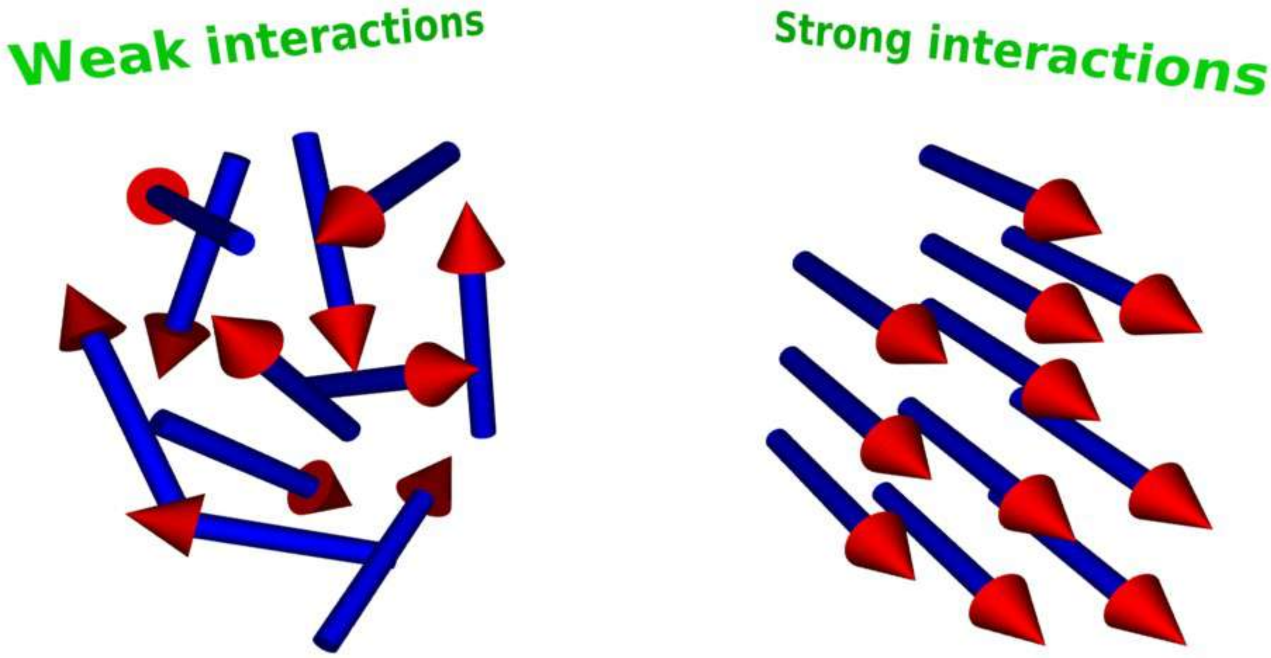
A repulsive atomic gas on the border of itinerant ferromagnetism



Gareth Conduit[1,2] , Ben Simons[3] & Ehud Altman[1]

1. Weizmann Institute, 2. Ben Gurion University, 3. University of Cambridge

G.J. Conduit & B.D. Simons, Phys. Rev. A **79**, 053606 (2009)

G.J. Conduit, A.G. Green & B.D. Simons, Phys. Rev. Lett. **103**, 207201 (2009)

G.J. Conduit & B.D. Simons, Phys. Rev. Lett. **103**, 200403 (2009)

G.J Conduit & E. Altman, arXiv: 0911.2839

Ferromagnetism in iron and nickel

- Typical ferromagnets undergo a *second order* transition

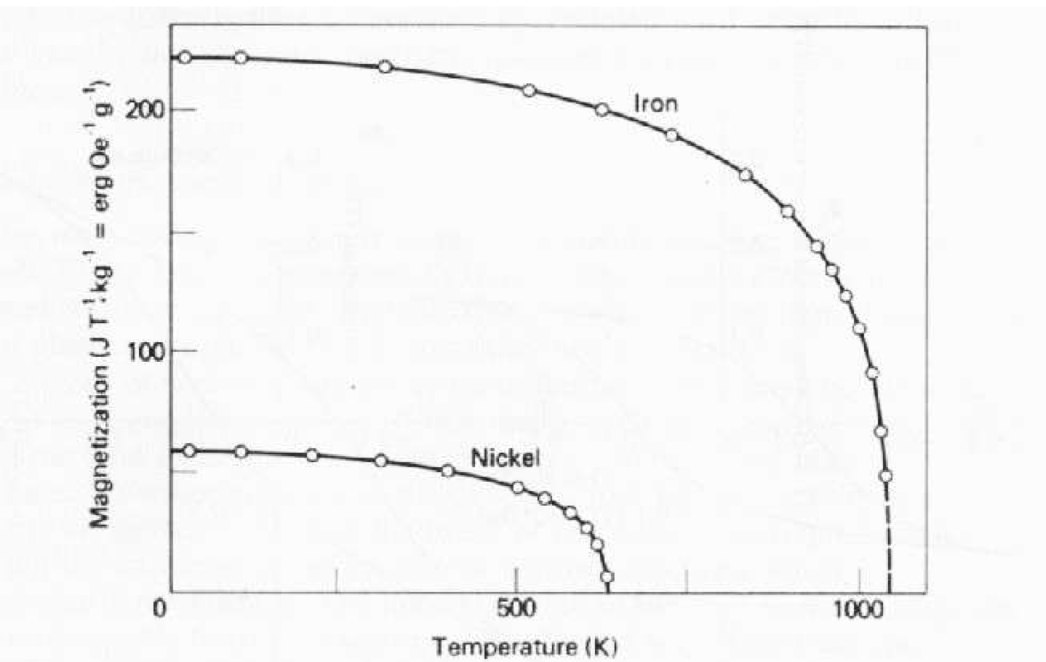
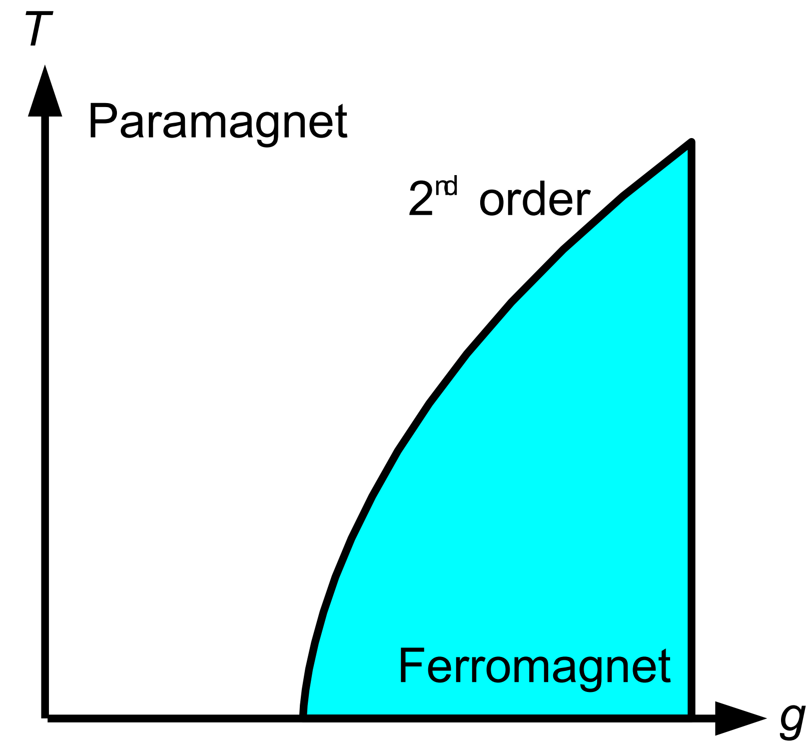
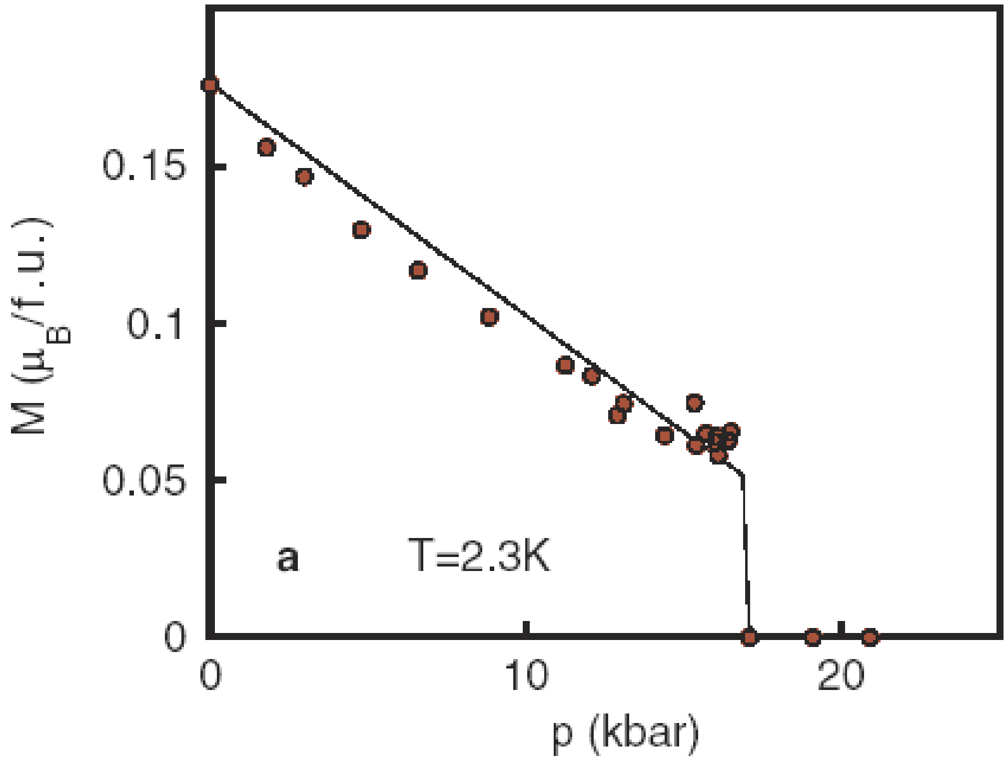


