LECTURE 2

← Weak interactions: mean field
Slater-Saxton → numerics

- Strong interactions
  - Possibility for ferromagnetic transitions

- Mean field Stoner transition calculation
  - Cold atoms language
  - Second order
  - Behaviour with temperature

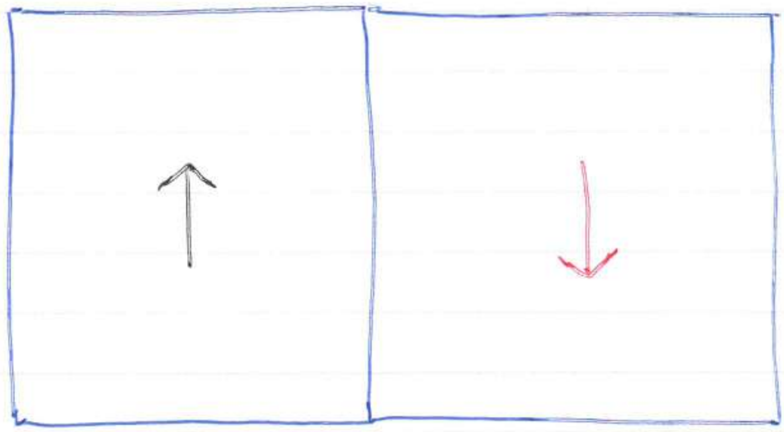
- Observations:
  Solid state
  Cold atoms

- Going beyond mean field theory
  - Order-by-disorder
  - Spiral, p-wave

- Polaron limit.

LECTURE 2

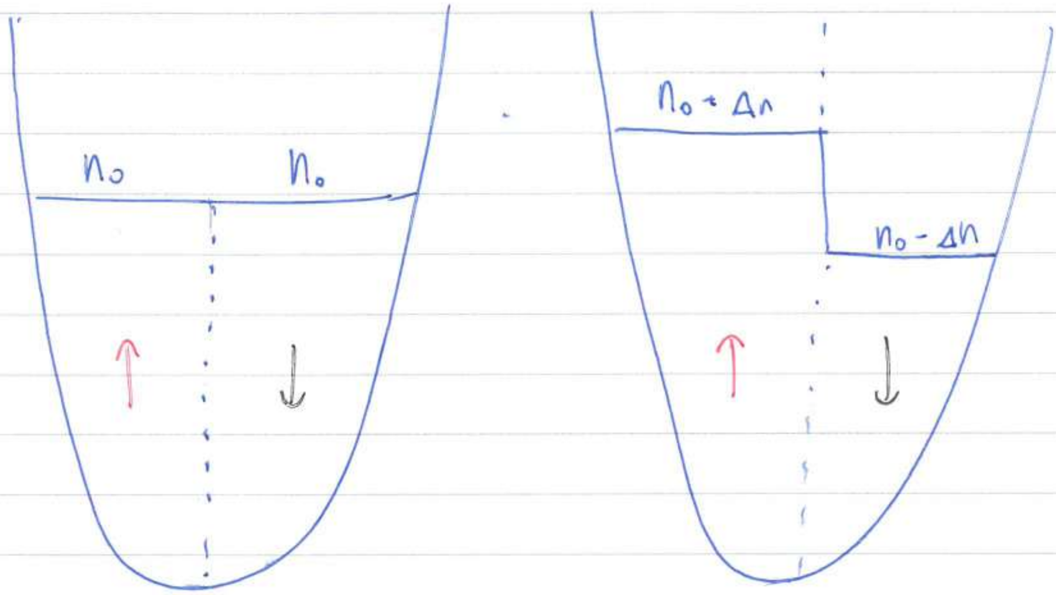
Repulsive interactions : split into domains



