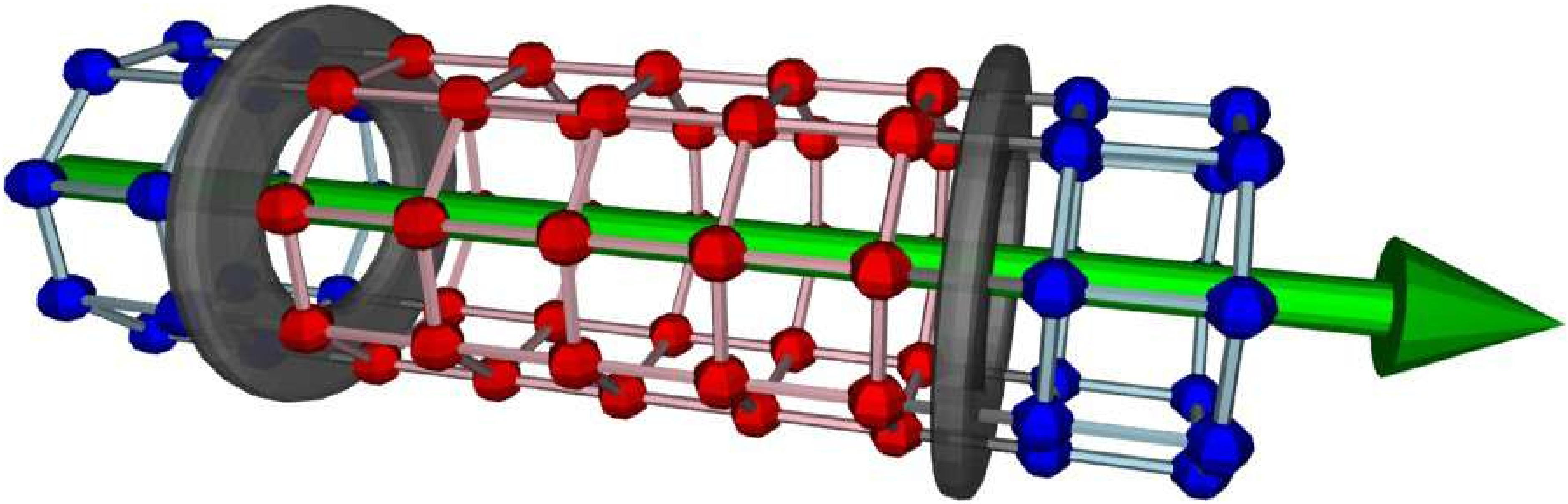
Modelling the magnetoresistance of disordered superconducting films