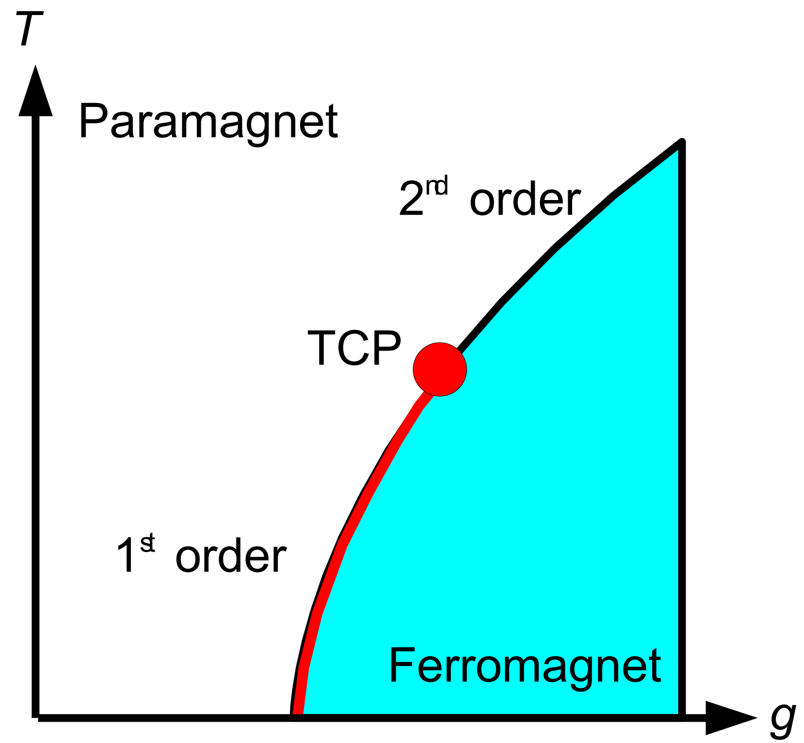
Figure 1.2 Spontaneous magnetization plotted against temperature for iron and nickel.