No repulsion but kinetic energy cost of squeezing spins into smaller box

$KE \sim n^{2/3}$ so $\times\ 2^{2/3}$

To search for ferromagnetic transition



$$KE = \int_0^{E_F} E \cdot \nu_{F0} \sqrt{\frac{E}{E_{F0}}} \, dE$$

$$= \frac{\nu_{F0}}{\sqrt{E_{F0}}} \frac{2}{5} E_F^{5/2}$$

$$= \frac{\nu_{F0}}{\sqrt{E_{F0}}} \cdot \frac{2}{5} \cdot \left(\frac{3}{2} \cdot n \cdot \frac{\sqrt{E_{F0}}}{\nu_{F0}} \right)^{5/3}$$

$$= \left(\frac{\sqrt{E_{F0}}}{\nu_{F0}} \right)^{2/3} \frac{2}{5} \left(\frac{3}{2} \right)^{5/3} \cdot n^{5/3}$$

$$n = \int_0^{E_F} \nu_{F0} \frac{\sqrt{E}}{\sqrt{E_{F0}}} \, dE$$

$$= \frac{\nu_{F0}}{\sqrt{E_{F0}}} \cdot \frac{2}{3} E_F^{3/2}.$$

$$E_F^{1/2} = \left(\frac{3}{2} \cdot n \cdot \frac{\sqrt{E_{F0}}}{\nu_{F0}} \right)^{1/3}$$

$$E = \left(\frac{\sqrt{E_{F0}}}{\nu_{F0}} \right)^{2/3} \frac{2}{5} \left(\frac{3}{2} \right)^{5/3} \cdot n^{5/3} \left((1+\Delta n)^{5/3} + (1-\Delta n)^{5/3} \right) + g (n+\Delta n)(n-\Delta n)$$

$$= \left(\frac{\sqrt{E_{F0}}}{\nu_{F0}} \right)^{2/3} \frac{4}{5} \left(\frac{3}{2} \right)^{5/3} \cdot n^{5/3} \cdot \left(1 + \frac{1}{2} \cdot \frac{5}{3} \cdot \frac{2}{3} \cdot \Delta n^2 \right) + g n^2 \left(1 - \Delta n^2 \right)$$

$$= \left(\frac{\sqrt{E_{F0}}}{\nu_{F0}} \right)^{2/3} \frac{4}{5} \cdot \left(\frac{3}{2} \right)^{5/3} \cdot n^{5/3} \cdot \left(1 + \frac{5}{9} \Delta n^2 \right) + g n^2 \left(1 - \Delta n^2 \right)$$

look at coefficient of Δn^2:

$$\left|\frac{\sqrt{E_{F0}}}{v_{F0}}\right|^{2/3} \cdot \frac{4}{\cancel{5}} \left|\frac{3}{2}\right|^{5/3} \cdot \cancel{n^{5/3}} \cdot \frac{\cancel{5}}{9} = g \cancel{n^2} \cdot n^{1/3}$$

$$\left|\frac{\sqrt{E_{F0}}}{v_{F0}}\right|^{2/3} \cdot \frac{4}{9} \left|\frac{3}{2}\right|^{5/3} = g \cdot \left|\frac{v_{F0}}{\sqrt{E_{F0}}}\right|^{1/3} \cdot \left|\frac{2}{3}\right|^{1/3} \cdot E_{F0}^{1/2}$$

$$\frac{4}{9} \left|\frac{3}{2}\right|^2 = g v_{F0}$$

$$1 = g v_{F0}.$$

So second order ferromagnetic transition at $g v_F = 1$

Suitable wavefunction

- Perturbation theory o. Slater Determinants
- $\Psi = D_\uparrow D_\downarrow$ with different #'s of $\uparrow$ and $\downarrow$
- Problem with spin uncertainty

  $$\langle \Delta S_x \times \Delta S_y \rangle \geq \frac{\hbar}{2} |S_z|$$

  • Only ok for unpolarised and fully polarised.
- Jastrow Form $e^J D_\uparrow D_\downarrow$ allows further correlations to be captured
- Strategy : plot $E(M)$

Beyond mean field: fluctuations increase partition function & lower the energy

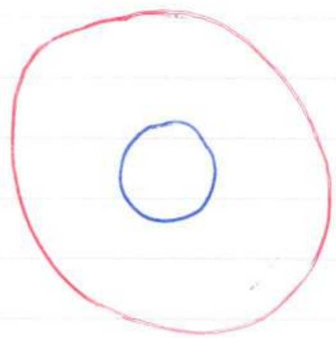
$$F = -kT\ln Z, \qquad Z = \sum_i e^{-\beta E_i}$$