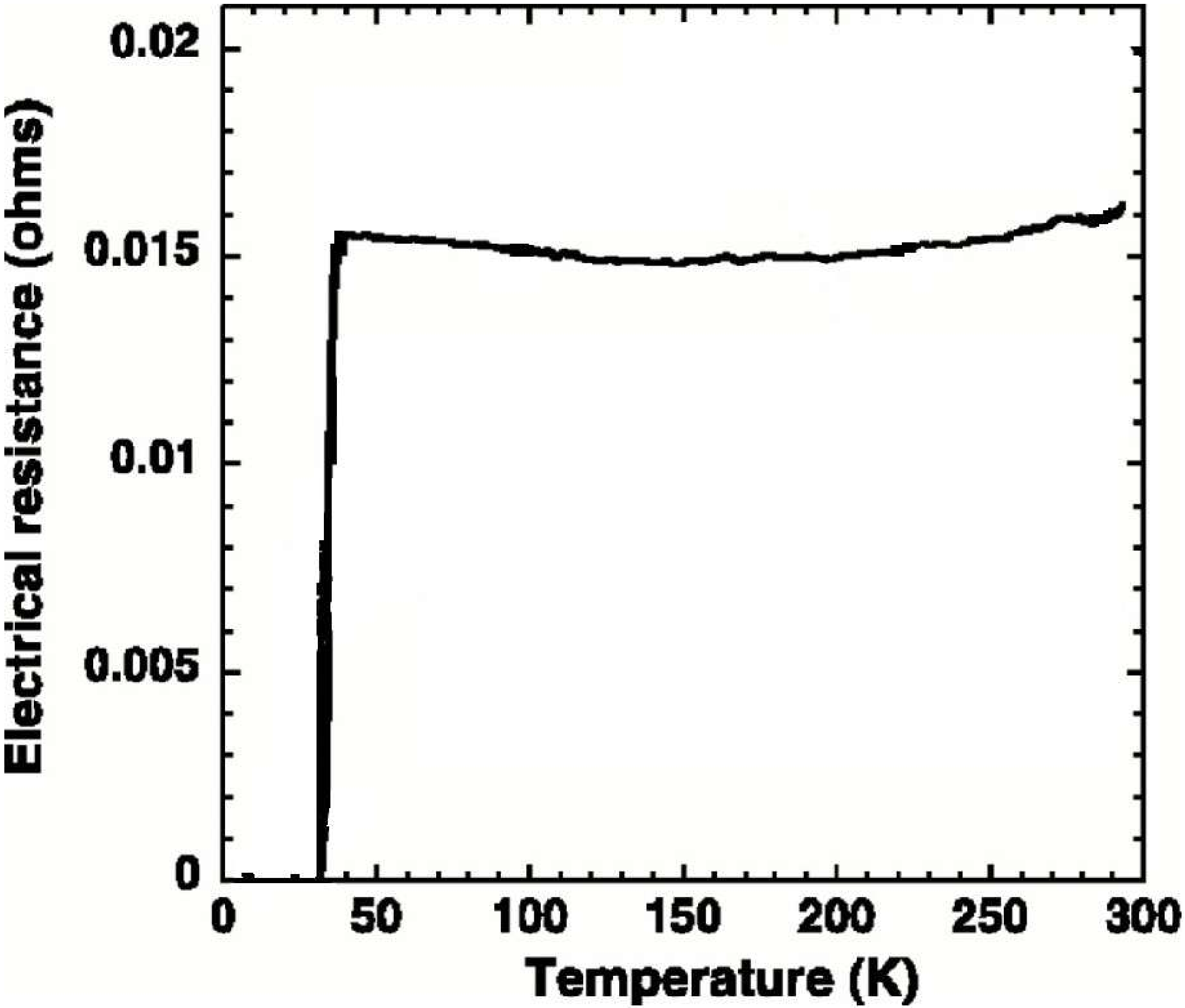


Gareth Conduit, Yigal Meir

University of Cambridge & Ben Gurion University

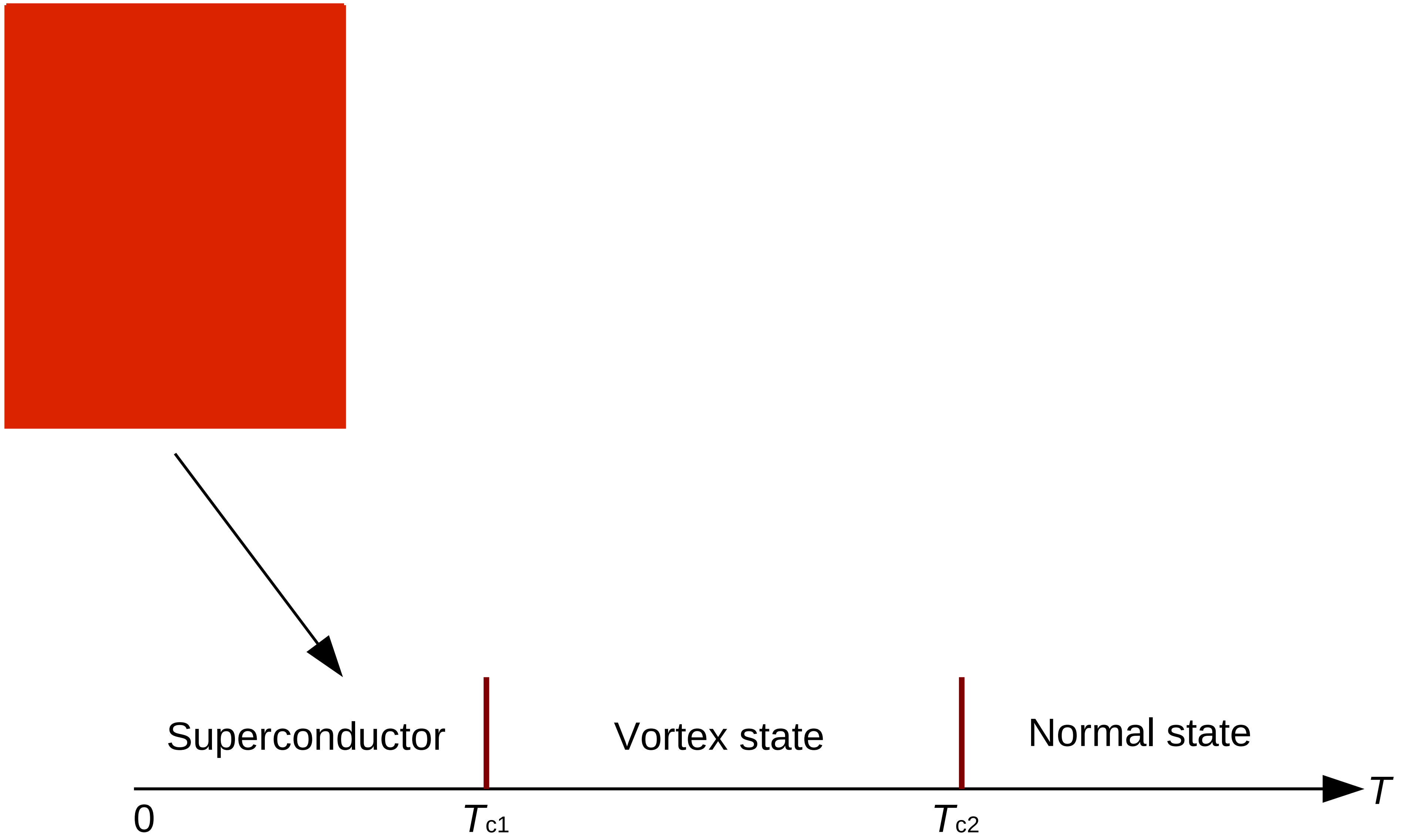
PRB **84**, 064513 (2011); accepted for publication in Phys. Rev. Lett.

BCS superconductivity in MgB_2



Monteverde et al., Science **292**, 75 (2001)

Kosterlitz Thouless transition



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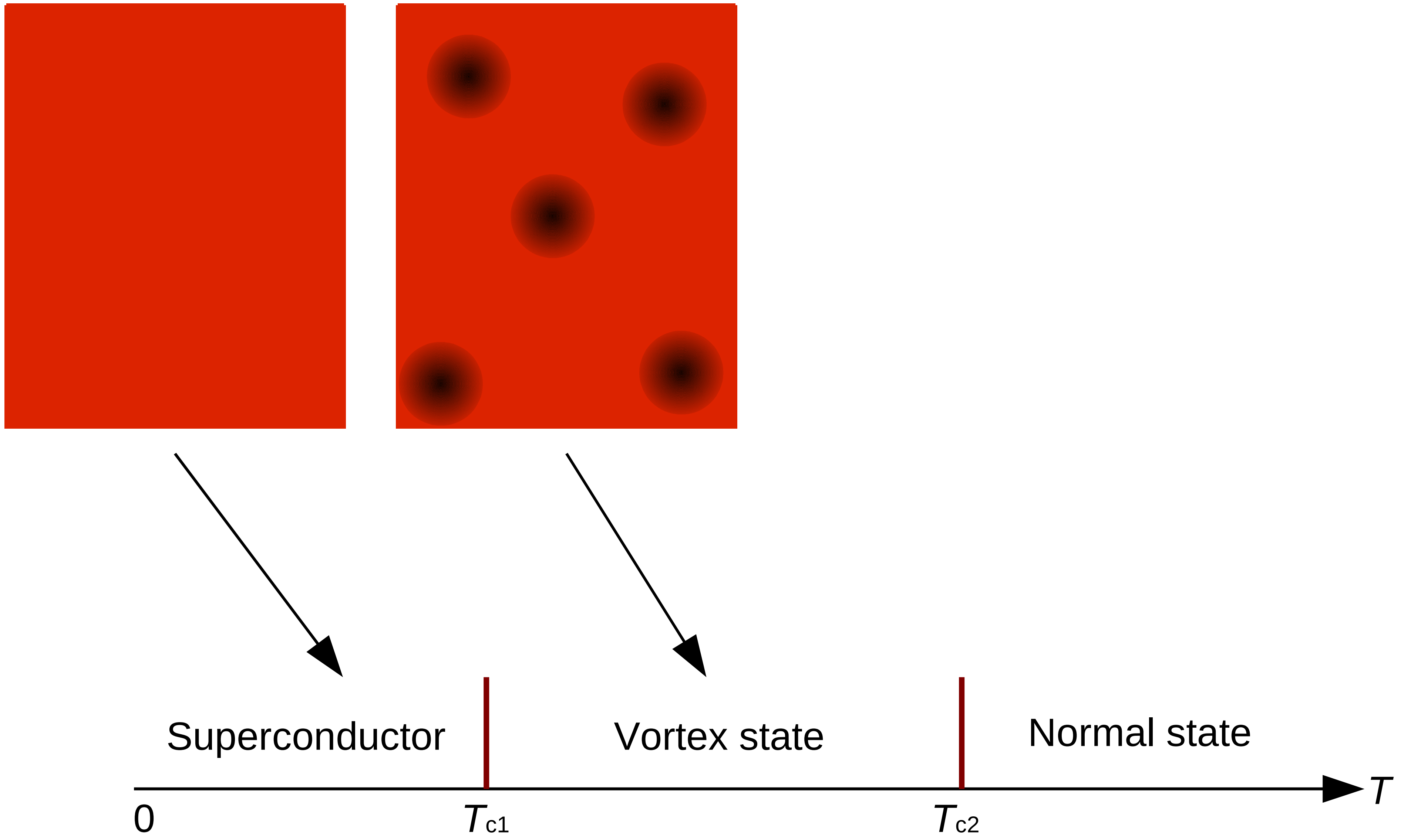

