Space time

Between two events A, B

$$\Delta x = \gamma\left(\Delta x' + v\Delta t'\right) \qquad \Delta x' = \gamma\left(\Delta x - v\Delta t\right)$$

$$\Delta t = \gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right) \qquad \Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

Frame S' moves at $+v$ w.r.t S

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \geq 1$$

$$c^2\Delta t^2 - \Delta x^2 = c^2\Delta t'^2 - \Delta x'^2$$

Identify one quantity do not want to know and another $= 0$ eg

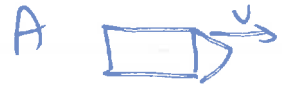
length contraction : $\Delta t = 0$

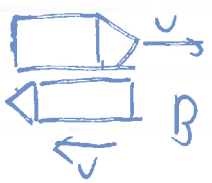
gives $\Delta x = \frac{\Delta x'}{\gamma}$

time dilation : $\Delta x' = 0$

gives $\Delta t' = \frac{\Delta t}{\gamma}$

twin paradox





$\Delta x = 0$
$\Delta t = 0$
so $\Delta t' = 0$

$\Delta x'_{AB} = 0 \qquad \Delta t'_{AB} = \Delta t_{AB} / \gamma$

$\Delta x'_{BC} = 0 \qquad \Delta t'_{BC} = \Delta t_{BC} / \gamma$

$$\Delta t'_{AC} = \Delta t'_{AB} + \Delta t'_{BC}$$

$$= \frac{\Delta t_{AB} + \Delta t_{BC}}{\gamma}$$

$$= \frac{\Delta t_{AC}}{\gamma}.$$

- Mass energy

$E^2 = p^2c^2 + m^2c^4$ $\qquad \frac{p}{E} = \frac{v}{c^2}$

$E = \gamma mc^2$, $p = \gamma mv$ $\qquad \rightarrow E = pc$ for photon

$KE = (\gamma - 1)mc^2$ $\qquad E^2 - p^2c^2$ Frame invariant.

- Conservation of energy, rest mass, momentum means best to avoid equations containing γ.