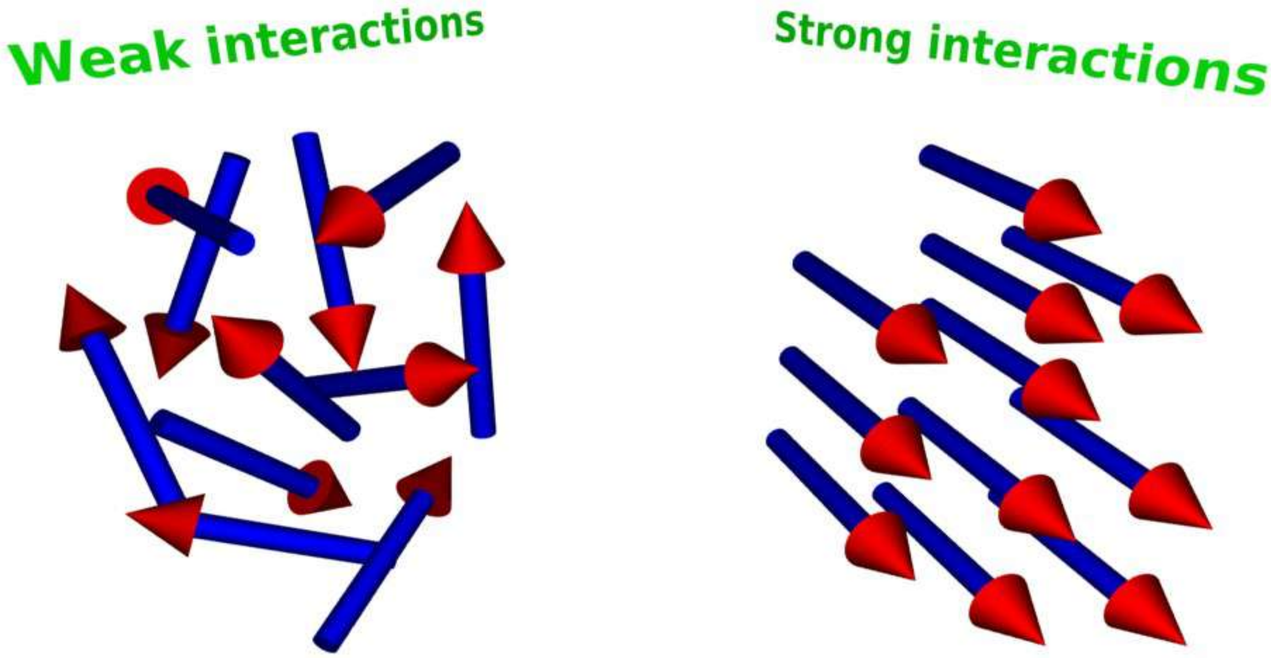
A repulsive atomic gas on the border of itinerant ferromagnetism



Gareth Conduit[1, 2] & Ben Simons[3]

1. Weizmann Institute of Science, 2. Ben Gurion University, 3. University of Cambridge

G.J. Conduit & B.D. Simons, Phys. Rev. A **79**, 053606 (2009)

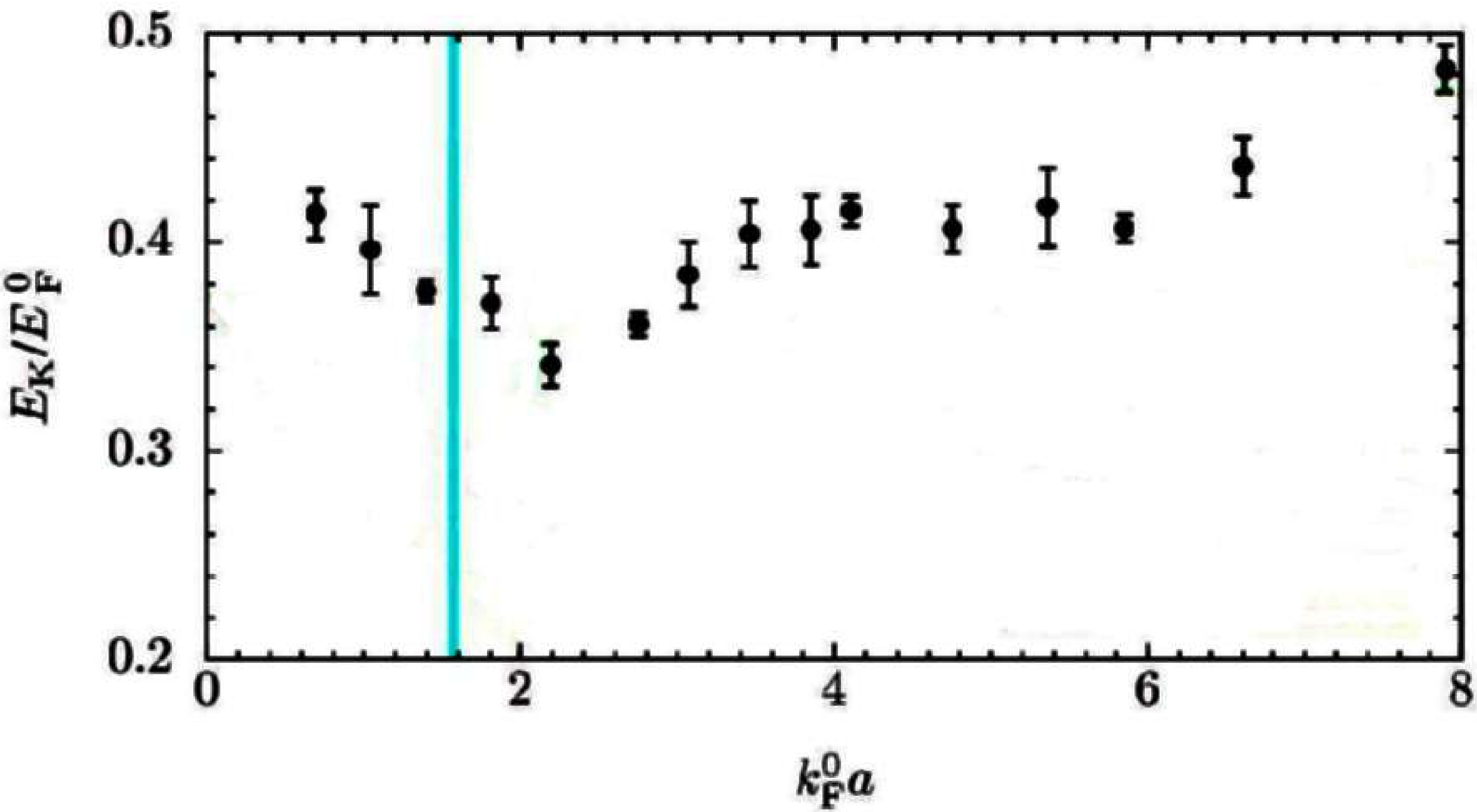
G.J. Conduit, A.G. Green & B.D. Simons, Phys. Rev. Lett. **103**, 207201 (2009)

G.J. Conduit & B.D. Simons, Phys. Rev. Lett. **103**, 200403 (2009)