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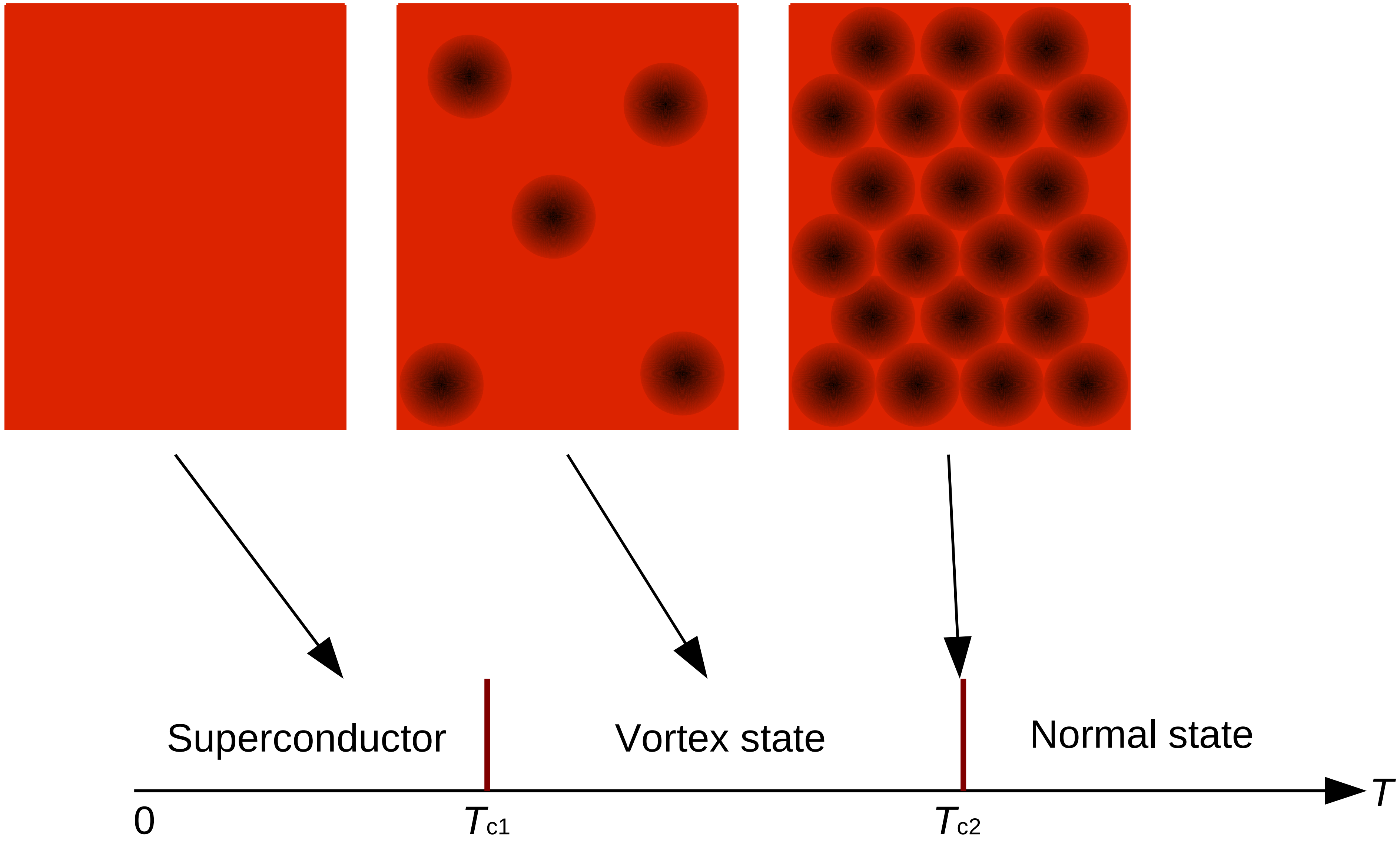

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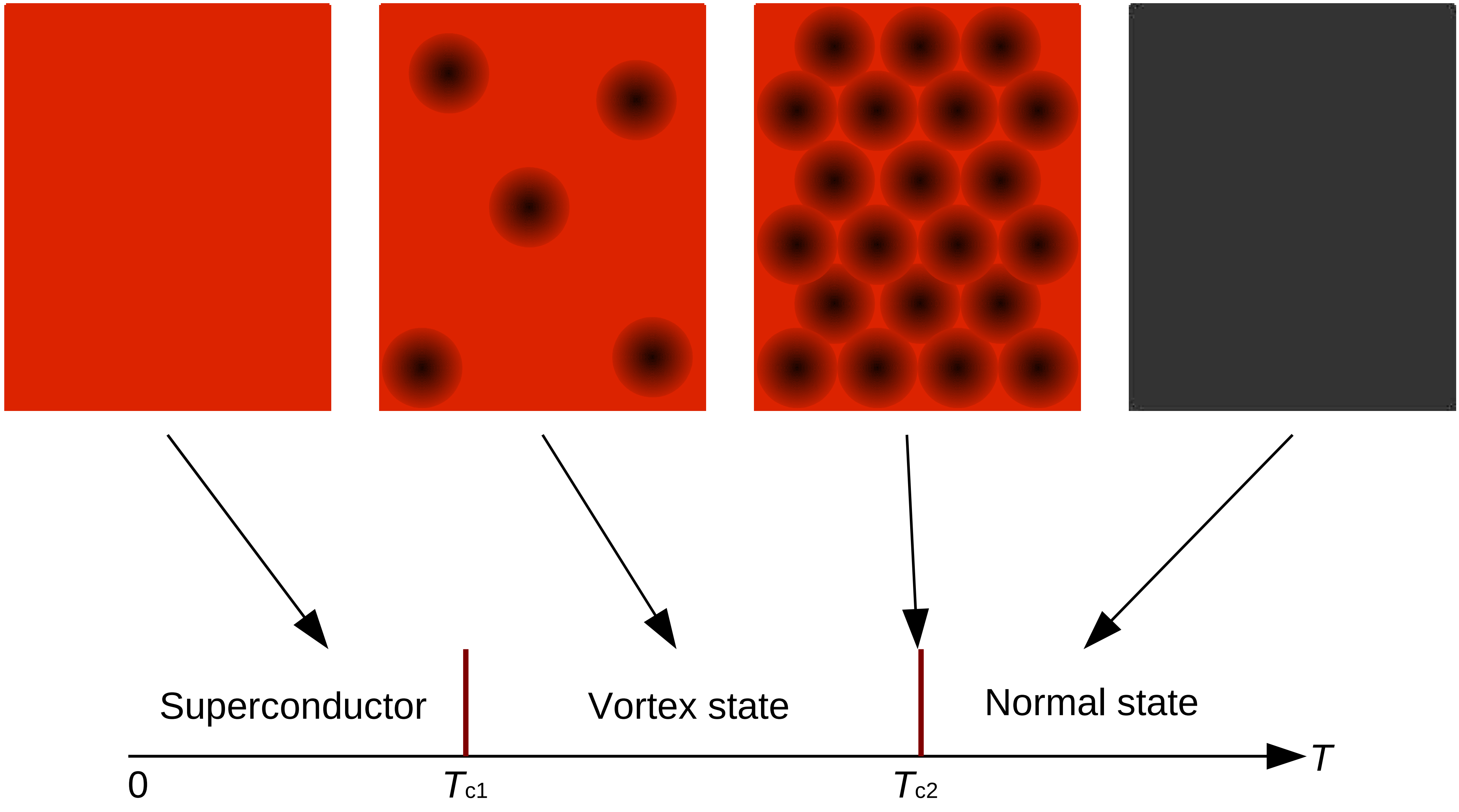
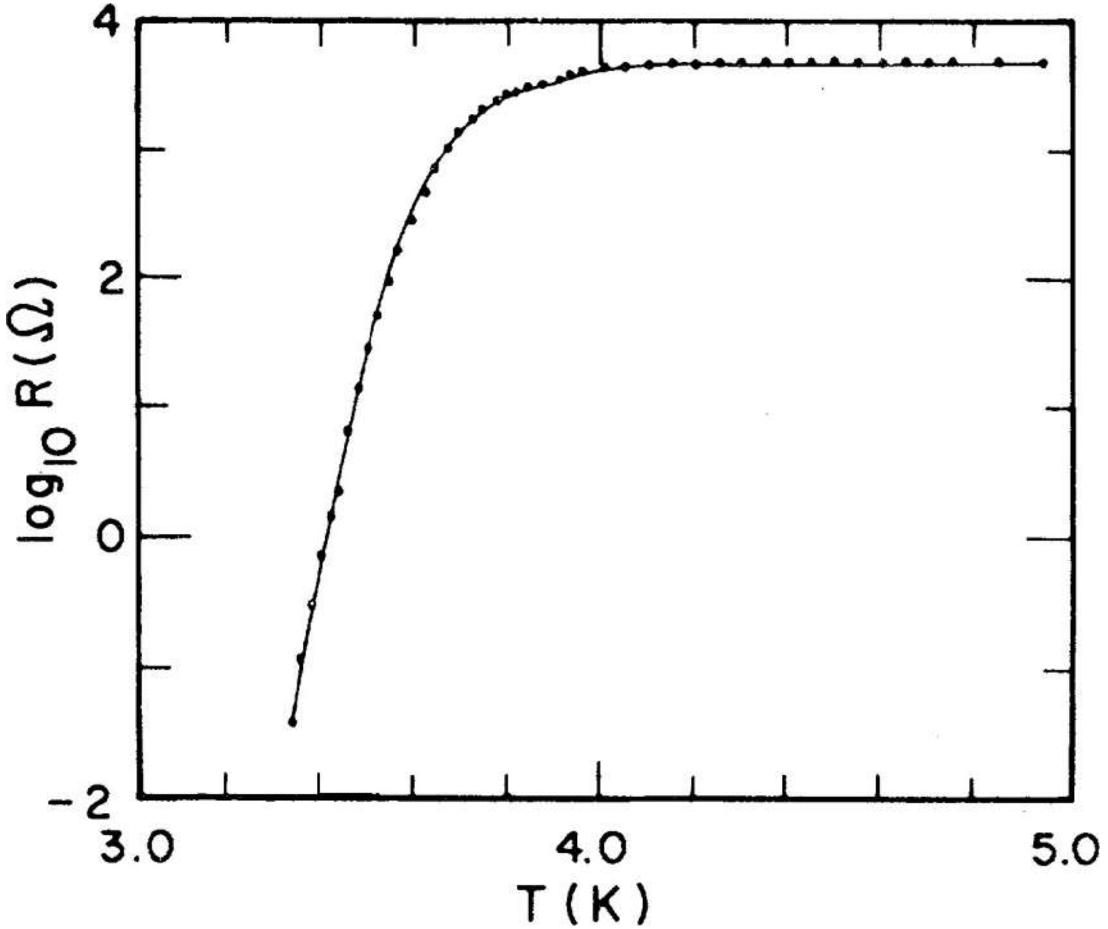


