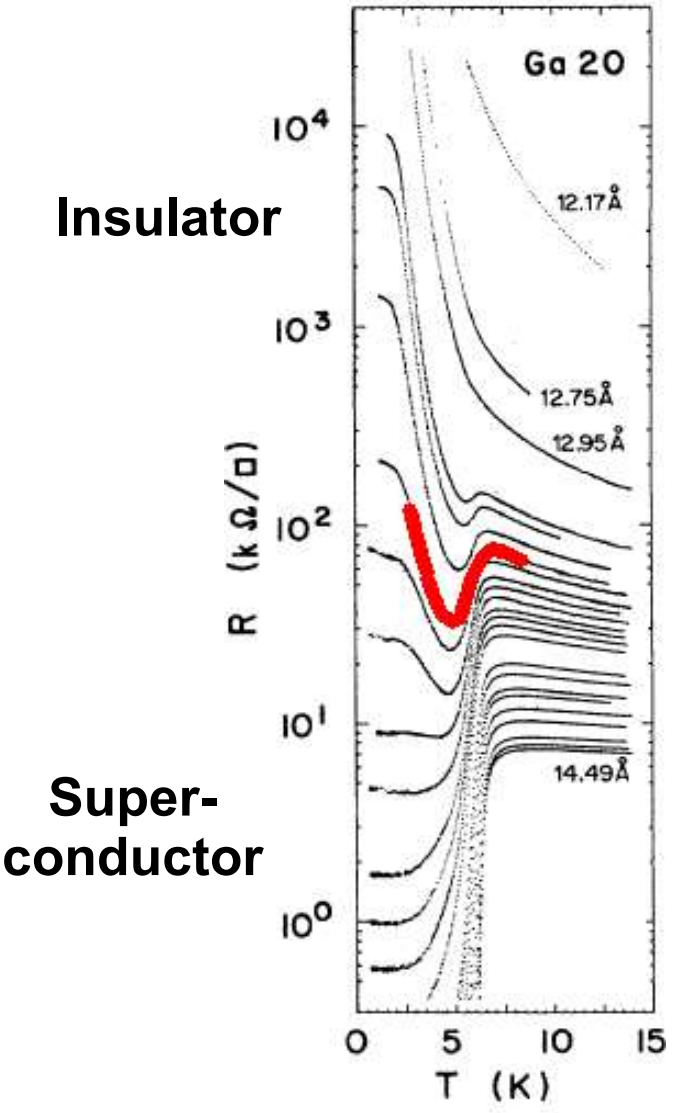
BCS superconductivity



Superconductor-insulator transition

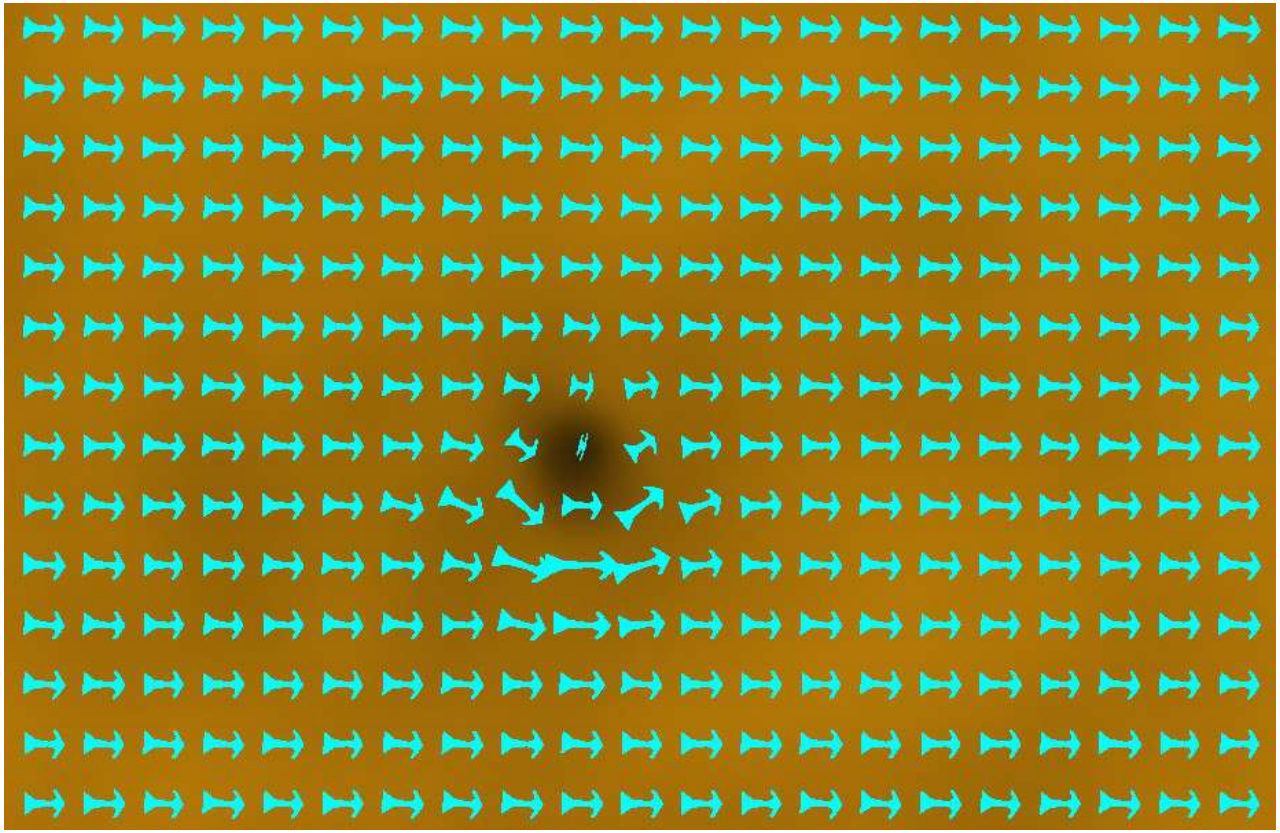
- Non-monotonic R-T curves [Jaeger 89, Baturina 07]





Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear *IV* characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



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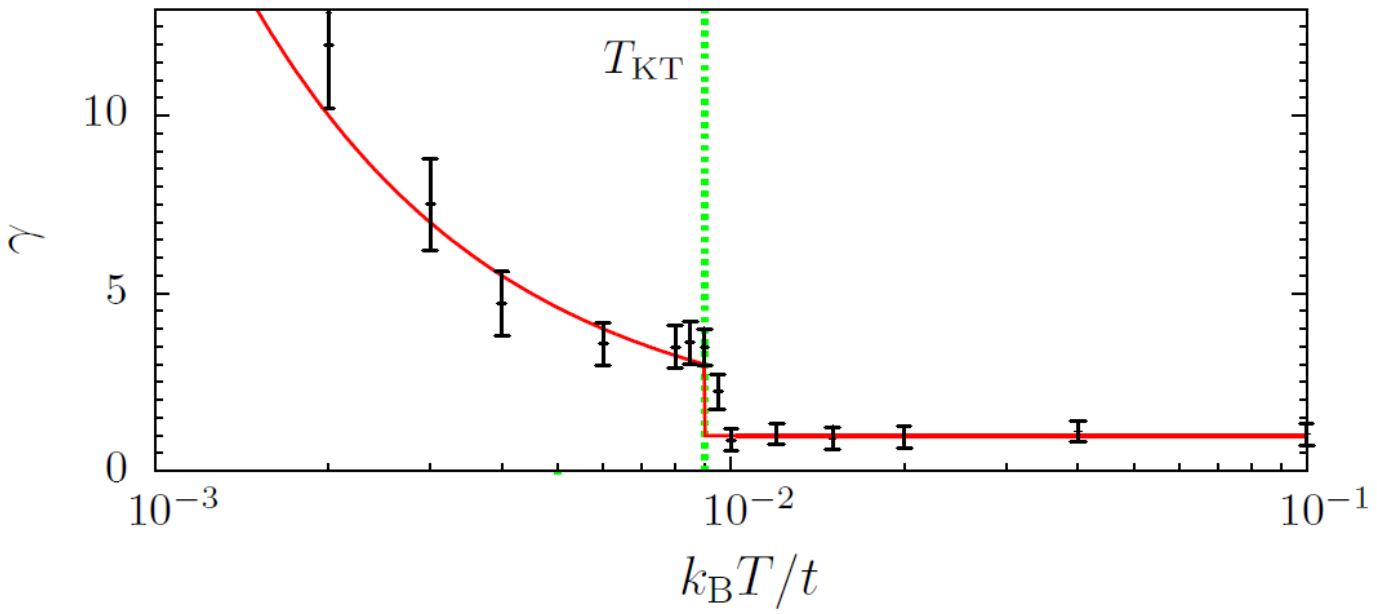