G.J Conduit & E. Altman, arXiv: 0911.2839

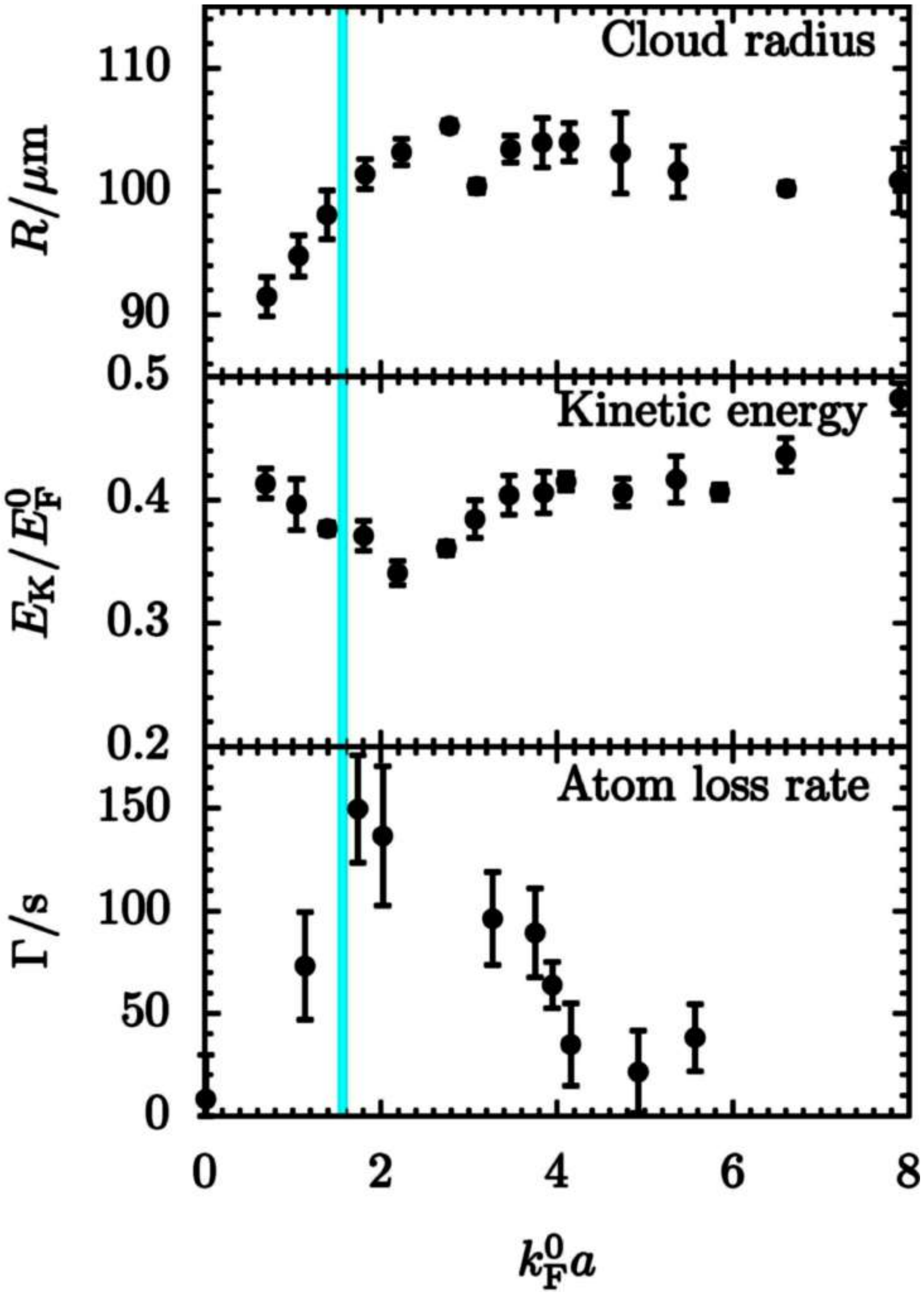
Experimental evidence for ferromagnetism

- Minimum in kinetic energy at $k_F a \approx 2.2$



Jo, Lee, Choi, Christensen, Kim, Thywissen, Pritchard & Ketterle, Science **325**, 1521 (2009)

Further key experimental signatures



$$E_K \propto n^{5/3}$$

$$\Gamma \propto (k_F a)^6 n_\uparrow n_\downarrow (n_\uparrow + n_\downarrow)$$

Jo, Lee, Choi, Christensen, Kim, Thywissen, Pritchard & Ketterle, Science **325**, 1521 (2009)

Equilibrium study of ferromagnetism

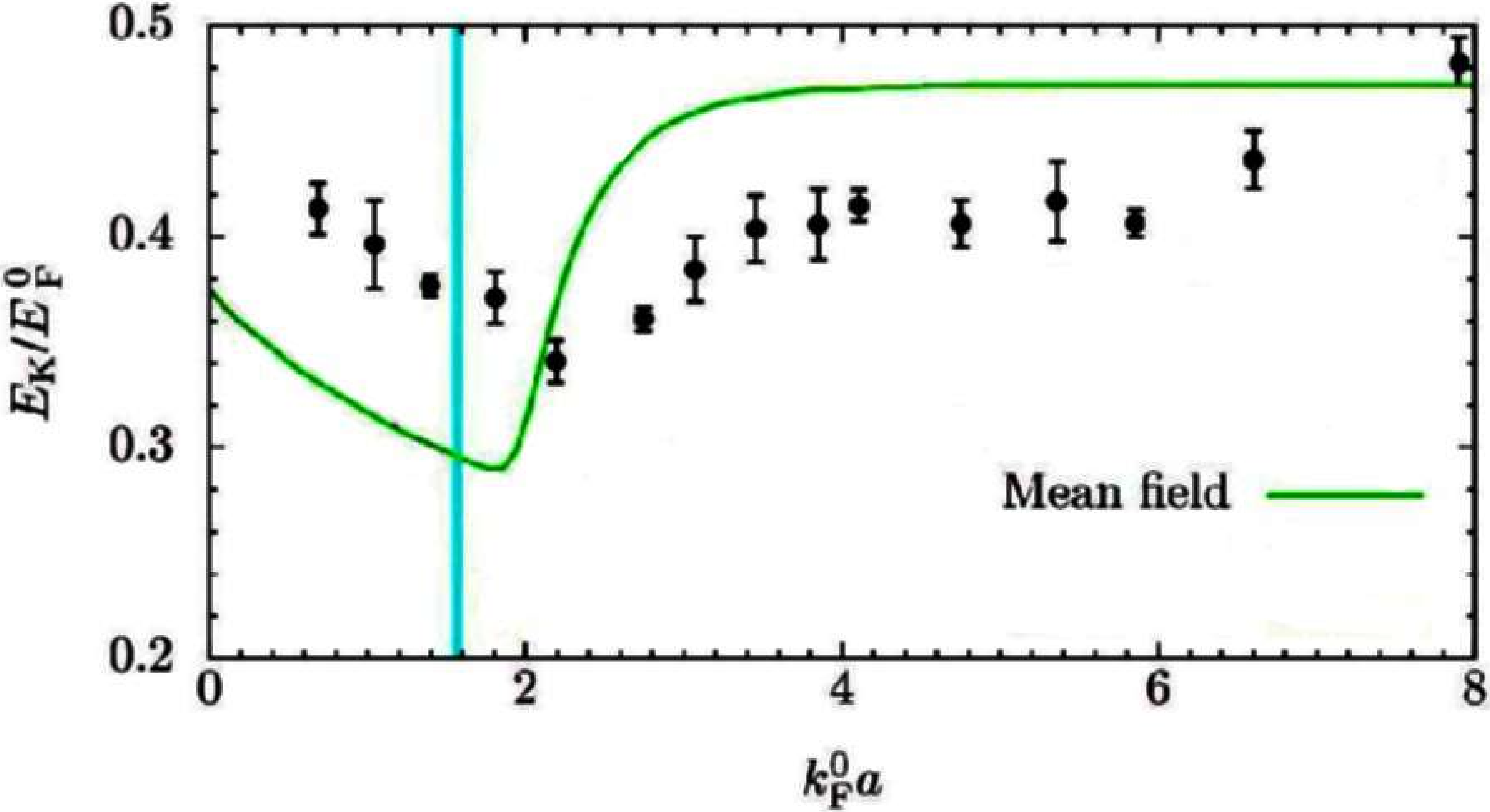
$$Z=\int D\psi \exp\left(-\iint d\tau\, d\boldsymbol{r} \sum_{\sigma} \bar{\psi}_{\sigma}(-i\hat{\omega}+\hat{\epsilon}-\mu)\psi_{\sigma}-g\,\bar{\psi}_{\uparrow}\bar{\psi}_{\downarrow}\psi_{\downarrow}\psi_{\uparrow}\right)$$