KT transition conductivity



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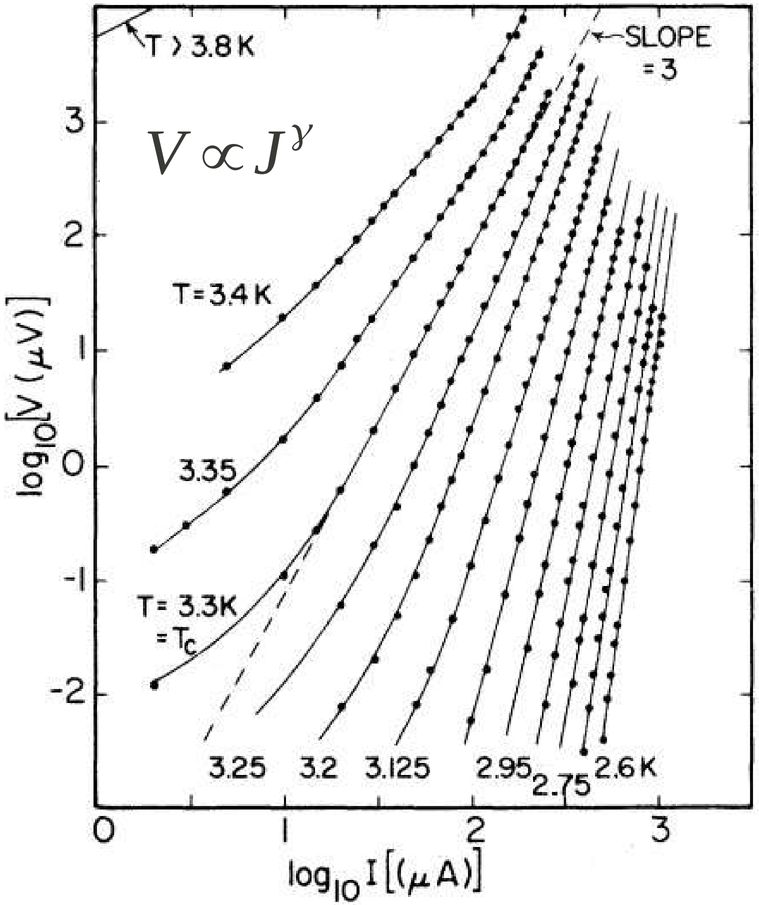
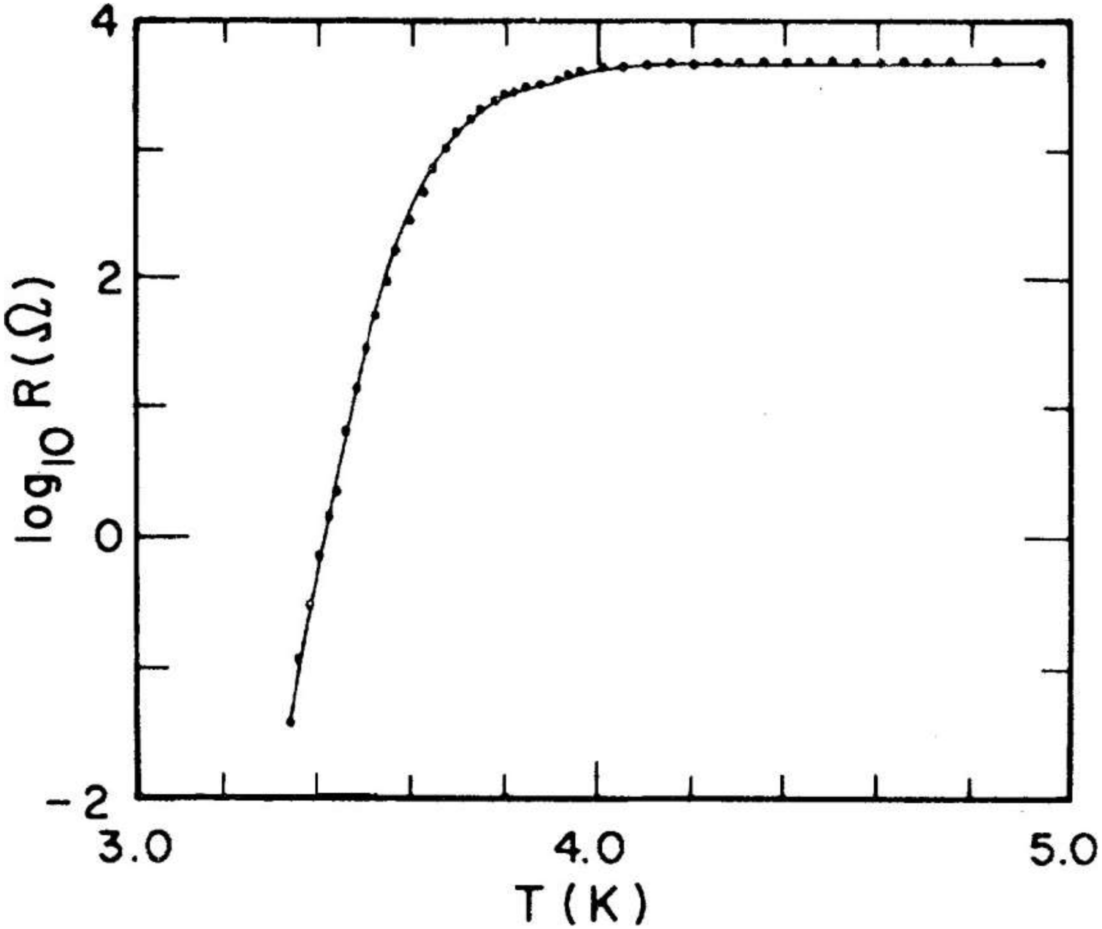
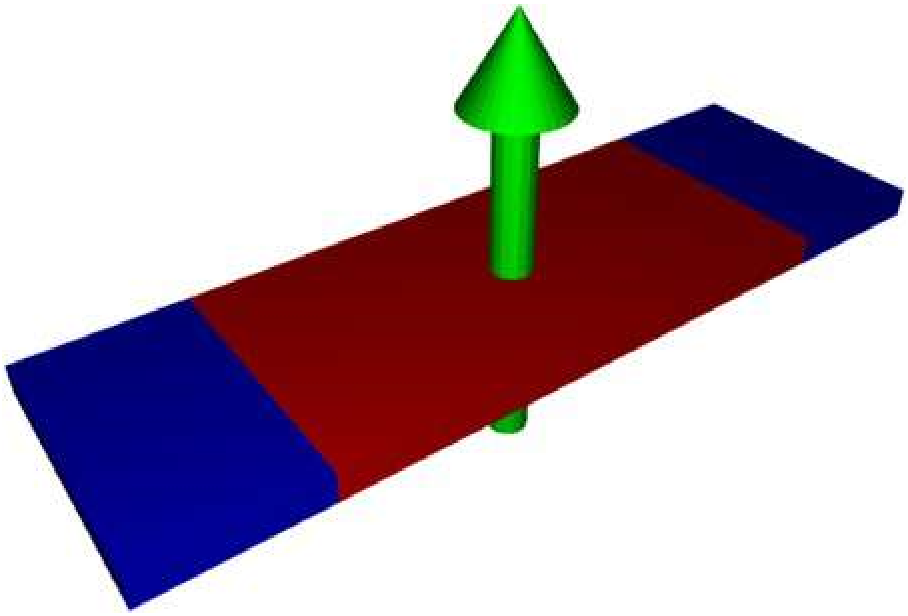
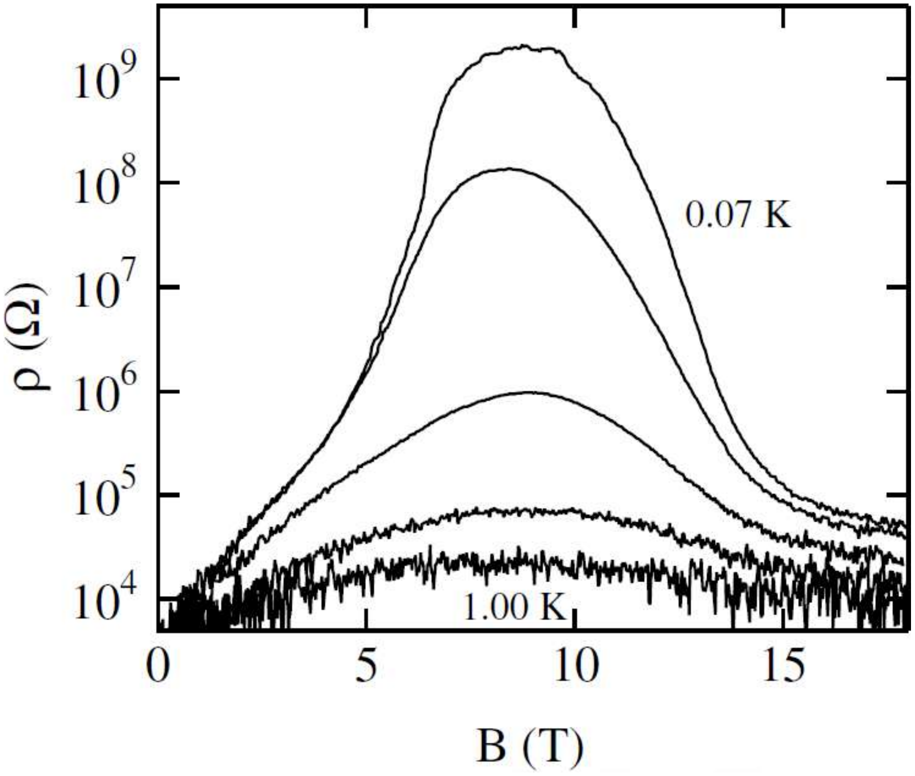