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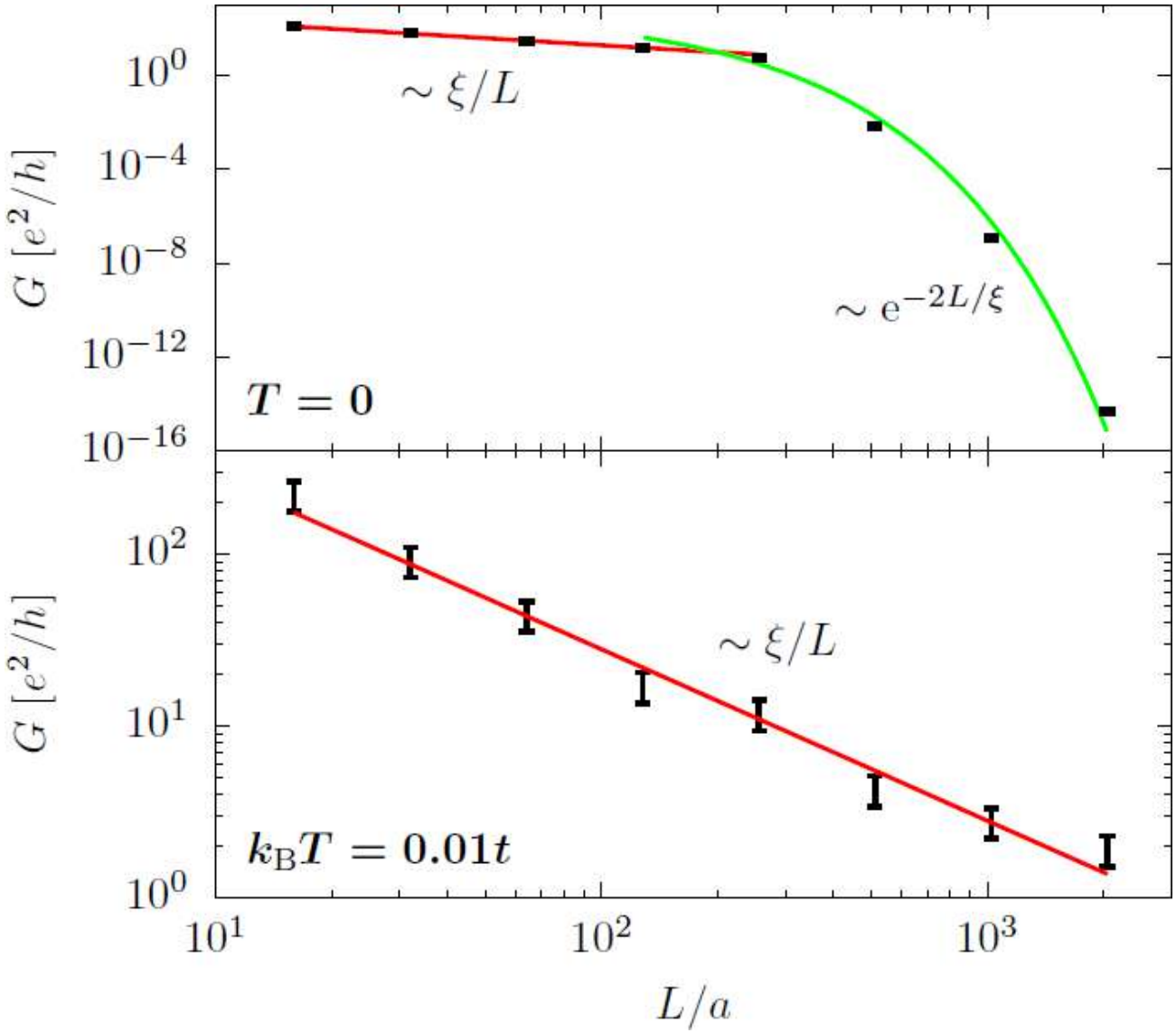