- Decouple with the average magnetisation *m* gives the Stoner criterion

$$F=F_0+\frac{1-g\nu}{2\nu}m^2+um^4+vm^6$$

Mean-field analysis & consequences of trap

- Correct qualitative behavior[1] but transition at $k_F a \approx 1.8$ instead of $k_F a \approx 2.2$



[1]LeBlanc, Thywissen, Burkov & Paramekanti, Phys. Rev. A **80**, 013607 (2009) & Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

Fluctuation corrections

$$Z=\int D\psi \exp\left(-\iint d\tau\, d\mathbf{r} \sum_{\sigma} \bar{\psi}_{\sigma}(-i\hat{\omega}+\hat{\epsilon}-\mu)\psi_{\sigma} - g\,\bar{\psi}_{\uparrow}\bar{\psi}_{\downarrow}\psi_{\downarrow}\psi_{\uparrow}\right)$$

- Integrate over fluctuations in magnetization and density and develop a perturbation theory in interaction strength

$$F=F_0+\frac{1-g\nu}{2\nu}m^2+um^4+vm^6+g^2\left(r\,m^2+w\,m^4\ln|m|\right) \qquad k_F a_{\text{crit}}=1.05$$

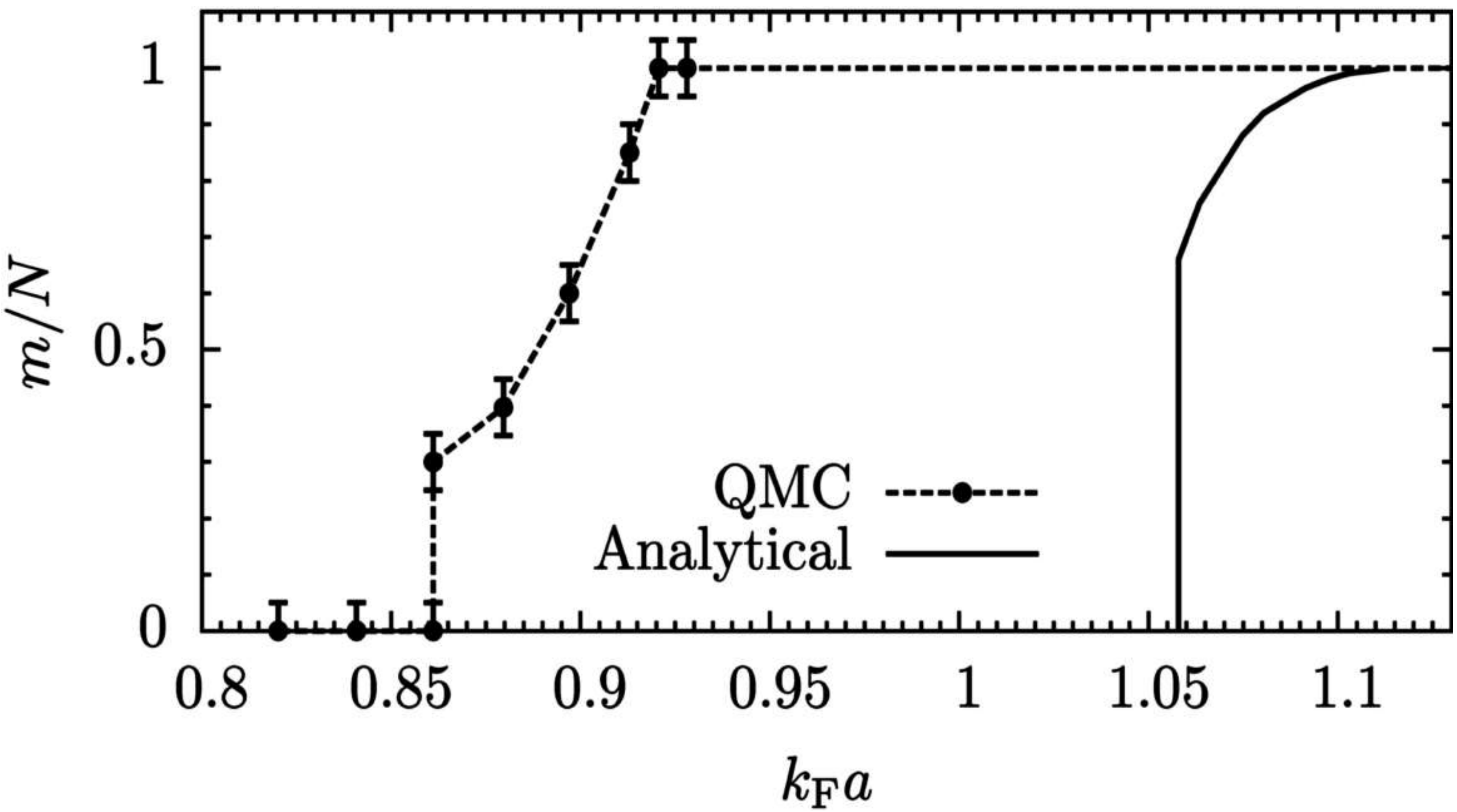
- First order transition[1]

[1]Abrikosov (1958), Duine & MacDonald (2005), Belitz *et al*. (1997) & Conduit & Simons (2009)

Quantum Monte Carlo

$$F = F_0 + \frac{1-g\nu}{2\nu} m^2 + u m^4 + v m^6 + g^2 \left(r\, m^2 + w\, m^4 \ln|m| \right) \qquad k_{\mathrm{F}} a_{\mathrm{crit}} = 1.05$$