Transition in disordered systems

- Magnetoresistance peak [Sambandamurthy 04]

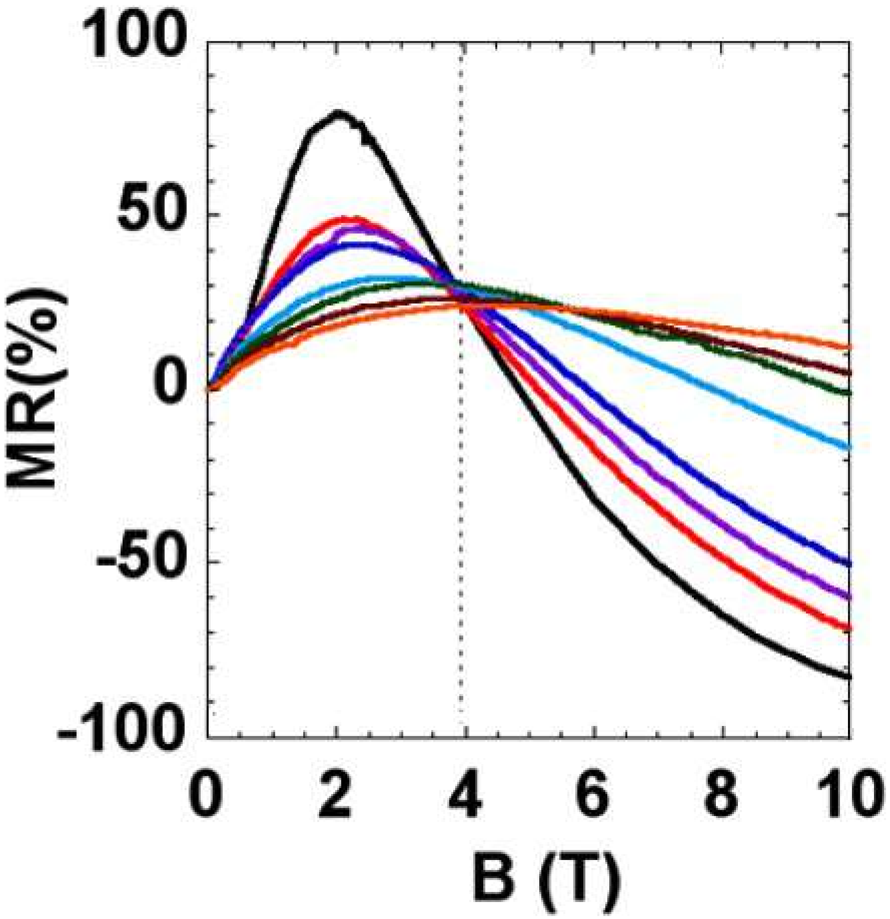


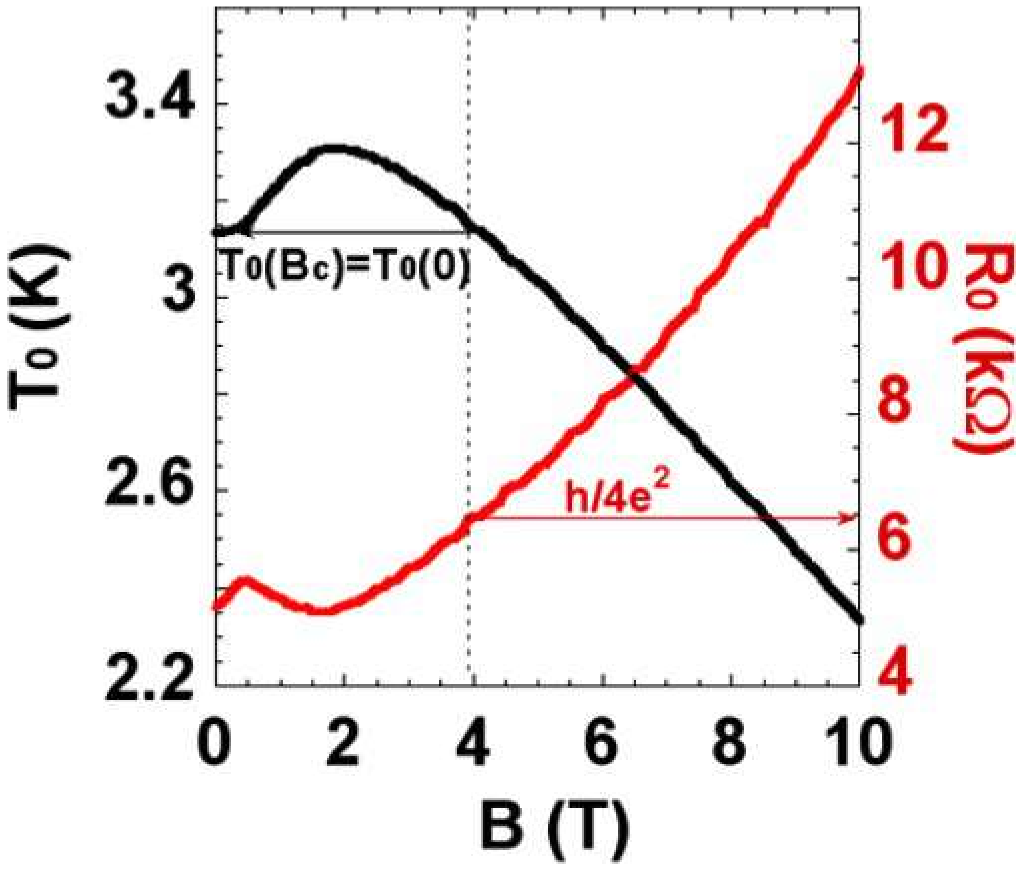


Transition in highly disordered systems

$$MR(B,T)=\frac{R(B,T)-R(0,T)}{R(0,T)}$$

[Lin & Goldman 11]





