

LECTURE 1

- Condensed matter physics juxtaposition of interactions and quantum mechanics
- Many body interactions : stars in galaxies
- Quantum mechanics - diffraction
- Combination gives counterintuitive phenomena.

- Real life solids have electrons travelling over lattice
- Basis sets for the electrons

Real space

Reciprocal space

- latter preferable for conducting freely moving electrons, known as itinerant
- Study physics, descriptors and consequences of interactions in these systems.

Non interacting system:

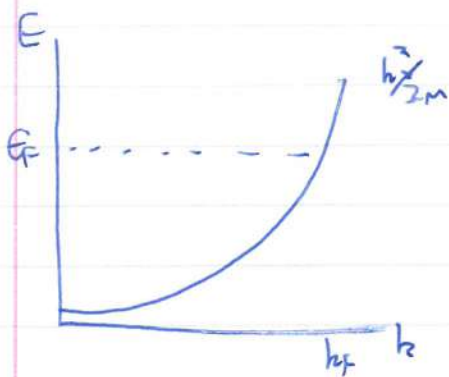
- Hamiltonian $\begin{vmatrix} e^{ik_1 r_{1T}} & e^{ik_2 r_{1T}} \\ e^{ik_1 r_{2T}} & e^{ik_2 r_{2T}} \end{vmatrix} \begin{vmatrix} e^{-ik_1 r_{1L}} & e^{-ik_1 r_{2L}} \\ e^{-ik_2 r_{1L}} & e^{-ik_2 r_{2L}} \end{vmatrix} = \begin{vmatrix} \phi(r_{1T}-r_{1L}) & \phi(r_{1T}-r_{2L}) \\ \phi(r_{2T}-r_{1L}) & \phi(r_{2T}-r_{2L}) \end{vmatrix}$

- Found in alkali metals, cold atom gases

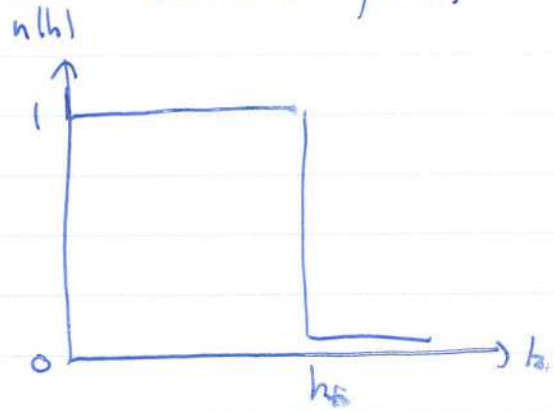
- Representation:
State determined
Scalr quantized

- kinetic energy $\frac{\frac{1}{(2\pi)^3} \int \frac{1}{2m} k^2 \cdot 4\pi k^2 \cdot dk}{\frac{1}{(2\pi)^3} \int 4\pi k^2 \cdot dk} = \frac{3}{5} E_F$

- Band structure



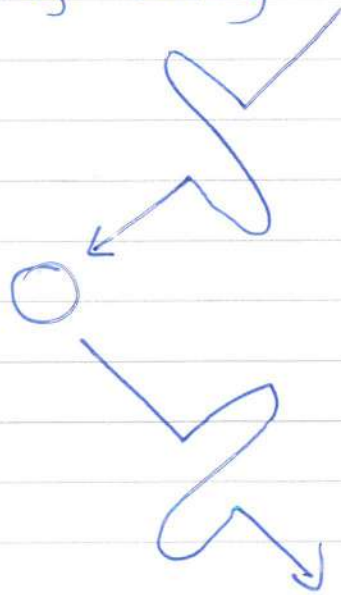
Distribution function



Weak interactions

- Hamiltonian, contact
- Found in screened electrons - heavy fermion materials
 - Opposite spin interactions only as fermions
- Found in cold atom gases - opposite spin only.