First order phase behavior — $ZrZn_2$

- At low temperature and high pressure $ZrZn_2$ has a first order transition

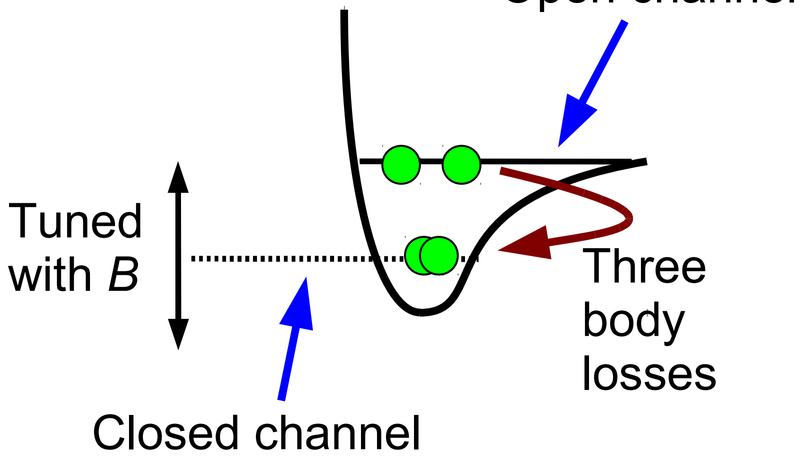


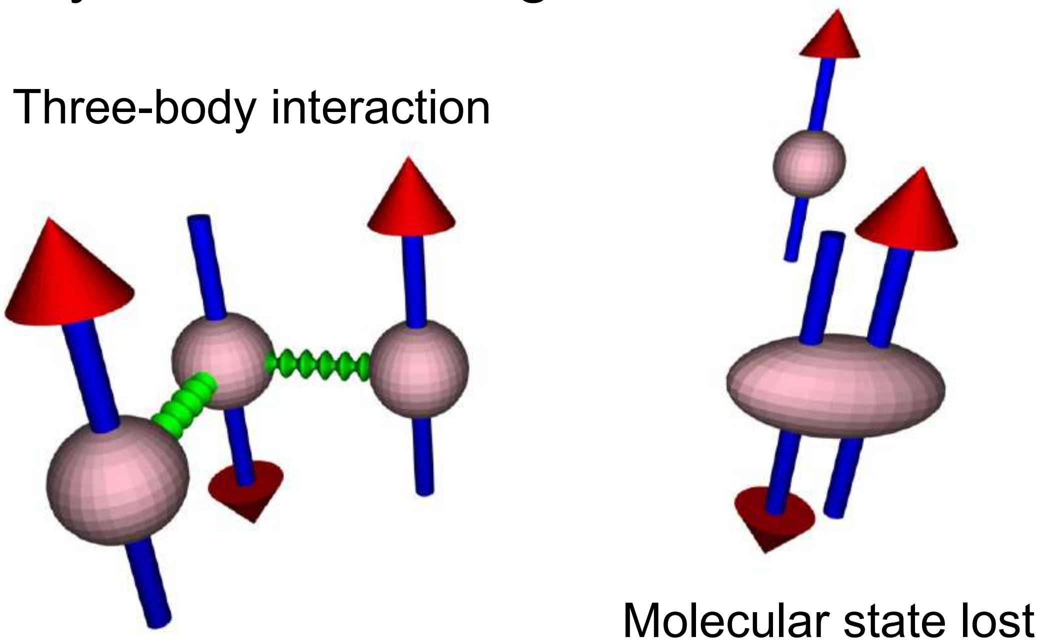


Uhlarz *et al.*, PRL 2004

Three body losses

- Three body losses inhibit the stability of the ferromagnetic state





- To reduce three-body losses the interaction strength is ramped rapidly
- In boson systems, three-body scattering can give rise to hard-core interactions and drive the formation of a Tonks-Girardeau gas [Syassen *et al*., Science **320**, 1329 (2009)]
- The experiment is an opportunity to study not only ferromagnetism but also three-body loss and dynamics

Itinerant ferromagnetism in cold atom gases

- Use two ^{6}Li states to represent pseudo up and down-spin electrons

$$H = \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}} c^{\dagger}_{\sigma\mathbf{k}} c_{\sigma\mathbf{k}} + g\sum_{\mathbf{k}} c^{\dagger}_{\uparrow\mathbf{k}} c^{\dagger}_{\downarrow\mathbf{k}} c_{\downarrow\mathbf{k}} c_{\uparrow\mathbf{k}}$$

$$E \approx \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}} n_{\sigma}(\varepsilon_{\mathbf{k}}) + g N_{\uparrow} N_{\downarrow}$$

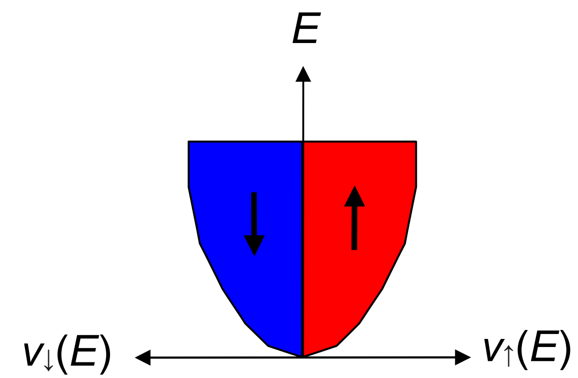
- A ΔE shift in the Fermi surface causes:

  (1) Kinetic energy increase of $\frac{1}{2}v\Delta E^2$

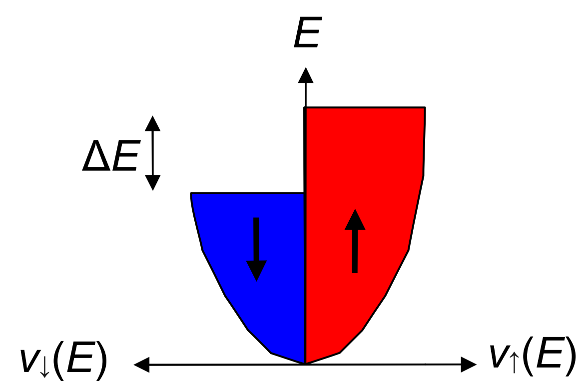
  (2) Reduction of repulsion of $-\frac{1}{2}gv^2\Delta E^2$

- Total energy shift is $\frac{1}{2}v\Delta E^2(1-gv)$ so a ferromagnetic transition occurs if $gv>1$

Not magnetised



Partially magnetised



Conduit & Simons, Phys. Rev. A **79**, 053606 (2009)

Jo, Lee, Choi, Christensen, Kim, Thywissen, Pritchard & Ketterle, Science **325**, 1521 (2009)

Cold atomic gases — spin

- Two fermionic atom species have a *pseudo-spin*:

^{6}Li	$m_F=1/2$	maps to	spin 1/2
^{6}Li	$m_F=-1/2$	maps to	spin -1/2

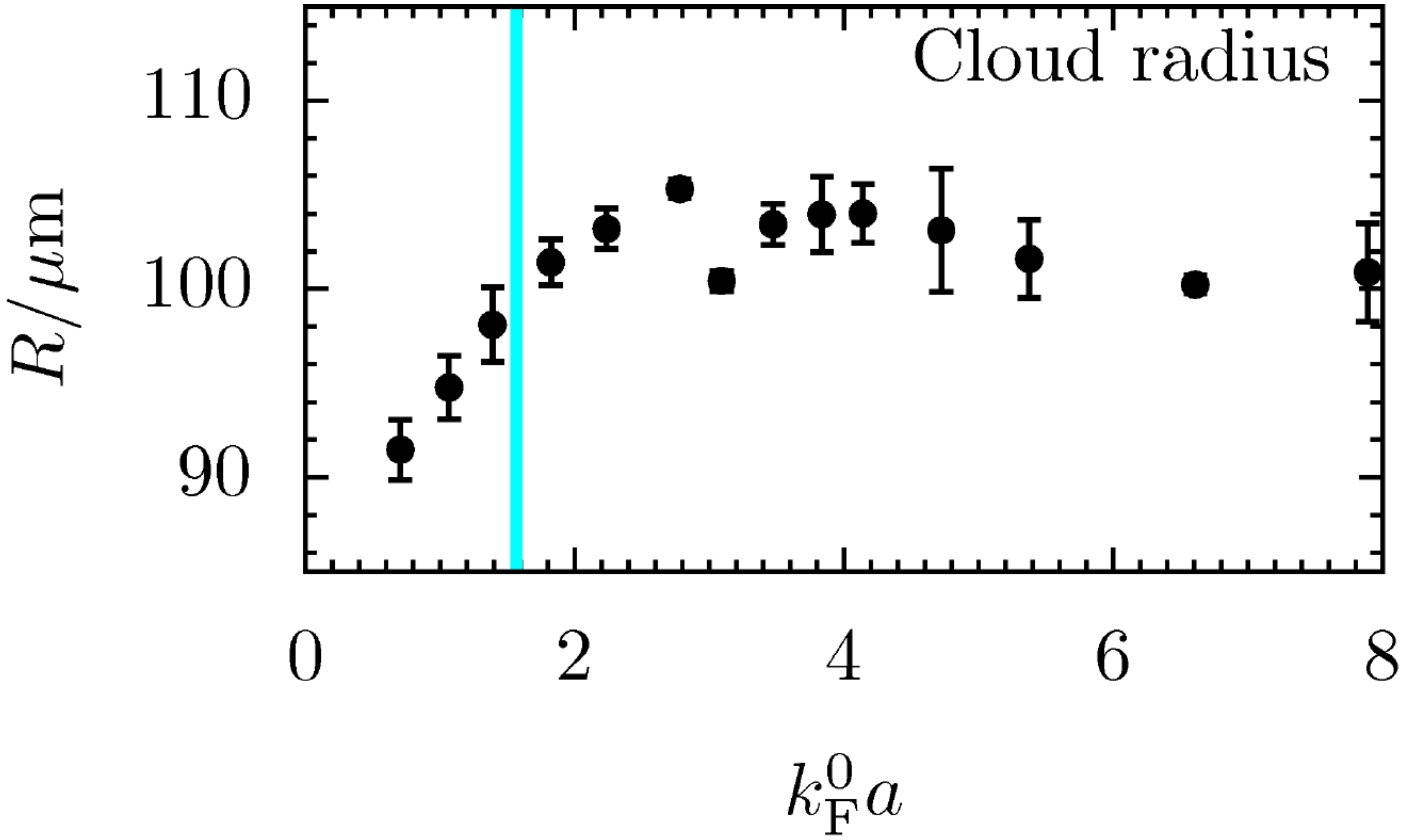
- The up-and down spin particles *cannot* interchange — population imbalance is fixed. Possible spin states are:

$(\lvert\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\rangle)/\sqrt{2}$	$S=0$, $S_z=0$	Non-magnetic state
$\lvert\uparrow\uparrow\rangle$	$S=1$, $S_z=1$	State not possible as S_z has changed
$\lvert\downarrow\downarrow\rangle$	$S=1$, $S_z=-1$	State not possible as S_z has changed
$(\lvert\uparrow\downarrow\rangle + \lvert\downarrow\uparrow\rangle)/\sqrt{2}$	$S=1$, $S_z=0$	Magnetic moment in plane

- Ferromagnetism, if favourable, must form in-plane

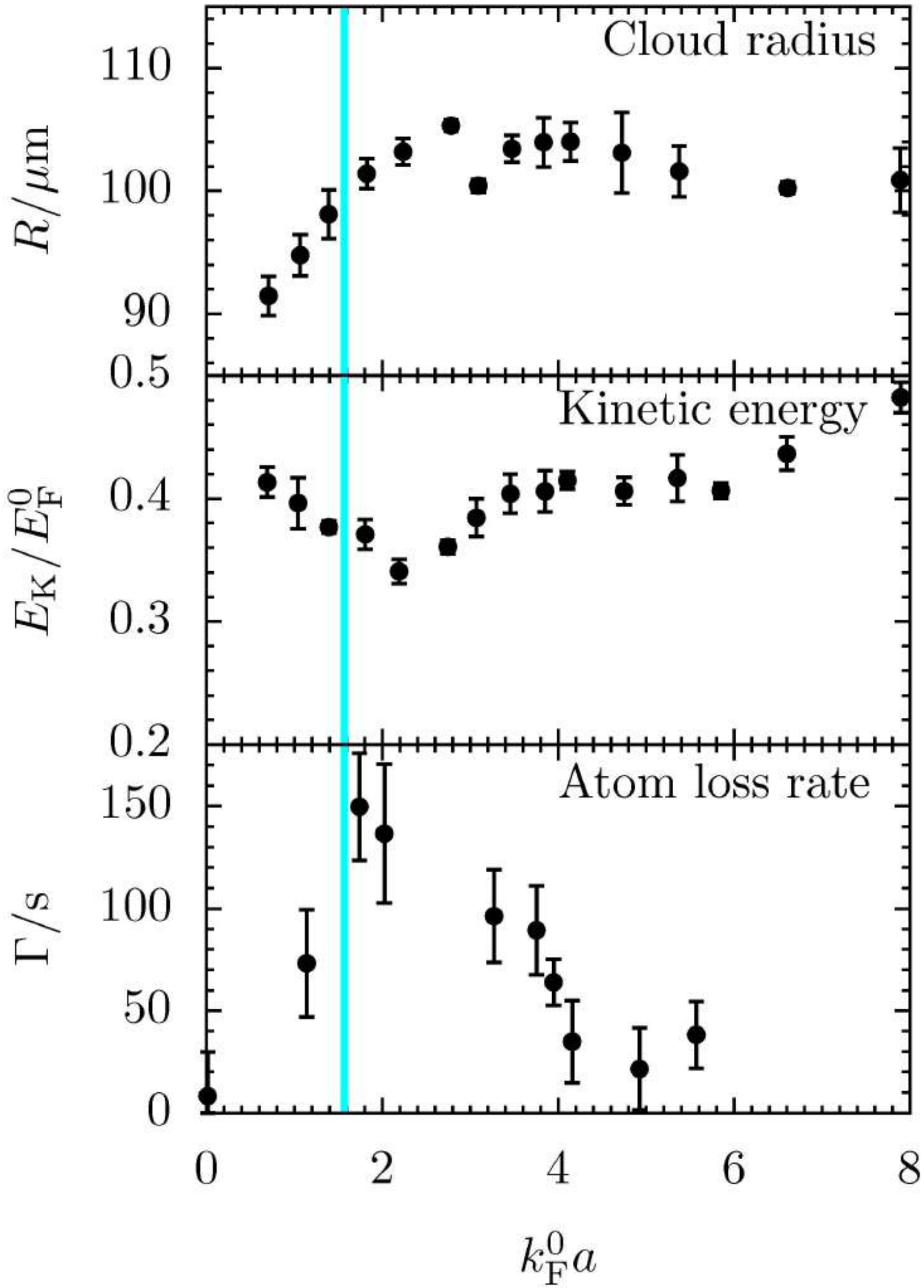
Experimental evidence for ferromagnetism

- Experimental points display same qualitative behavior but transition at $k_F a$=2.2



Jo, Lee, Choi, Christensen, Kim, Thywissen, Pritchard & Ketterle, Science **325**, 1521 (2009)