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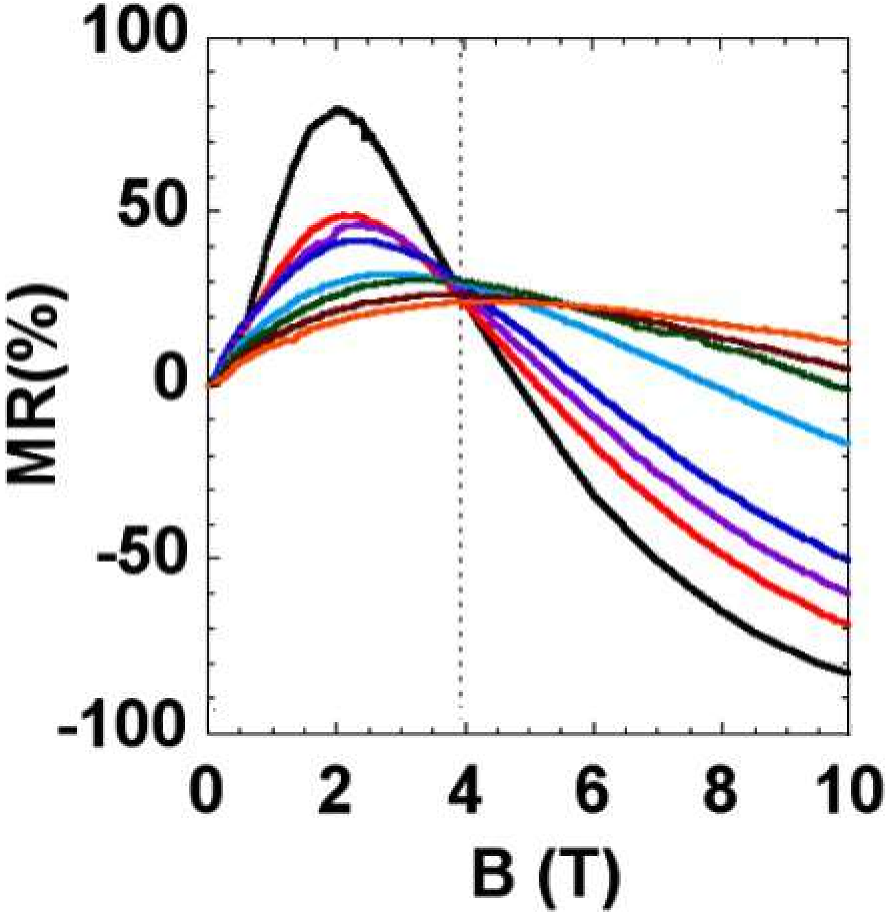
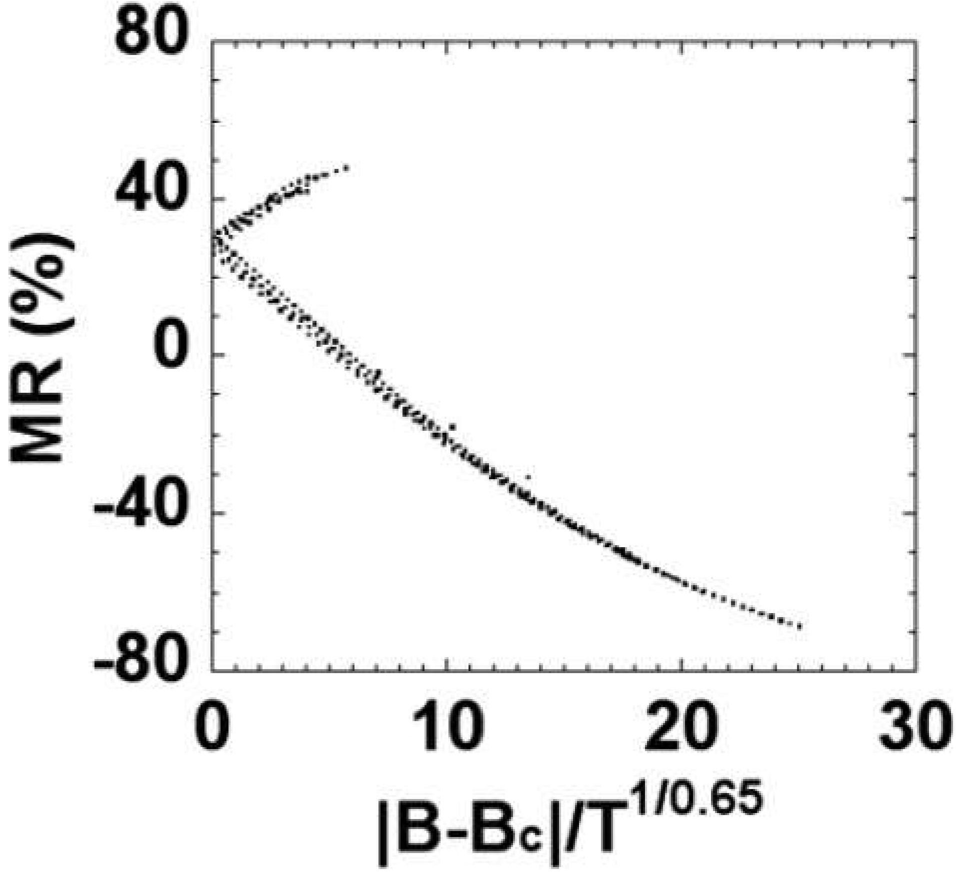