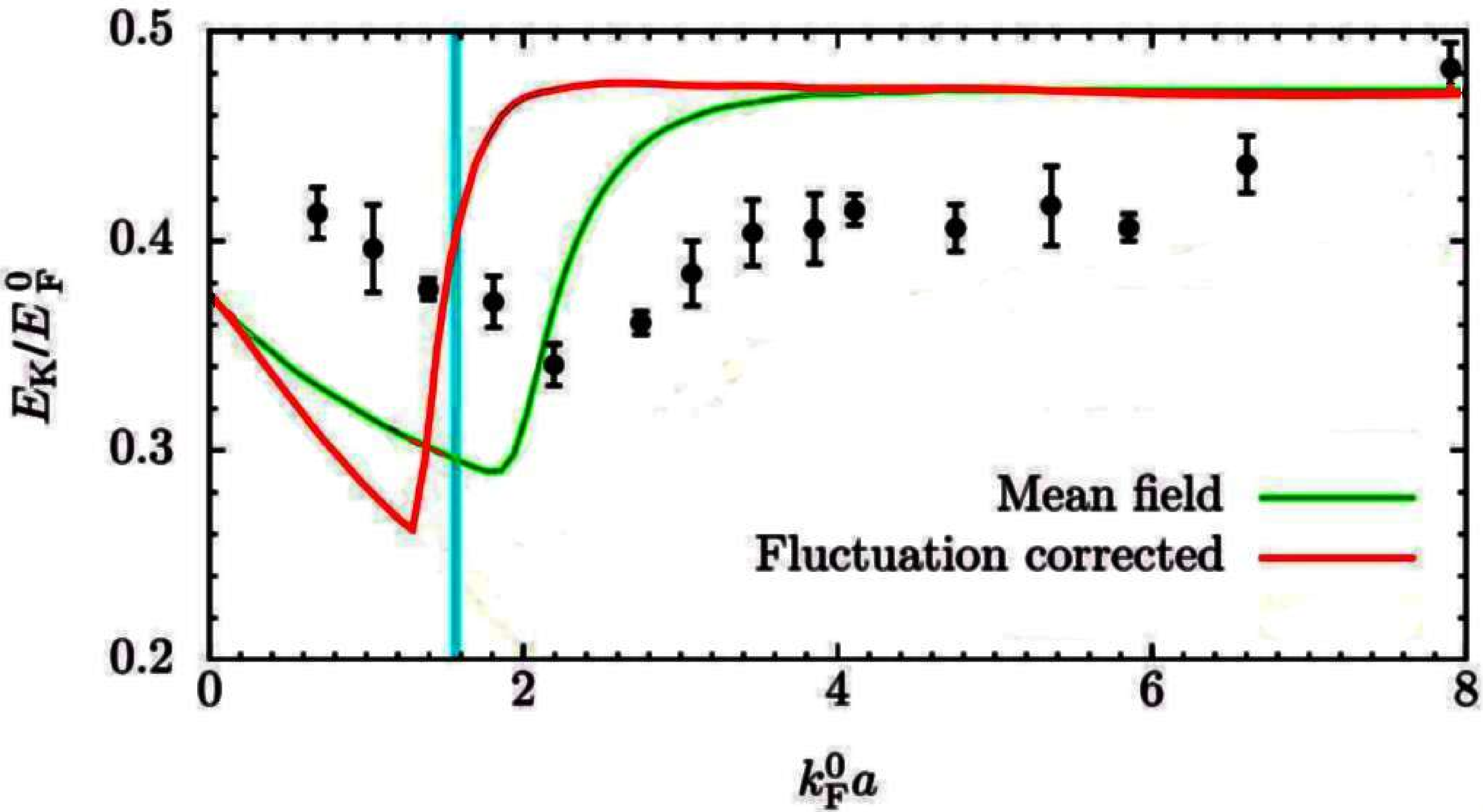
- First order transition verified with *ab initio* Quantum Monte Carlo[2]



[1]Conduit & Simons, Phys. Rev. Lett. **103**, 207201 (2009)

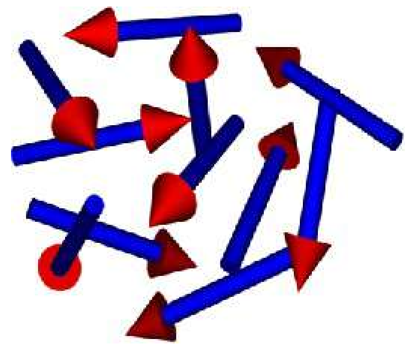
Fluctuation corrections in trap

- Extend theory through fluctuation corrections

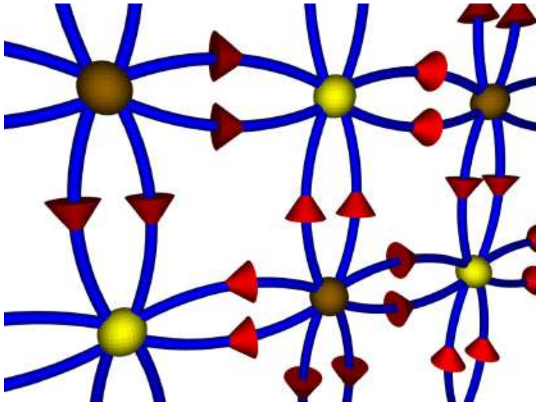


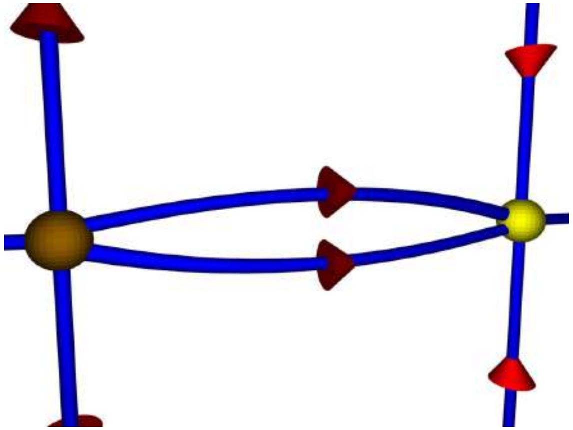
Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