Further key experimental signatures



$$E_K \propto n^{5/3}$$

$$\Gamma \propto (k_F a)^6 n_\uparrow n_\downarrow (n_\uparrow + n_\downarrow)$$

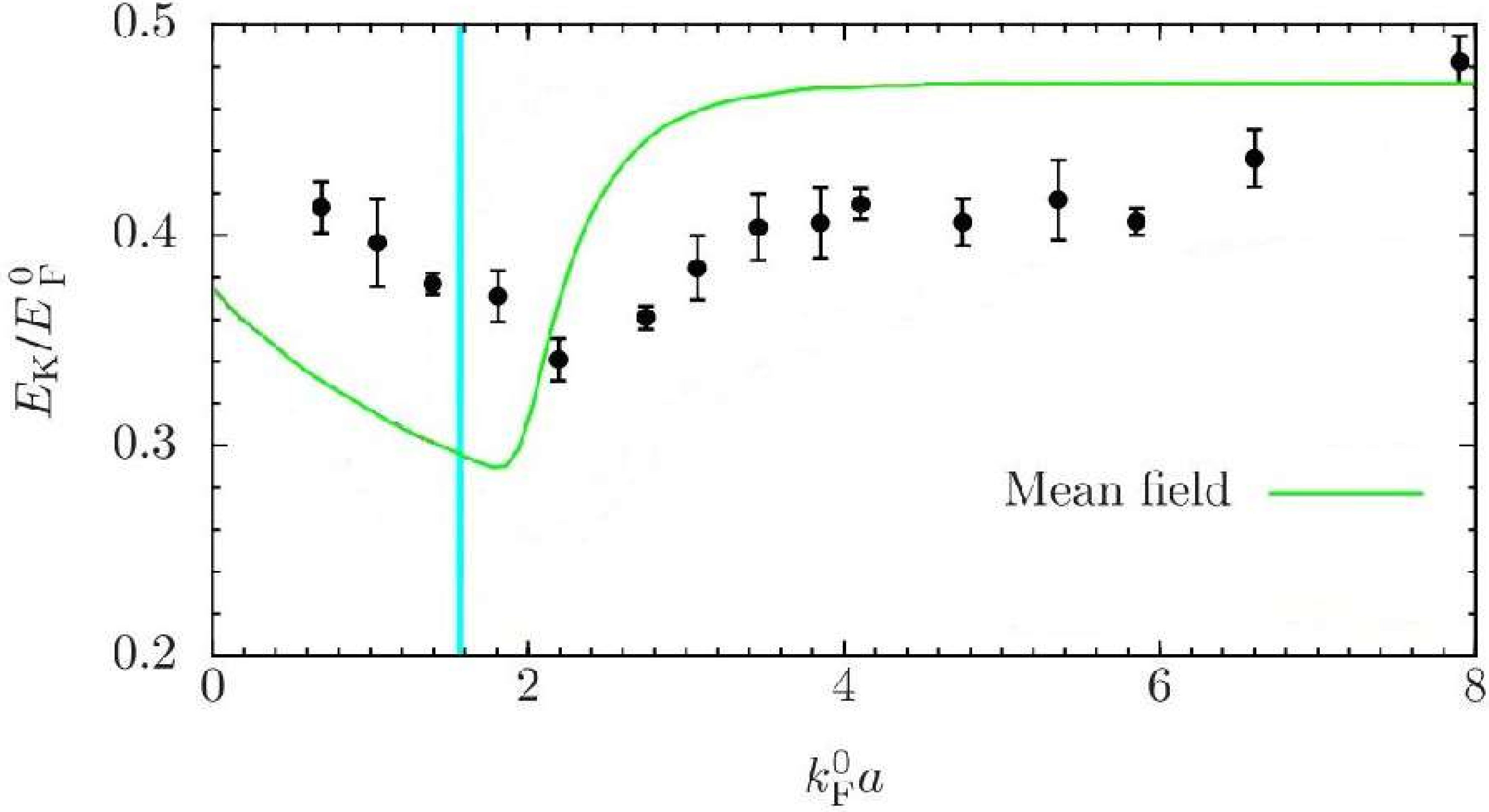
Jo, Lee, Choi, Christensen, Kim, Thywissen, Pritchard & Ketterle, Science **325**, 1521 (2009)

Synopsis of theoretical analysis

- To evaluate the experimental results we
  1) Employ a mean-field approximation to expose the consequences of a trapped geometry
  2) Consider how fluctuation corrections affect the transition
  3) Introduce new formalism that addresses atom loss
  4) Analyze how the mutual annihilation of defects inhibits the formation of a ferromagnetic state
- Active research on other possibilities
  1) Spin pattern formation [Berdnikov *et al.*, PRB **79**, 224403 (2009)]
  2) Trapped geometry & texture [LeBlanc *et al.*, PRA **80**, 013607 (2009)]
  3) Domain formation [Babadi *et al.*, arXiv:0908.3483]
  4) Other strongly correlated state [Zhai, PRA **80**, 051605(R) (2009)]
  5) First order transition [Duine & MacDonald, PRL **95**, 230403 (2005)]

Mean-field analysis & consequences of trap

- Recovers qualitative behavior[1] but transition at $k_F a$=1.8 instead of $k_F a$=2.2



[1]LeBlanc, Thywissen, Burkov & Paramekanti, Phys. Rev. A **80**, 013607 (2009) & Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

Fluctuation corrections

$$Z=\int D\psi \exp\left(-\iint d\tau\, d\boldsymbol{r} \sum_\sigma \bar{\psi}_\sigma(-i\omega+\epsilon-\mu)\psi_\sigma - g\bar{\psi}_\uparrow\bar{\psi}_\downarrow\psi_\downarrow\psi_\uparrow\right)$$

- Decouple with the average magnetisation gives the Stoner criterion

$$F=\sum_{\sigma,\boldsymbol{k}} \epsilon_{\boldsymbol{k}}^\sigma n(\epsilon_{\boldsymbol{k}}^\sigma)+gN^\uparrow N^\downarrow \propto (1-g\nu)m^2$$

- Integrate over fluctuations in magnetization and density and develop a perturbation theory in interaction strength for the free energy[1]