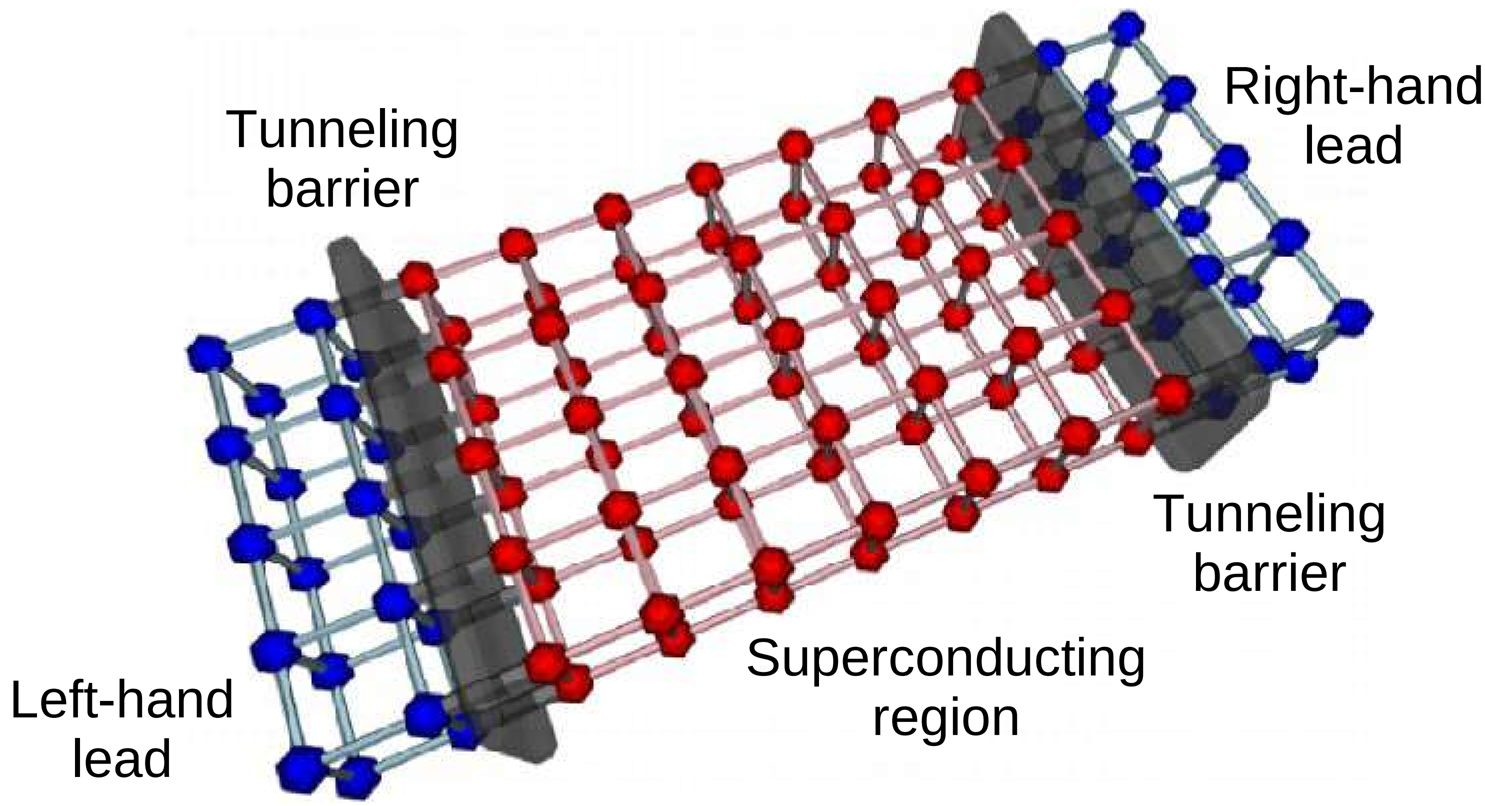
Strategy to study superconductors

- Develop new formalism to:
  - Calculate exact net current flow
  - Extract the microscopic current flow
  - Account for phase and amplitude fluctuations
  - Develop algorithm that permits access to large systems
- Test the formalism against a series of well-established results
- Study the magnetoresistance in thin-film superconductors

How to calculate the current

- General expression for the current [Meir & Wingreen, PRL 1992]

$$J = \frac{ie}{2h} \int d\epsilon \Big[\mathrm{Tr}\left\{ \left(f_L(\epsilon)\Gamma^L - f_R(\epsilon)\Gamma^R \right) \left(G^r_{e\sigma} - G^{a\sigma}_e \right) \right\} + \mathrm{Tr}\left\{ (\Gamma^L - \Gamma^R) G^<_{e\sigma} \right\} \Big]$$



Decoupling the interactions

- Negative *U* Hubbard model

$$\hat{H}_{\text{Hubbard}} = \sum_{i,\sigma} \epsilon_{i\sigma} c_{i\sigma}^{\dagger} c_{i\sigma} - \sum_{i} U_i c_{i\uparrow}^{\dagger} c_{i\downarrow}^{\dagger} c_{i\downarrow} c_{i\uparrow}$$

$$-\sum_{\langle i,j \rangle,\sigma} \left(t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + t_{ij}^{*} c_{j\sigma}^{\dagger} c_{i\sigma} \right)$$

- Decouple in density and Cooper pair channels

$$\rho_{i\sigma} = -|U_i| c_{i\sigma}^{\dagger} c_{i\sigma} \qquad \Delta_i = |U_i| \; c_{i\downarrow} c_{i\uparrow}$$

- Hamiltonian now contains single-body operators

$$\hat{\mathcal{H}}_{\text{BdG}} = \sum_{i,\sigma} (\epsilon_i + \rho_i) c_{i\sigma}^{\dagger} c_{i\sigma} - \sum_{\langle i,j \rangle,\sigma} \left(t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + t_{ij}^{*} c_{j\sigma}^{\dagger} c_{i\sigma} \right)$$

$$+ \sum_{i} \left(\Delta_i c_{i\uparrow}^{\dagger} c_{i\downarrow}^{\dagger} + \bar{\Delta}_i c_{i\downarrow} c_{i\uparrow} \right) + \sum_{i} \frac{|\Delta_i|^2 + \rho_i^2}{U_i}$$

Diagonalizing the Hamiltonian

- Hamiltonian now contains single-body operators

$$\hat{\mathcal{H}}_{\mathrm{BdG}} = \sum_{i,\sigma} (\epsilon_i + \rho_i) c^\dagger_{i\sigma} c_{i\sigma} - \sum_{\langle i,j \rangle,\sigma} \left(t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + t^*_{ij} c^\dagger_{j\sigma} c_{i\sigma} \right)$$

$$+ \sum_i \left(\Delta_i c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} + \bar{\Delta}_i c_{i\downarrow} c_{i\uparrow} \right) + \sum_i \frac{|\Delta_i|^2 + \rho_i^2}{U_i}$$

- Energy eigenstates can be found from diagonalization of

$$\hat{\mathcal{H}}_{\mathrm{BgG}} = \frac{|\Delta|^2 + \rho^2}{U} + \begin{pmatrix} c^\dagger_\uparrow & c_\downarrow \end{pmatrix} \begin{pmatrix} \epsilon + \rho & \Delta \\ \bar{\Delta} & -(\epsilon + \rho) \end{pmatrix} \begin{pmatrix} c_\uparrow \\ c^\dagger_\downarrow \end{pmatrix} + \epsilon + \rho$$

Accelerated Metropolis sampling

- To perform thermal sum calculate

$$\langle J \rangle = \sum_{\Delta,\rho} J[\Delta,\rho] e^{-\beta(E[\Delta,\rho]-E_0)}$$

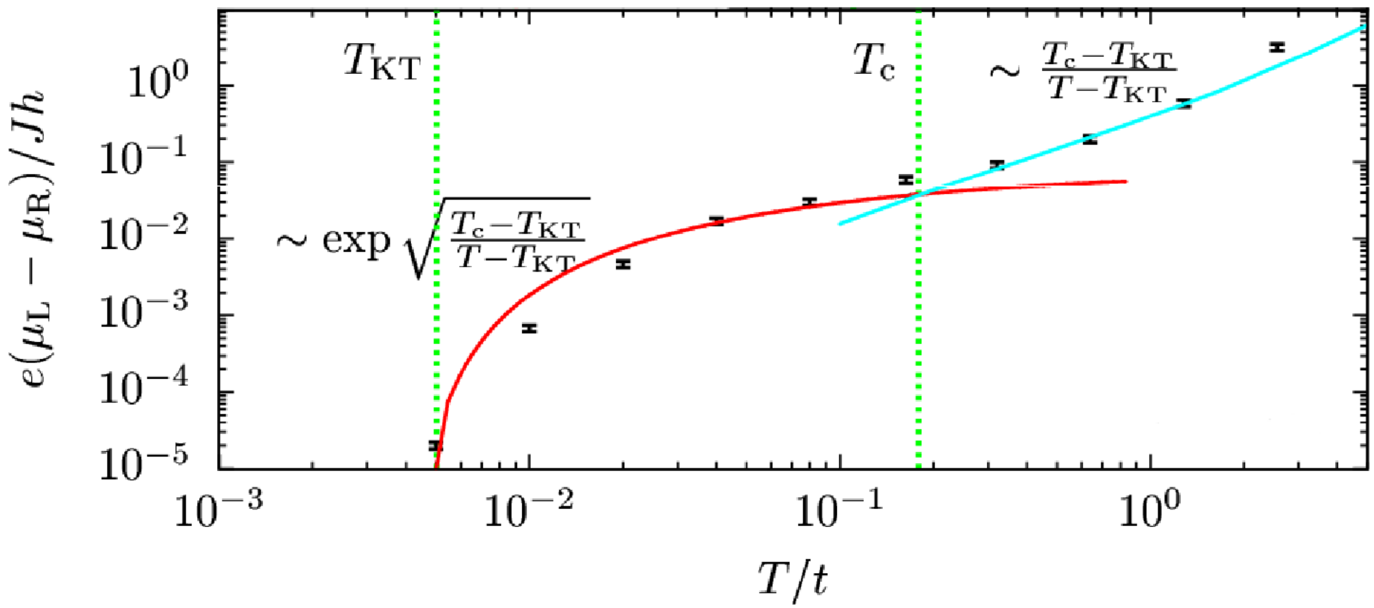
- Propose new configuration of Δ and ρ, accept with probability

$$\exp(\beta E[\Delta_{\text{old}},\rho_{\text{old}}]-\beta E[\Delta_{\text{new}},\rho_{\text{new}}])$$