Condensation of topological defects



Ramp up interactions



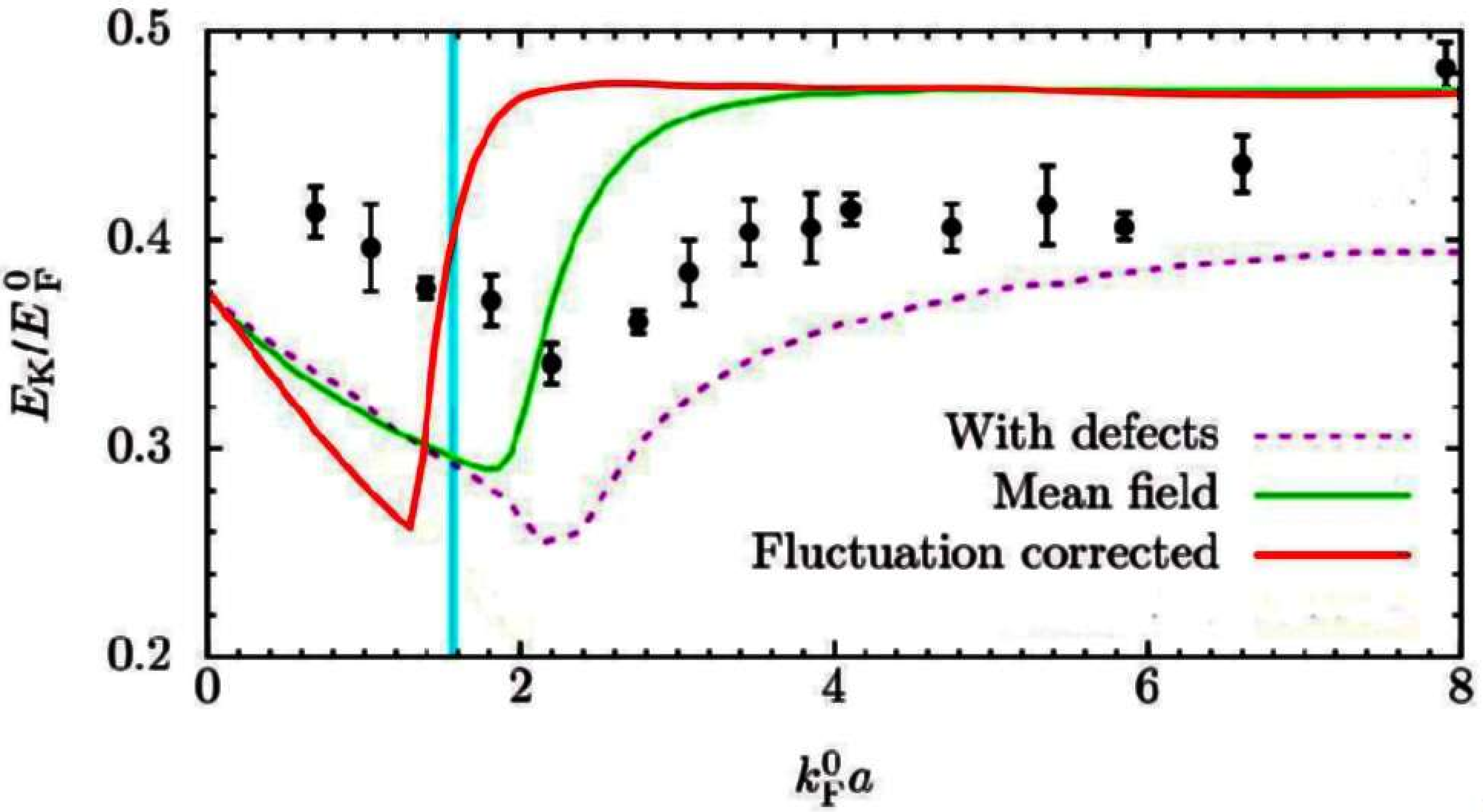


Mutual annihilation of defects

- Defects freeze out from paramagnetic state
- Defects grow as $L \sim t^{1/2}$
  [Bray, Adv. Phys. **43**, 357 (1994)]

Consequences of defect annihilation

- Defect annihilation raises required interaction strength



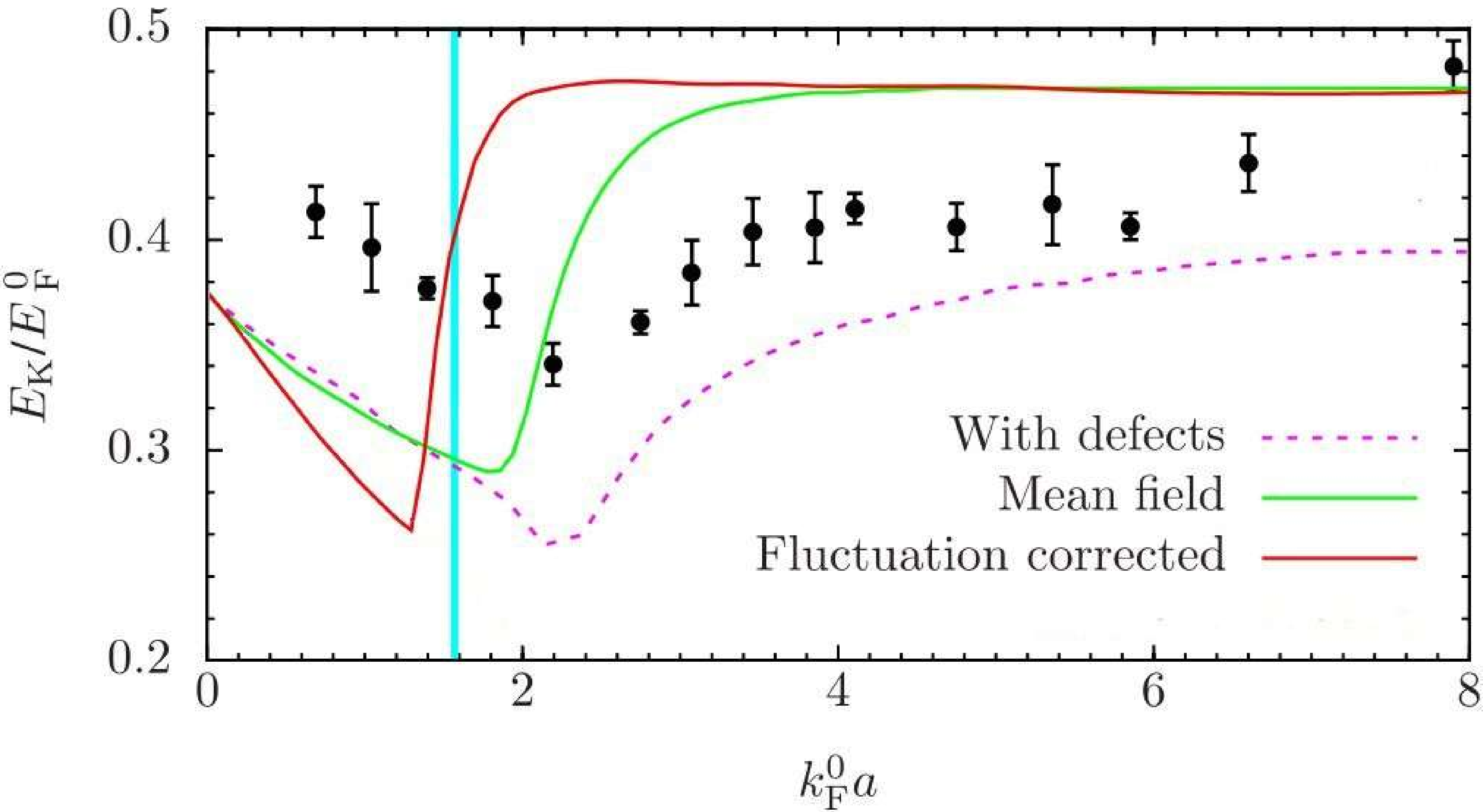
Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009), Babadi *et al*. arXiv 0908.3482 (2009)

Summary

- Mean-field theory provides a reasonable qualitative description of the transition
- Fluctuation corrections drive the transition first order, backed up by Quantum Monte Carlo calculations
- Discrepancy in the interaction strength could be accounted for by the non-equilibrium formation of the ferromagnetic phase
- Atom loss can damp fluctuations and renormalize the interaction strength upwards [Y3100014]

Condensation of topological defects

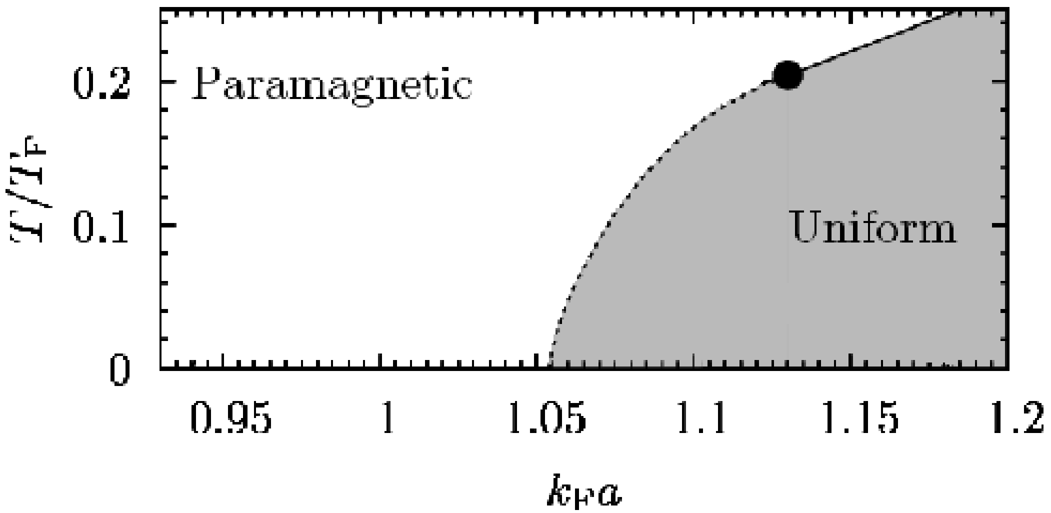
- Condensation of defects inhibits the transition