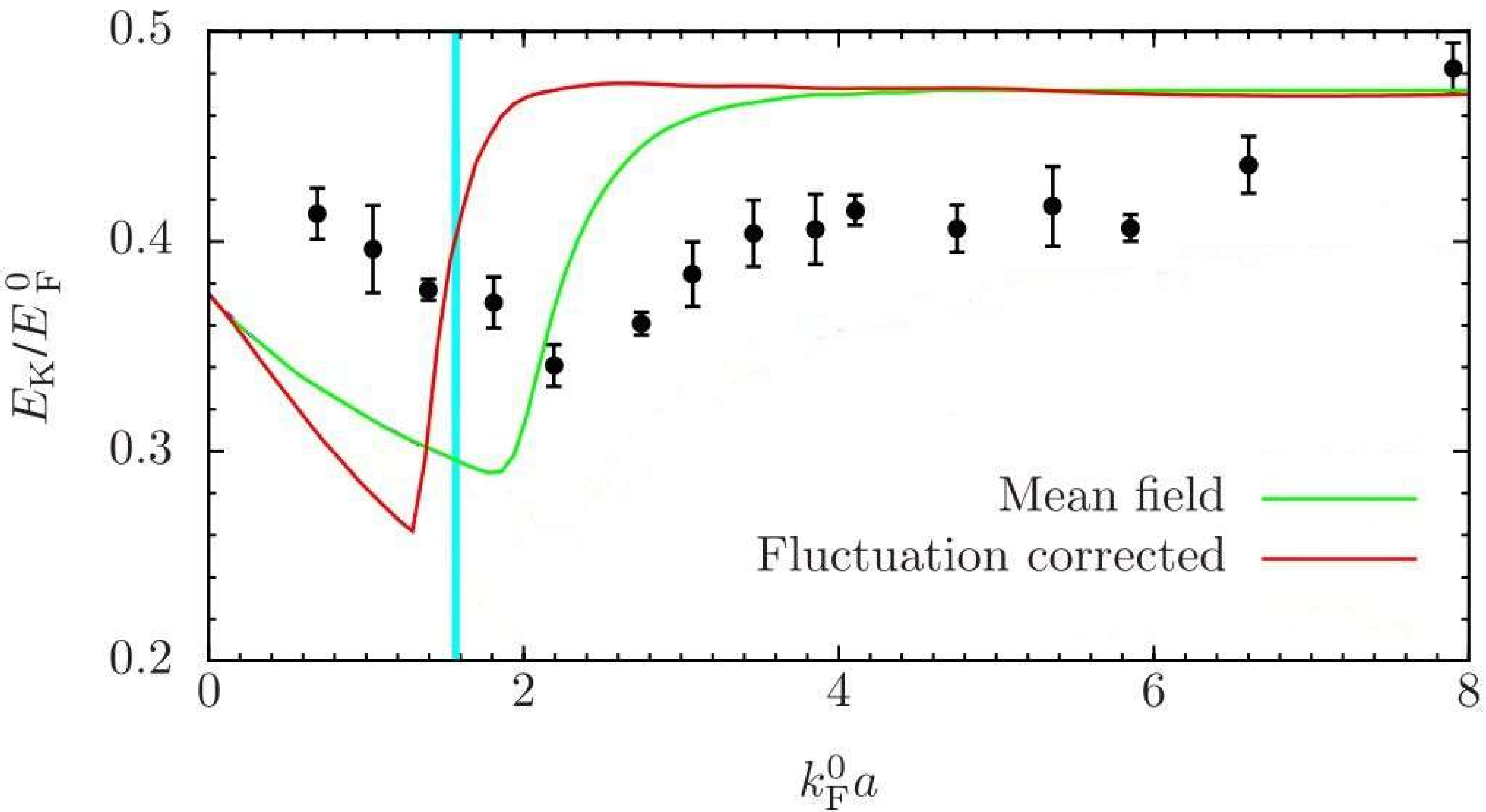
$$F=\sum_{\sigma,\boldsymbol{k}} \epsilon_{\boldsymbol{k}}^\sigma n(\epsilon_{\boldsymbol{k}}^\sigma)+gN^\uparrow N^\downarrow - \frac{2g^2}{V^3}\sum_{\boldsymbol{k}_{1,2,3,4}} \frac{n(\epsilon_{\boldsymbol{k}_1}^\uparrow)n(\epsilon_{\boldsymbol{k}_2}^\downarrow)[n(\epsilon_{\boldsymbol{k}_3}^\uparrow)+n(\epsilon_{\boldsymbol{k}_3}^\downarrow)]}{\epsilon_{\boldsymbol{k}_1}^\uparrow+\epsilon_{\boldsymbol{k}_2}^\downarrow-\epsilon_{\boldsymbol{k}_3}^\uparrow-\epsilon_{\boldsymbol{k}_4}^\downarrow}$$

- Backed up by *ab initio* Quantum Monte Carlo calculations[2]
- Enhanced particle-hole phase space at zero magnetisation[2] leads to an anomalous term[3] $m^4 \ln|m|$ in the Landau expansion that drives the ferromagnetic transition first order at $k_F a=1.054$

[1]Abrikosov (1958), Duine & MacDonald (2005) & Conduit & Simons, Phys. Rev. A **79**, 053606 (2009)
[2]Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)
[3]Belitz *et al*. Z. Phys. B (1997)

Fluctuation corrections

- Extend theory through fluctuation corrections



Conduit & Simons, Phys. Rev. A **79**, 053606 (2009) &
Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

Including atom loss

- Atom loss rate

  $\lambda n_{\uparrow}(\mathbf{r}) n_{\downarrow}(\mathbf{r}) \chi(\mathbf{r}-\mathbf{r}')[n_{\uparrow}(\mathbf{r}') + n_{\downarrow}(\mathbf{r}')]$

  which in second quantized form is

  $\lambda c_{\uparrow}^{\dagger}(\mathbf{r}) c_{\downarrow}^{\dagger}(\mathbf{r}) c_{\downarrow}(\mathbf{r}) c_{\uparrow}(\mathbf{r}) \chi(\mathbf{r}-\mathbf{r}')[c_{\uparrow}^{\dagger}(\mathbf{r}') c_{\uparrow}(\mathbf{r}') + c_{\downarrow}^{\dagger}(\mathbf{r}') c_{\downarrow}(\mathbf{r}')]$

- With a mean-field approximation, $\overline{N} = c_{\uparrow}^{\dagger} c_{\uparrow} + c_{\downarrow}^{\dagger} c_{\downarrow}$

  $\lambda \overline{N} c_{\uparrow}^{\dagger}(\mathbf{r}) c_{\downarrow}^{\dagger}(\mathbf{r}) c_{\downarrow}(\mathbf{r}) c_{\uparrow}(\mathbf{r})$

  it appears on same footing as the interaction term

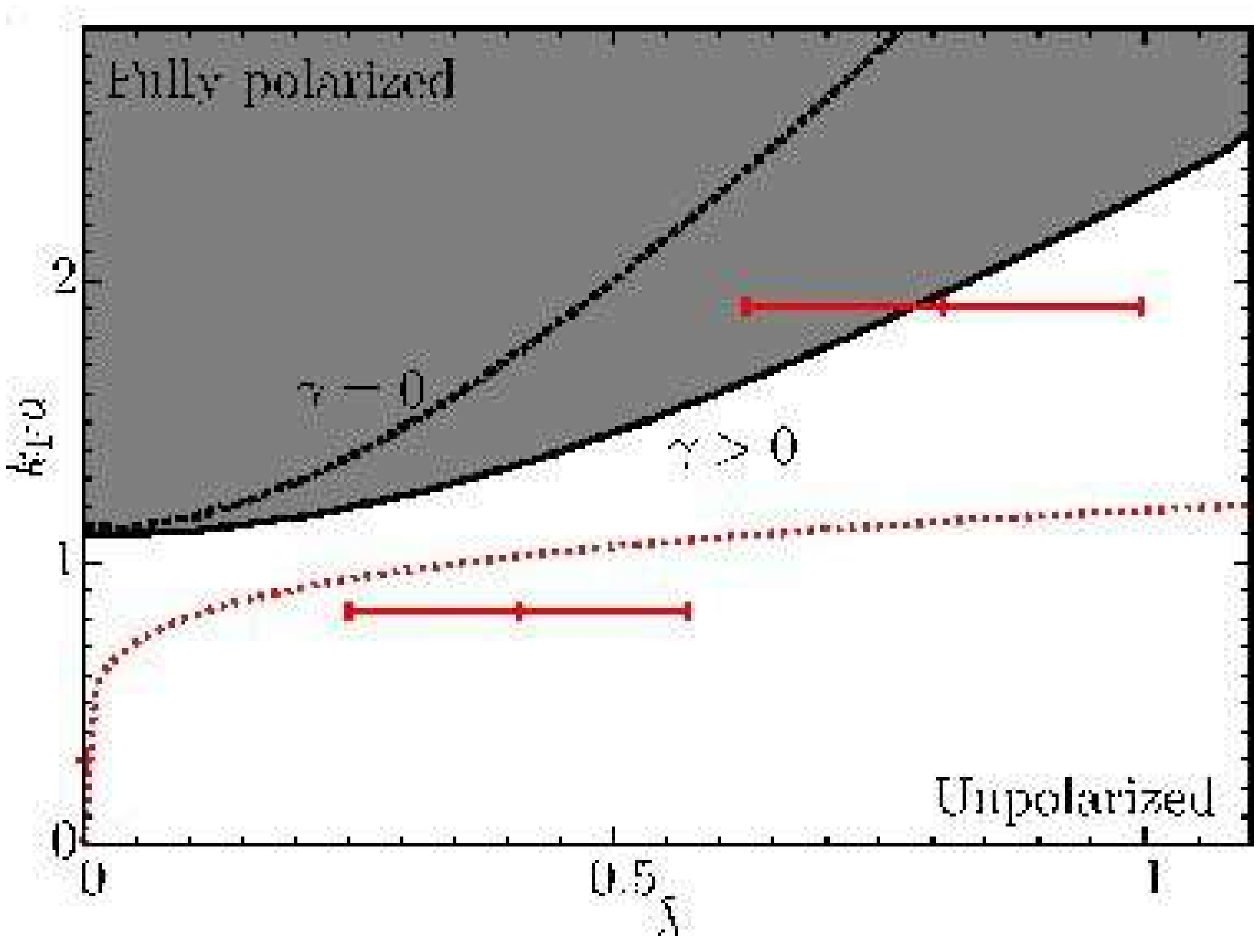
  $S_{\mathrm{int}} = (g + i\lambda \overline{N}) c_{\uparrow}^{\dagger}(\mathbf{r}) c_{\downarrow}^{\dagger}(\mathbf{r}) c_{\downarrow}(\mathbf{r}) c_{\uparrow}(\mathbf{r})$