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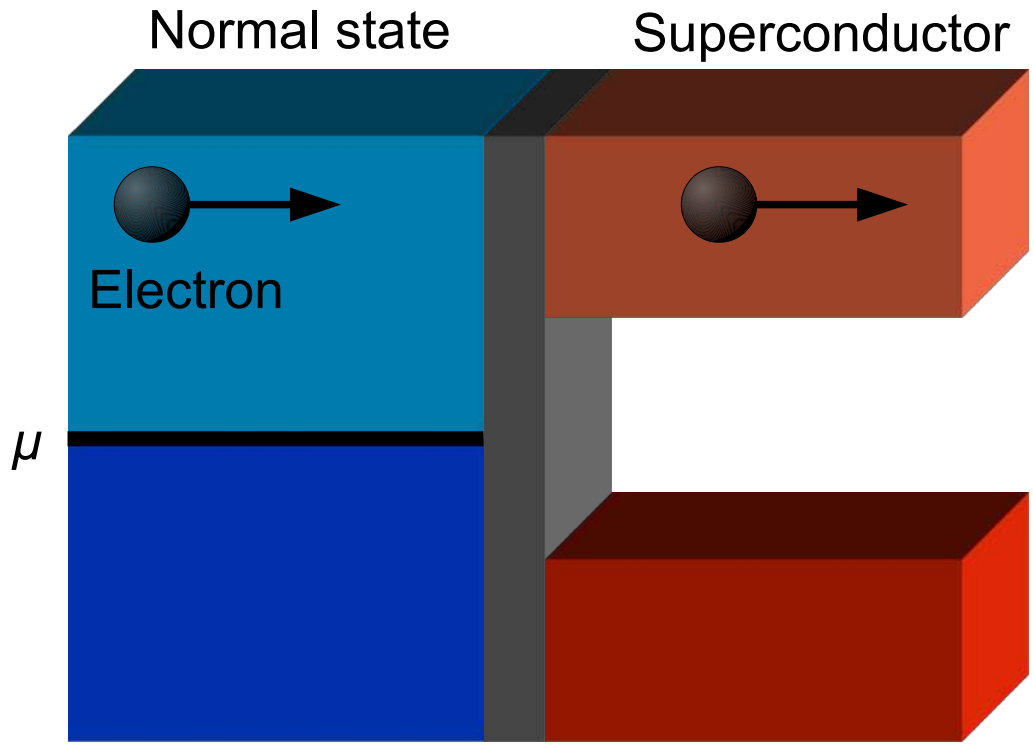
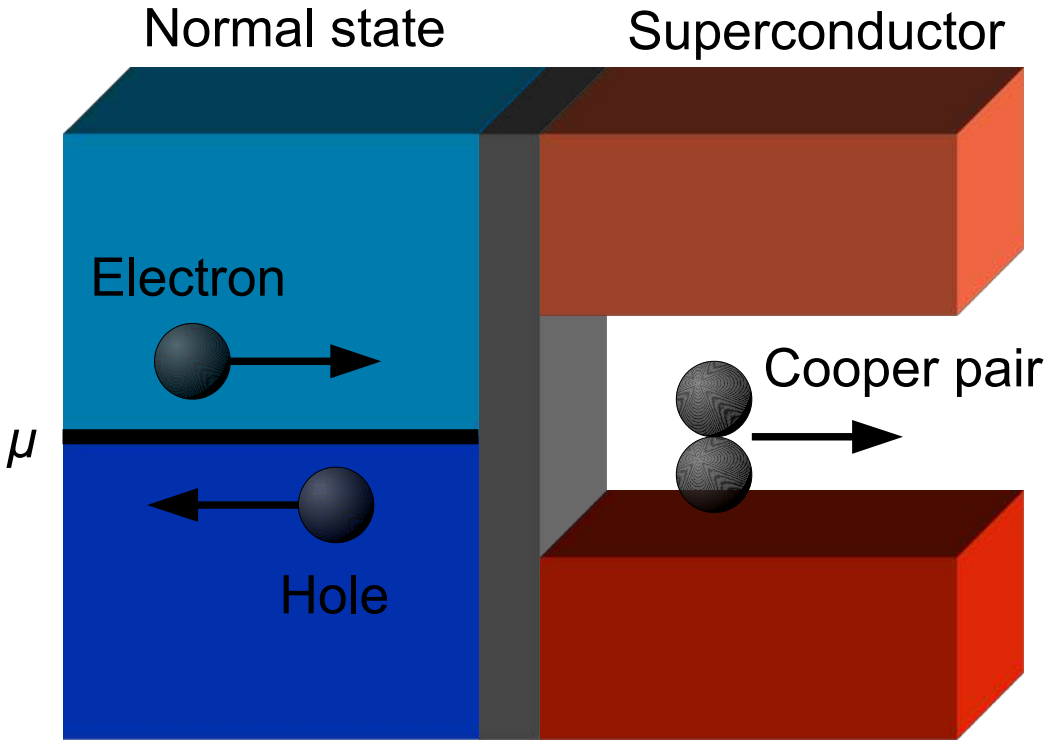