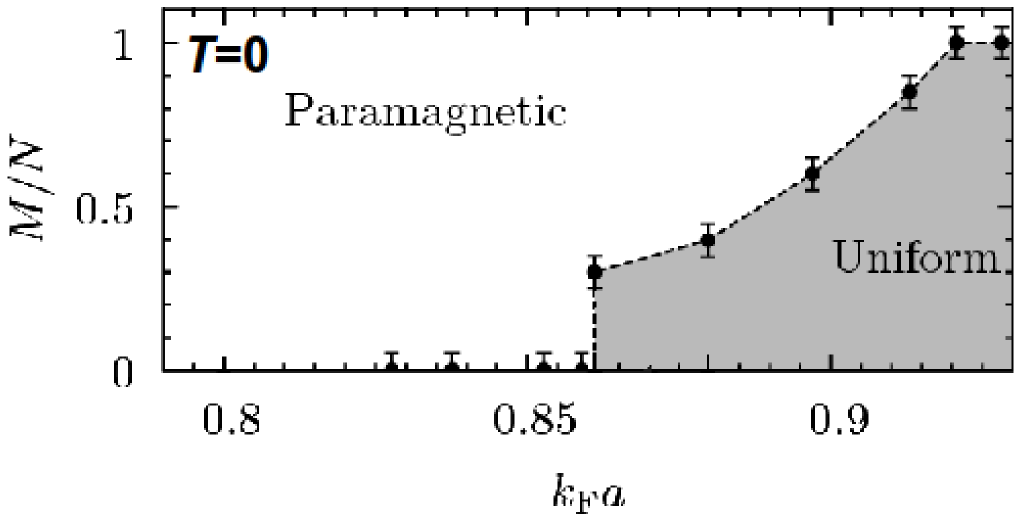
Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

First order phase transition and Quantum Monte Carlo verification

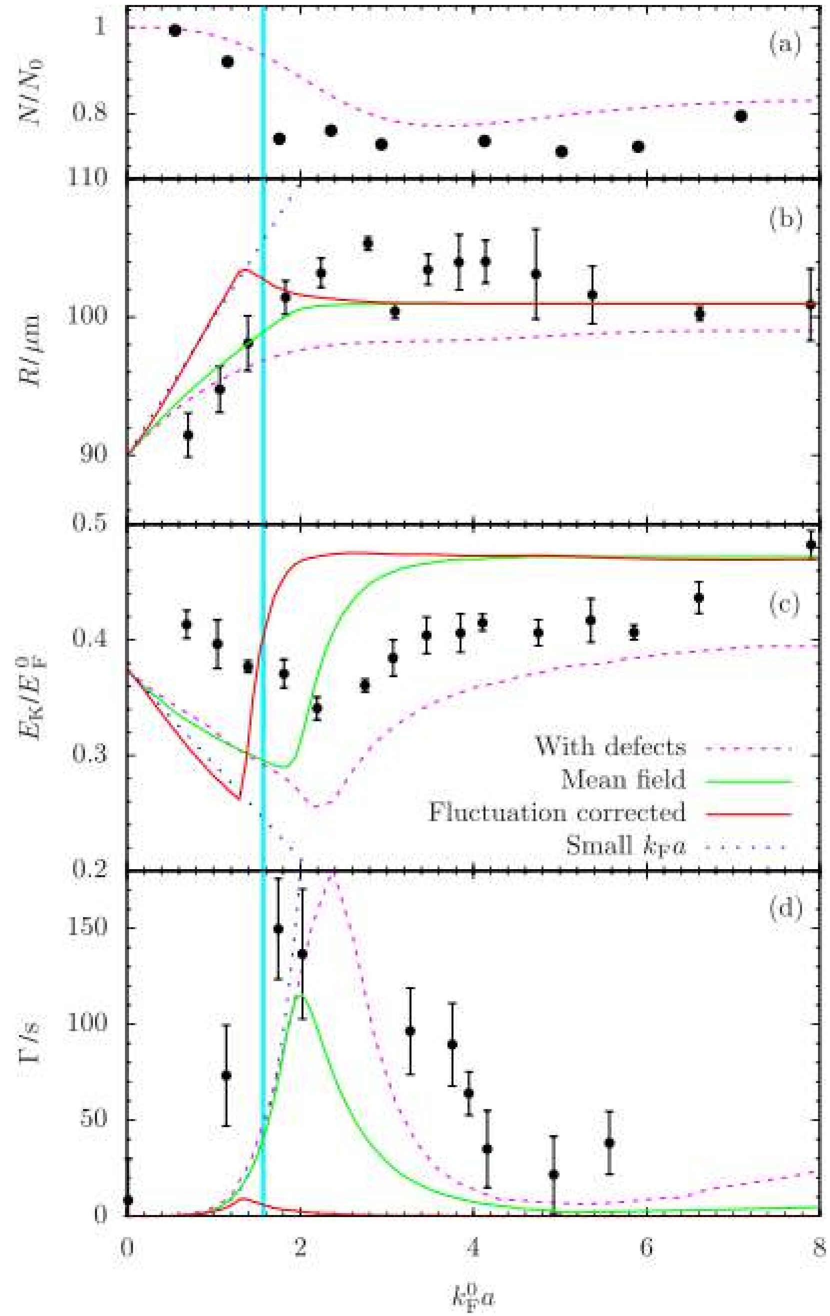
- First order transition into uniform phase with TCP



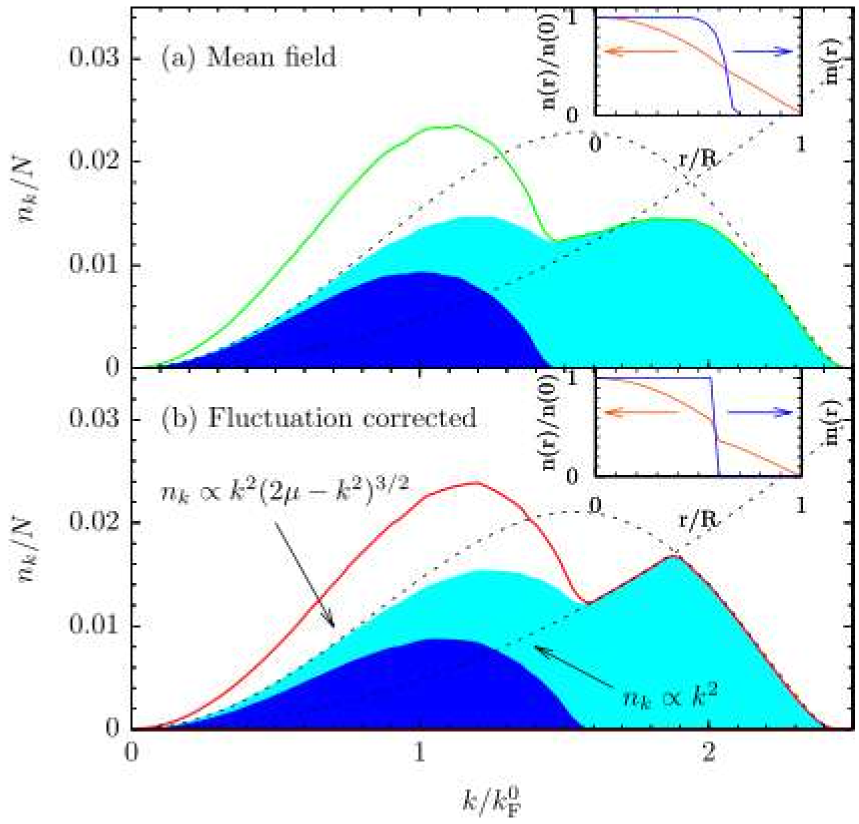
- QMC also sees first order transition



Summary of equilibrium results



Momentum distribution



New approach to fluctuation corrections

$$Z = \int D\psi \exp\left(-\int \sum_{\sigma} \bar{\psi}_{\sigma} (-i\omega + \epsilon - \mu) \psi_{\sigma} - g \int \bar{\psi}_{\uparrow} \bar{\psi}_{\downarrow} \psi_{\downarrow} \psi_{\uparrow} \right)$$