- Loss damps fluctuations so inhibits transition

$$F = \sum_{\sigma, \boldsymbol{k}} \epsilon_{\boldsymbol{k}}^{\sigma} n(\epsilon_{\boldsymbol{k}}^{\sigma}) + g N^{\uparrow} N^{\downarrow} - 2(g^2 - \lambda^2 \overline{N}^2) \Upsilon$$

Ramifications of atom loss

- Atom loss has the potential to raise the interaction strength required for a ferromagnetic transition



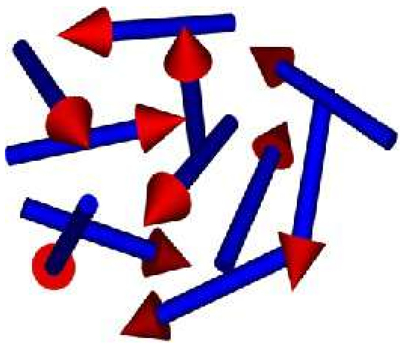
Conduit & Altman, arXiv: 0911.2839

Condensation of topological defects

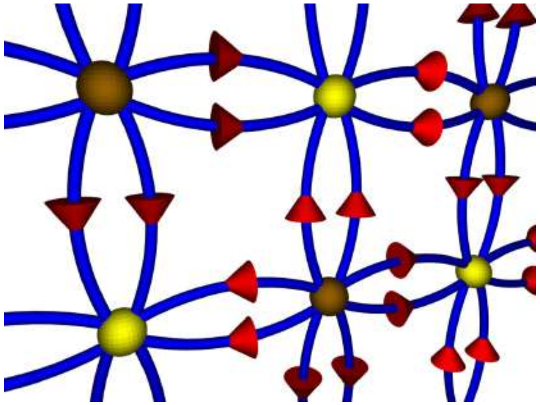
- Defects freeze out from disordered state

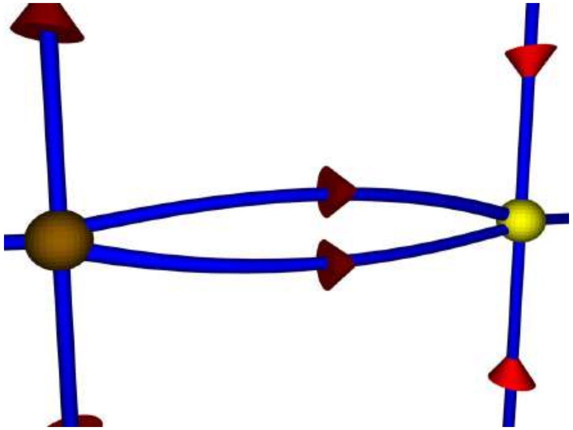
- Defect annihilation hinders the formation of the ferromagnetic phase thus raising the required interaction strength

- Defect radius $L \sim t^{½}$ [Bray, Adv. Phys. **43**, 357 (1994)]



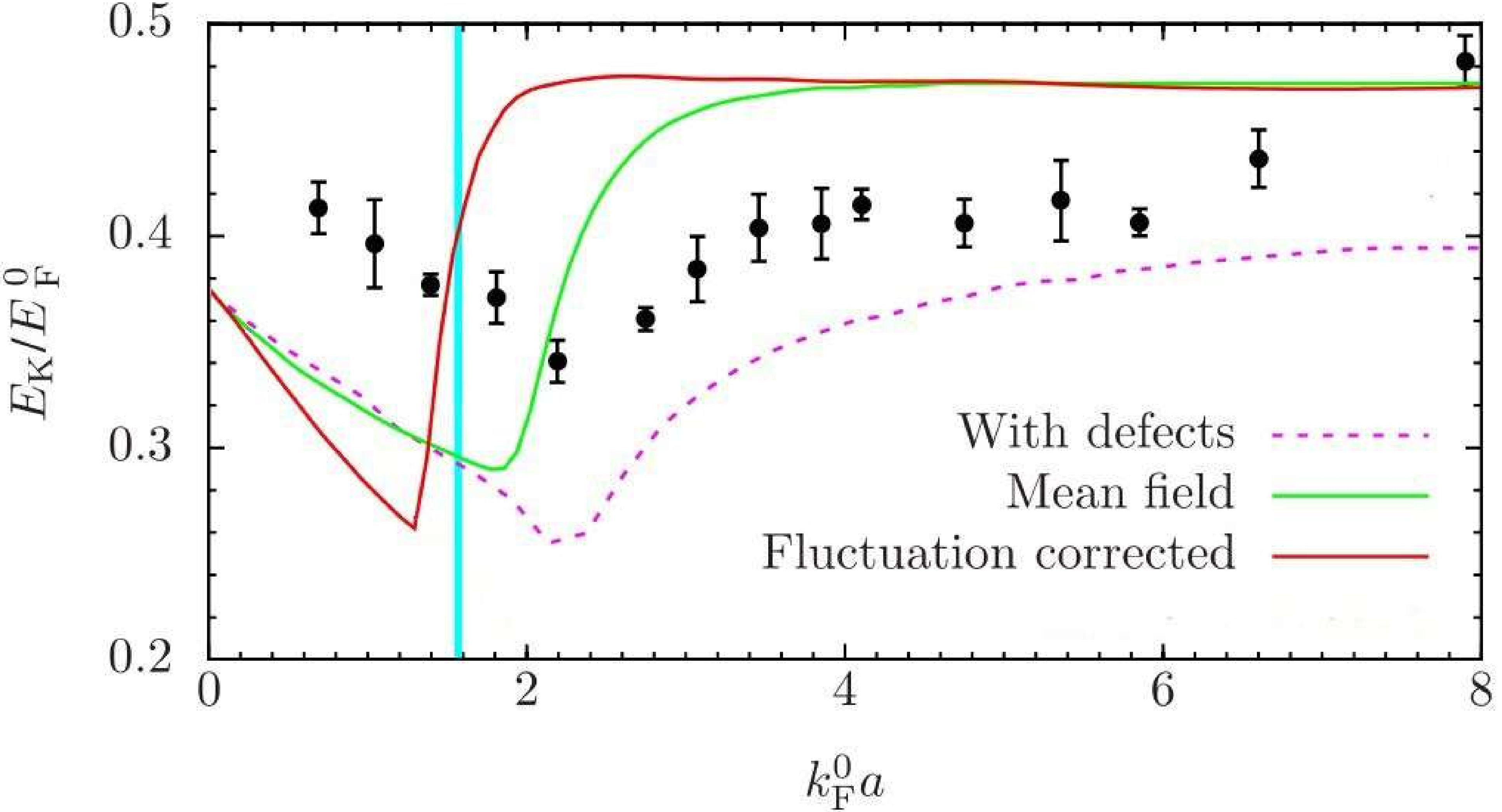
Start in paramagnetic state and ramp up interaction strength