Two body scattering:



$$\hat{H} = -\frac{1}{2r^2} \frac{d}{dr} \left(r^2 \frac{d\psi}{dr} \right) + \frac{1}{2r^2 \sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{d\psi}{d\theta} \right) + \frac{1}{2r^2 \sin^2\theta} \frac{d^2\psi}{d\phi^2}$$

with $\psi = \frac{u}{r}$ get

$$\hat{H} = -\frac{1}{2} \frac{d^2 u}{dr^2}$$

Non-interacting eigenstate

$$\frac{\sin kr}{r}$$

With interaction

$$\psi = \frac{\sin(kr + \delta)}{r}$$

$$r\psi = \sin(kr + \delta)$$

$$= \sin kr \cdot \cos\delta + \cos kr \cdot \sin\delta$$

$$= \sin\delta \left(\cot\delta \sin kr + \cos kr \right)$$

$$\sim \sin\delta \left(\cot\delta \cdot kr + 1 \right)$$

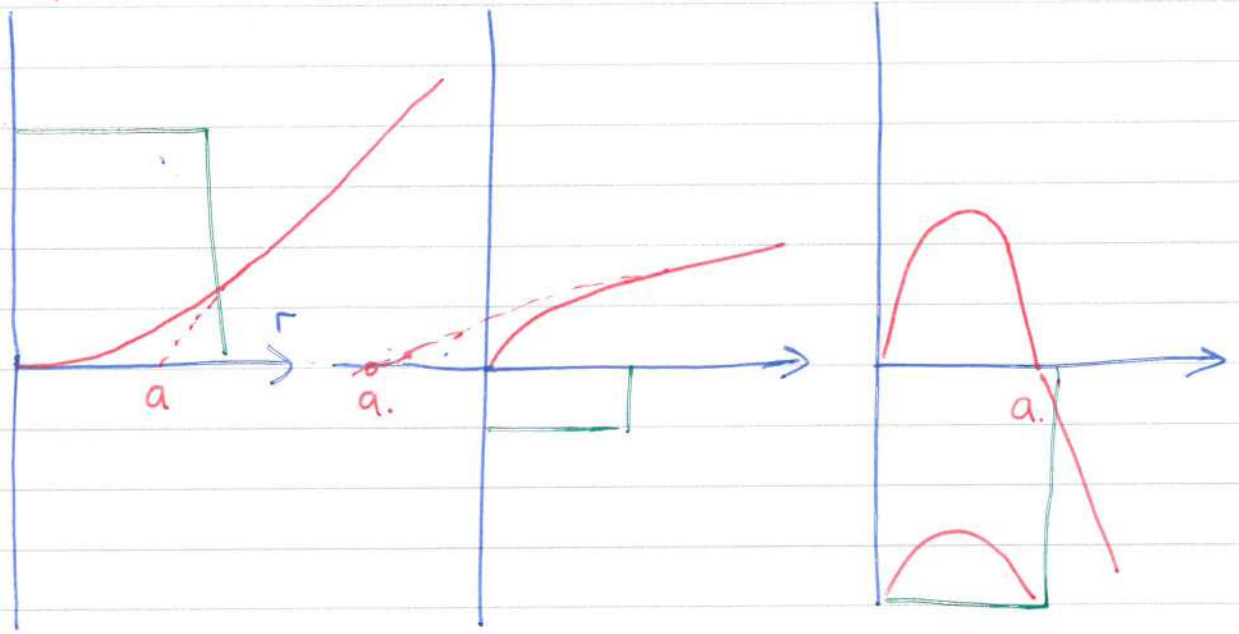
for small $k \ll \frac{1}{r}$

look at node; call a

$$\cot\delta = -\frac{1}{ka}$$

Graphical meaning: node is ψ .

✓ $r \psi$



Can make effective repulsive interaction with bound state underlying attractive well, but get

To enter bound state that has a long relaxation time / Pauli blocked. need 3-body interaction

Real cold atoms: ground state is lump of solid.

Next order term, effective range:

$$\cot \delta = -\frac{1}{ka} + \frac{1}{2} k r_{\text{eff}}$$