- Analytic strategy:

  1) Decouple in both the density and spin channels (previous approaches employ only spin)

  2) Integrate out electrons

  3) Expand about uniform magnetisation

  4) Expand density and magnetisation fluctuations to second order

  5) Integrate out density and magnetisation fluctuations
- Aim to uncover connection to second order perturbation theory
- Unravel origin of logarithmic divergence in energy and relation to tricritical point structure

Analytical method

- System free energy $F=-k_B T \ln Z$ is found via the partition function

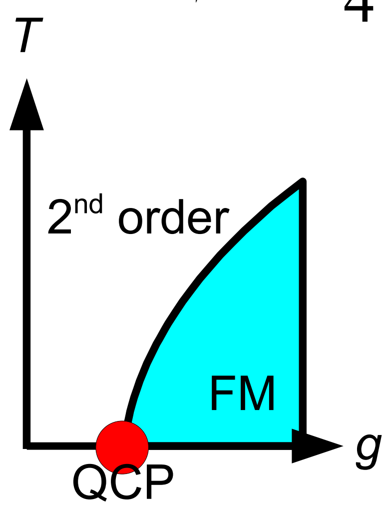
$$Z=\int D\psi \exp\left(-\int \sum_\sigma \bar{\psi}_\sigma(-i\omega+\epsilon-\mu)\psi_\sigma - g\int \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow\right)$$

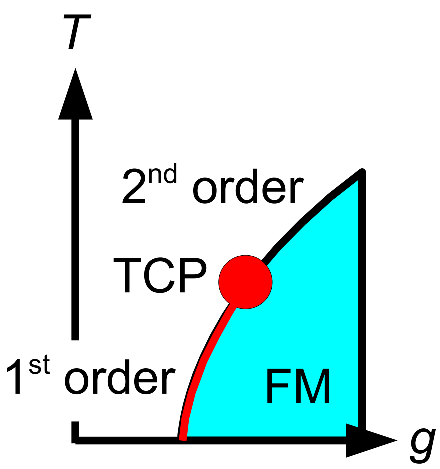
- Decouple using only the average magnetisation $m=\bar{\psi}_\downarrow \psi_\uparrow - \bar{\psi}_\uparrow \psi_\downarrow$

  gives $F \propto (1-g\nu)m^2$ i.e. the Stoner criterion

- Belitz-Kirkpatrick-Vojta (soft particle-hole & magnetisation) [Belitz *et al.* Z. Phys. B 1997]

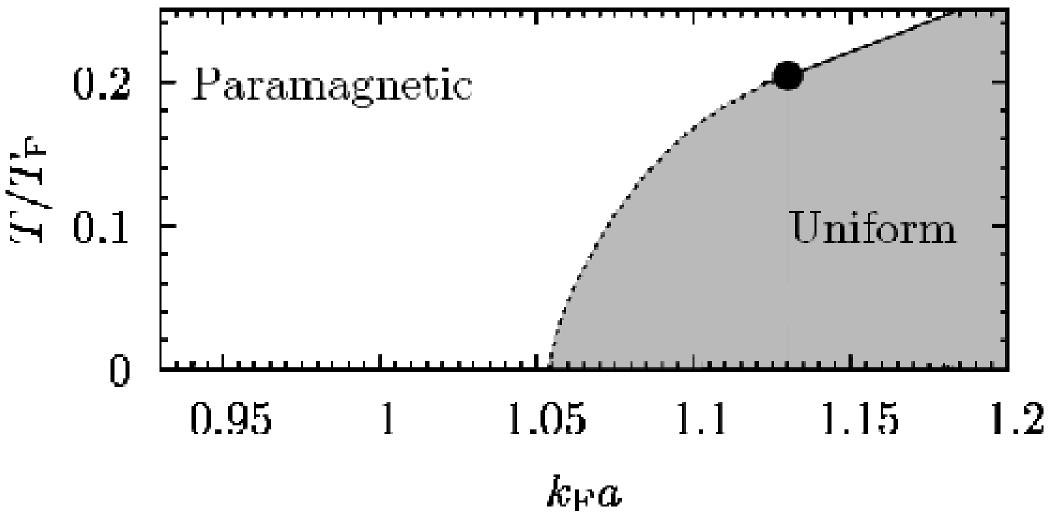
$$F=\frac{1}{2}\left(|\omega|/\Gamma_q + r + q^2\right)m^2 + \frac{u}{4}m^4 \ln(m^2+T^2) + \cdots - hm$$



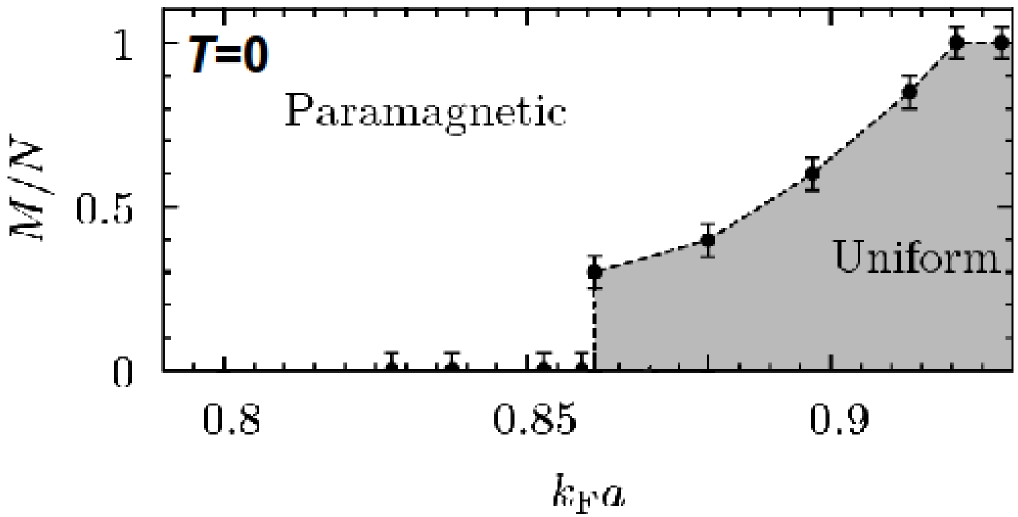


Quantum Monte Carlo verification

- First order transition into uniform phase with TCP



- QMC also sees first order transition



Cold atomic gases — spin

- Two fermionic atom species have a *pseudo-spin*:

^{6}Li	m_F=1/2	maps to	spin 1/2
^{6}Li	m_F=-1/2	maps to	spin -1/2

- The up-and down spin particles *cannot* interchange — population imbalance is fixed. Possible spin states are:

$\left\|\uparrow\uparrow\right\rangle$	S=1, S_z=1	State not possible as S_z has changed
$\left\|\downarrow\downarrow\right\rangle$	S=1, S_z=-1	State not possible as S_z has changed
$\left(\left\|\uparrow\downarrow\right\rangle+\left\|\downarrow\uparrow\right\rangle\right)/\sqrt{2}$	S=1, S_z=0	Magnetic moment in plane
$\left(\left\|\uparrow\downarrow\right\rangle-\left\|\downarrow\uparrow\right\rangle\right)/\sqrt{2}$	S=0, S_z=0	Non-magnetic state