$$\sigma \propto V^2$$

Andreev, JETP **19**, 1228 (1964)

Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear *IV* characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



$$\sigma \propto V^4$$

Andreev, JETP **19**, 1228 (1964)

Verification

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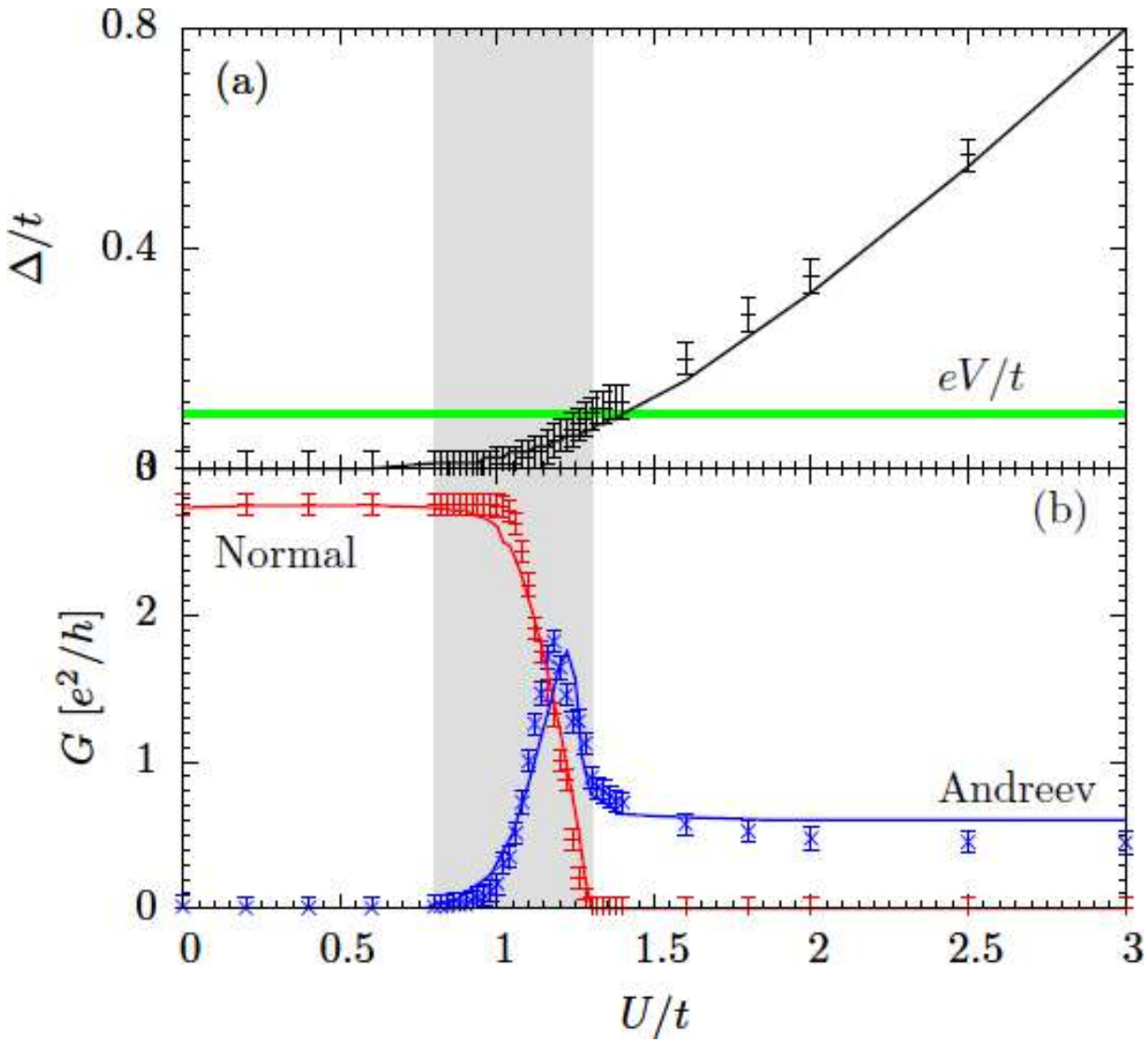
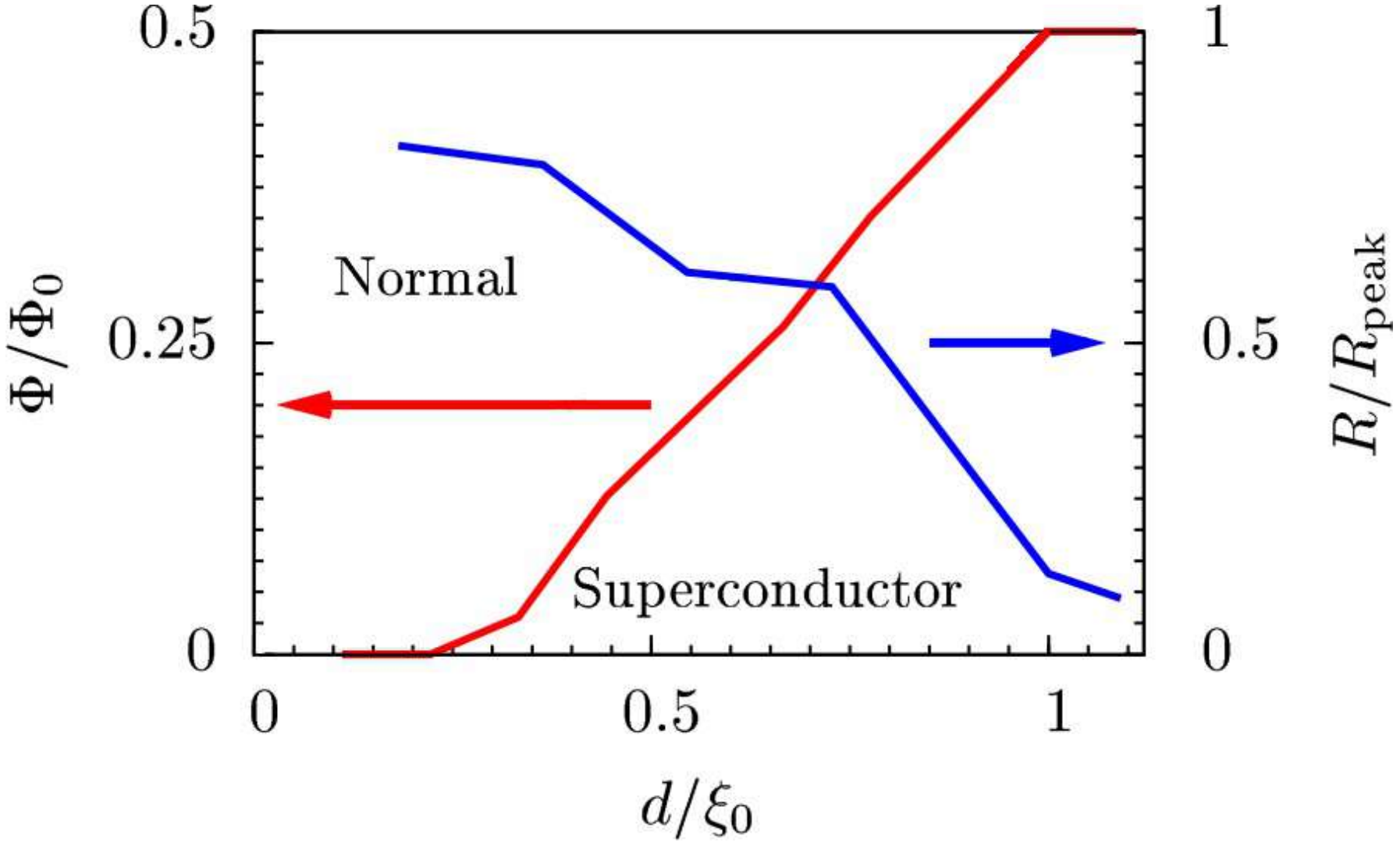
- Nonlinear *IV* characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder

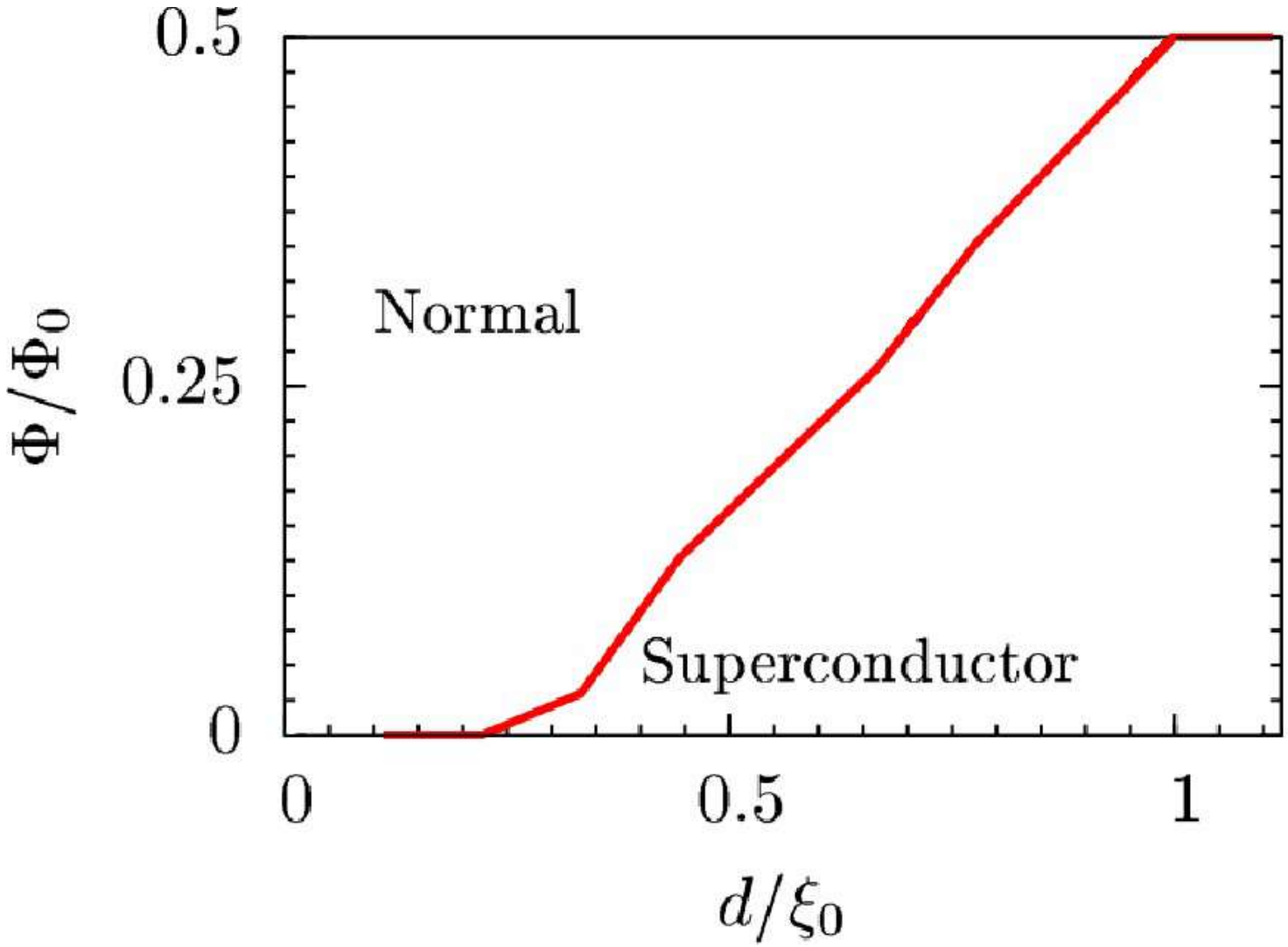


Blonder, Tinkham & Klapwijk, Phys. Rev. B **25**, 4515 (1982)

Variation with diameter



Variation with diameter



Model one-dimensional systems