Condensation of topological defects

- Condensation of defects inhibits the transition



Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

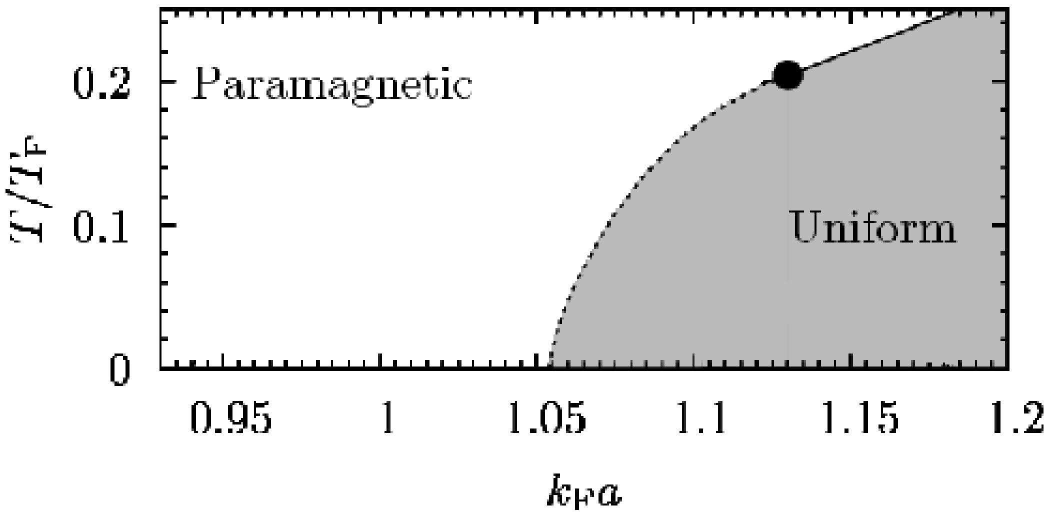
Summary

- Mean-field theory provides a reasonable qualitative description of the transition
- Discrepancy in the interaction strength could be accounted for by:

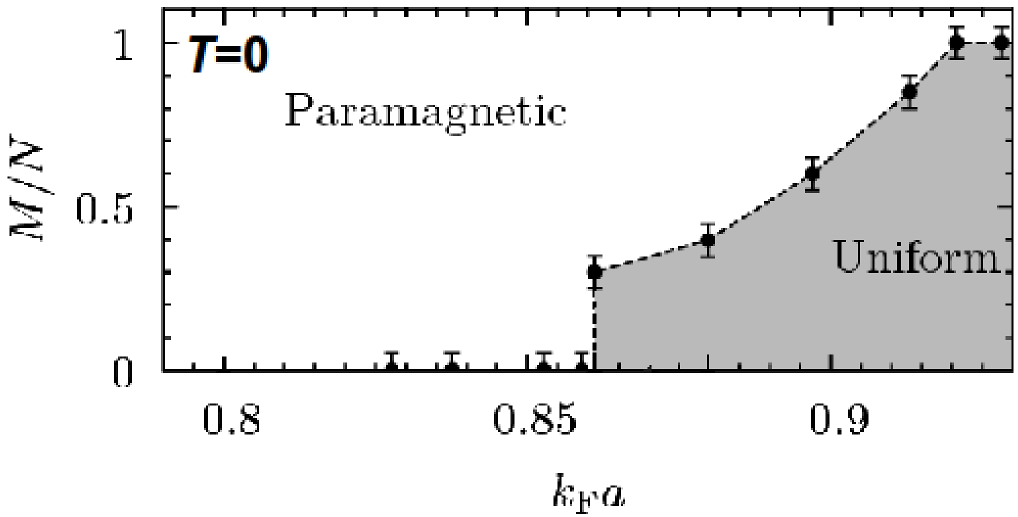
1) Renormalization of interaction strength due to atom loss
2) The mutual annihilation of defects inhibiting the formation of the ferromagnetic phase

First order phase transition and Quantum Monte Carlo verification

- First order transition into uniform phase with TCP



- QMC also sees first order transition



New approach to fluctuation corrections

$$Z=\int D\psi \exp\left(-\int \sum_{\sigma} \bar{\psi}_{\sigma}(-i\omega+\epsilon-\mu)\psi_{\sigma}-g\int \bar{\psi}_{\uparrow}\bar{\psi}_{\downarrow}\psi_{\downarrow}\psi_{\uparrow}\right)$$

- Analytic strategy:

  1) Decouple in both the density and spin channels (previous approaches employ only spin)

  2) Integrate out electrons

  3) Expand about uniform magnetisation

  4) Expand density and magnetisation fluctuations to second order

  5) Integrate out density and magnetisation fluctuations
- Aim to uncover connection to second order perturbation theory
- Unravel origin of logarithmic divergence in energy and relation to tricritical point structure

Analytical method

- System free energy F=-$k_B T$ln$Z$ is found via the partition function

$$Z = \int D\psi \exp\left(-\int \sum_\sigma \bar{\psi}_\sigma(-i\omega+\epsilon-\mu)\psi_\sigma - g\int \bar{\psi}_\uparrow\bar{\psi}_\downarrow\psi_\downarrow\psi_\uparrow\right)$$

- Decouple using only the average magnetisation $m=\bar{\psi}_\downarrow\psi_\uparrow-\bar{\psi}_\uparrow\psi_\downarrow$

  gives $F \propto (1-g\nu)m^2$ i.e. the Stoner criterion

- Belitz-Kirkpatrick-Vojta (soft particle-hole & magnetisation) [Belitz *et al.* Z. Phys. B 1997]

$$F = \frac{1}{2}\left(|\omega|/\Gamma_q + r + q^2\right)m^2 + \frac{u}{4}m^4\ln(m^2+T^2) + \cdots - hm$$