- Ferromagnetism, if favourable, must form in-plane

Particle-hole perspective

- To second order in g the free energy is

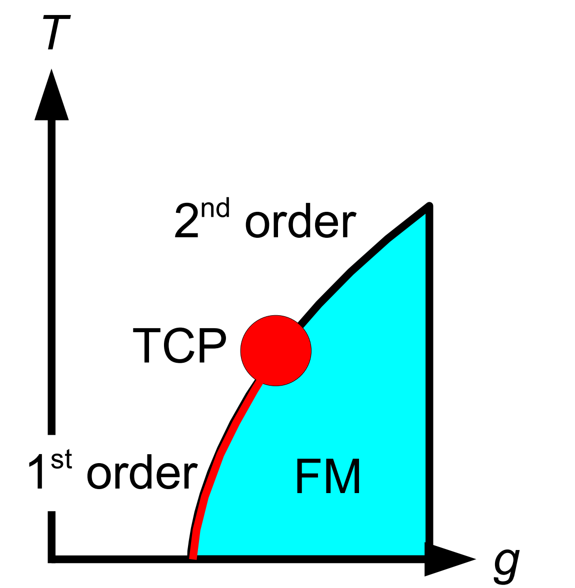
$$F=\sum_{\sigma,\boldsymbol{k}}\epsilon_{\boldsymbol{k}}^{\sigma}n(\epsilon_{\boldsymbol{k}}^{\sigma})+gN^{\uparrow}N^{\downarrow}$$

$$-\frac{2g^2}{V^3}\sum_{\boldsymbol{p}}\int\int\frac{\rho^{\uparrow}(\boldsymbol{p},\epsilon_{\uparrow})\rho^{\downarrow}(-\boldsymbol{p},\epsilon_{\downarrow})}{\epsilon_{\uparrow}+\epsilon_{\downarrow}}d\epsilon_{\uparrow}d\epsilon_{\downarrow}$$

$$+\frac{2g^2}{V^3}\sum_{\boldsymbol{k}_{1,2,3,4}}\frac{n(\epsilon_{\boldsymbol{k}_1}^{\uparrow})n(\epsilon_{\boldsymbol{k}_2}^{\downarrow})}{\epsilon_{\boldsymbol{k}_1}^{\uparrow}+\epsilon_{\boldsymbol{k}_2}^{\downarrow}-\epsilon_{\boldsymbol{k}_3}^{\uparrow}-\epsilon_{\boldsymbol{k}_4}^{\downarrow}}\delta(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}_3-\boldsymbol{k}_3)$$

with $\epsilon_{\boldsymbol{k}}^{\sigma}=\epsilon_{\boldsymbol{k}}+\sigma gm$ and a particle-hole density of states

$$\rho^{\sigma}(\boldsymbol{p},\epsilon)=\sum_{\boldsymbol{k}}n(\epsilon_{\boldsymbol{k}+\boldsymbol{p}/2}^{\sigma})\left[1-n(\epsilon_{\boldsymbol{k}-\boldsymbol{p}/2}^{\sigma})\right]\delta\left(\epsilon-\epsilon_{\boldsymbol{k}+\boldsymbol{p}/2}^{\sigma}+\epsilon_{\boldsymbol{k}-\boldsymbol{p}/2}^{\sigma}\right)$$

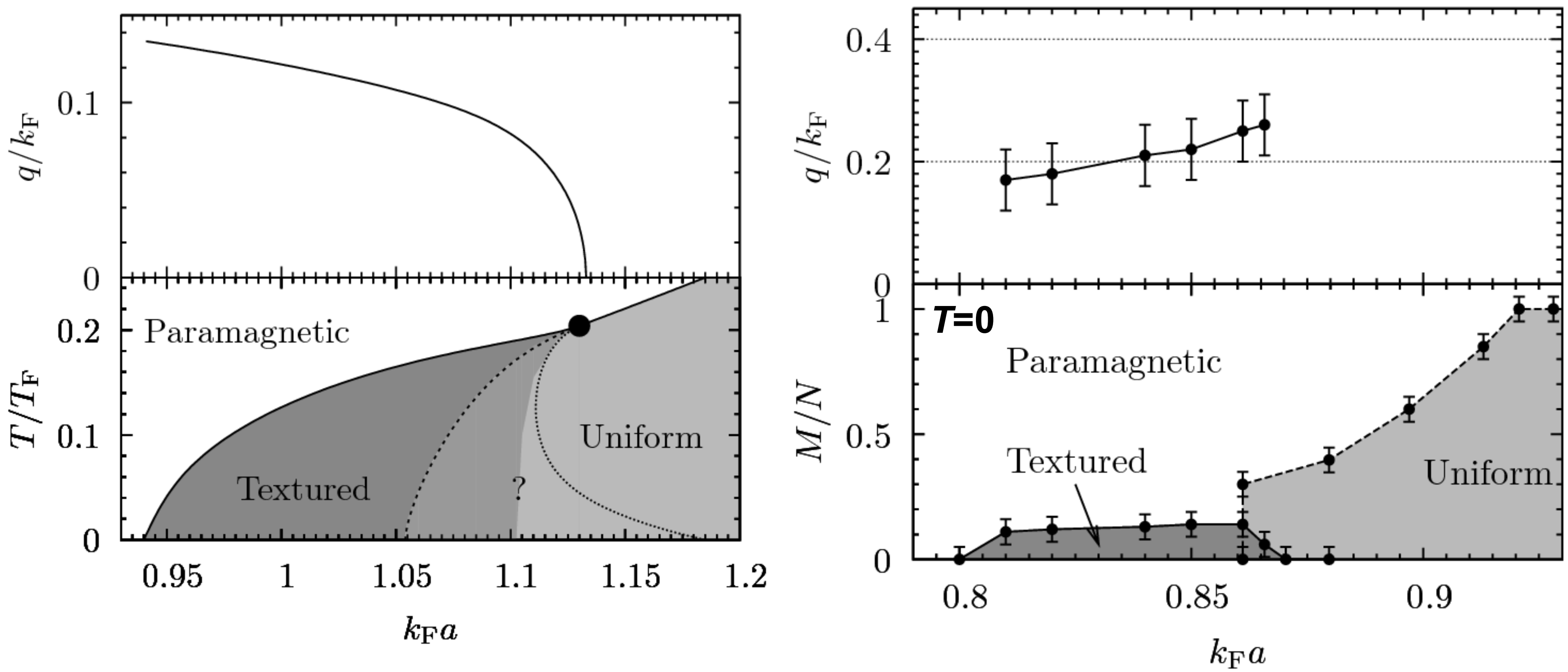


- Enhanced particle-hole phase space at zero magnetisation drives transition first order
- Recover $m^4 \ln m^2$ at T=0
- Links quantum fluctuation to second order perturbation approach[1]

[1]Abrikosov 1958 & Duine & MacDonald 2005

Quantum Monte Carlo: Textured phase

- Textured phase preempted transition with q=0.2k_F



Modified collective modes

- Collective mode dispersion

$$\Omega = \frac{q^2}{2}\left(1 - \frac{2^{5/3}3}{5k_{\mathrm{F}}a}\frac{1}{1+\tilde{\lambda}^2/(k_{\mathrm{F}}a)^2}\right)$$

- Collective mode damping

$$\Gamma = \frac{q^2}{2}\frac{2^{5/3}3\tilde{\lambda}}{5(k_{\mathrm{F}}a)^2}\frac{1}{1+\tilde{\lambda}^2/(k_{\mathrm{F}}a)^2}$$