Soft fluctuations M_1 are favorable

Consider Fermi surface

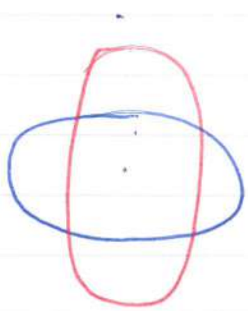
Paramagnet

Ferromagnet

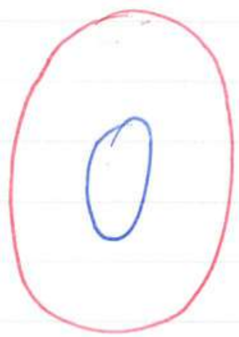


FIRST ORDER

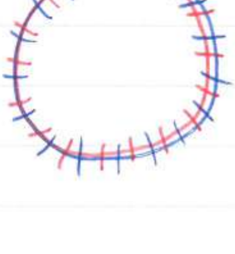
Nematic



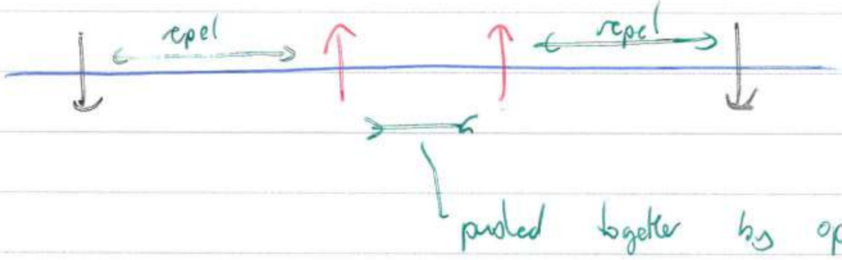
Spin spiral



Superconductor

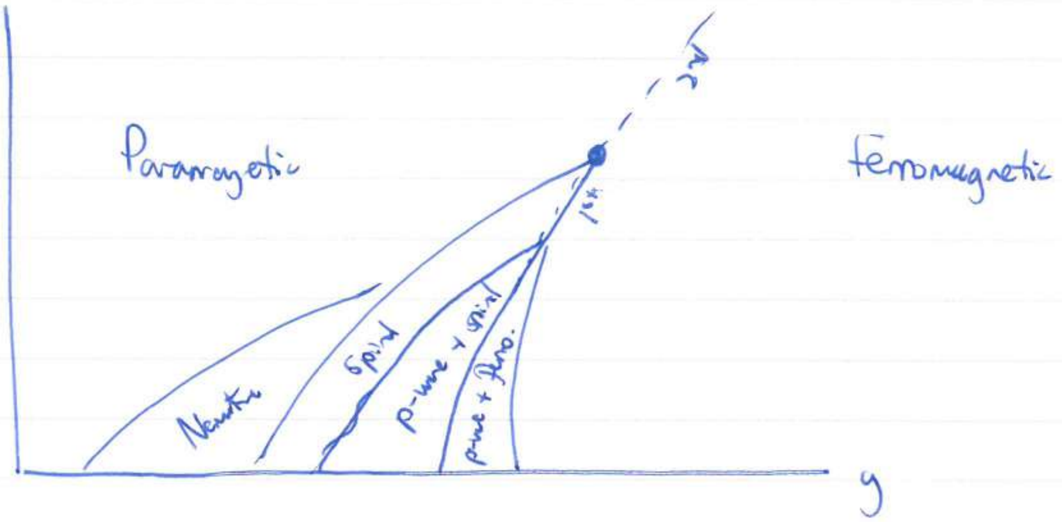


Superconductor pairing between equal spins



$\Delta_k = \langle C_k C_{-k} \rangle$ so $\Delta_k = -\Delta_{-k}$, so p-wave

Phase diagram



Observation

f-like conduction electrons screened by s,p to give contact interactions

Solid state : CeFePO —— SC
UGe$_2$ —— SC
$Sr_3Ru_2O_7$ —— Spiral
$NbFe_2$ —— Spiral

Cold atoms : Feltorle
Problems with losses
Two body losses with energy going into few surface
~~Zaccant~~ collide clouds

Summary

Seen how juxtaposition of quantum mechanics and interactions drive new phenomena

Scattering theory

Ferromagnetism, consequences of interactions