- Calculating $E[\Delta,\rho]$ costs $O(N^3)$, where N is the number of sites
- New method calculates $E[\Delta,\rho]-E[\Delta+\delta\Delta,\rho+\delta\rho]$ using a Chebyshev expansion [Weisse 09] in $O(N^{1.56})$ time

Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear *IV* characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



Halperin & Nelson, J. Low Temp. Phys 1979

Ambegaokar *et al*., PRB 1980

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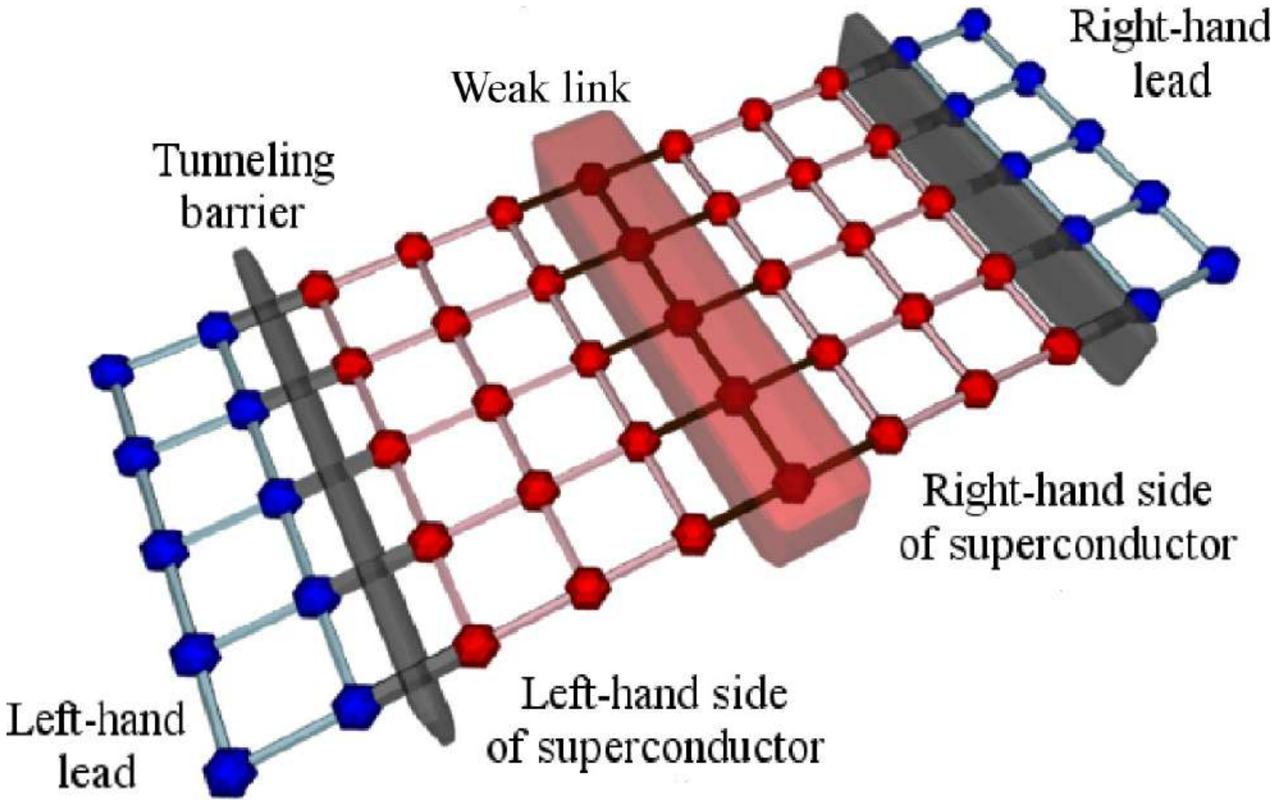

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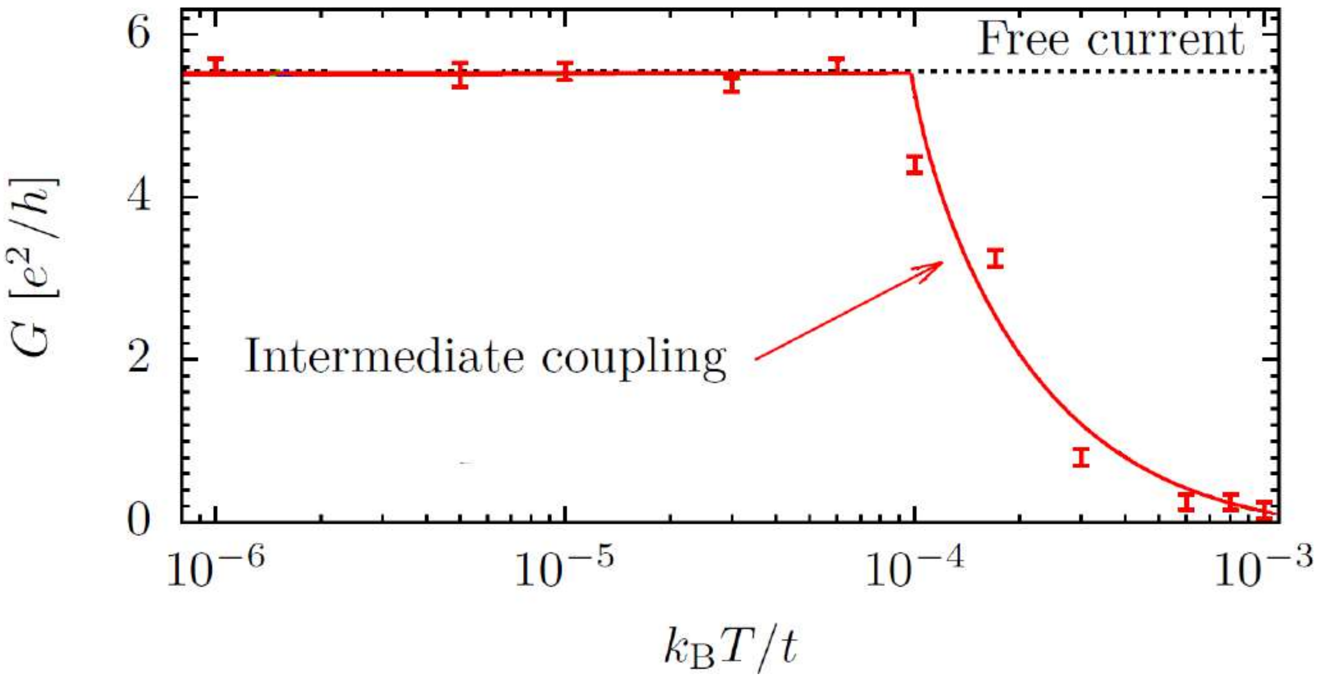

Ambegaokar & Baratoff, PRL **10**, 486 (1963)

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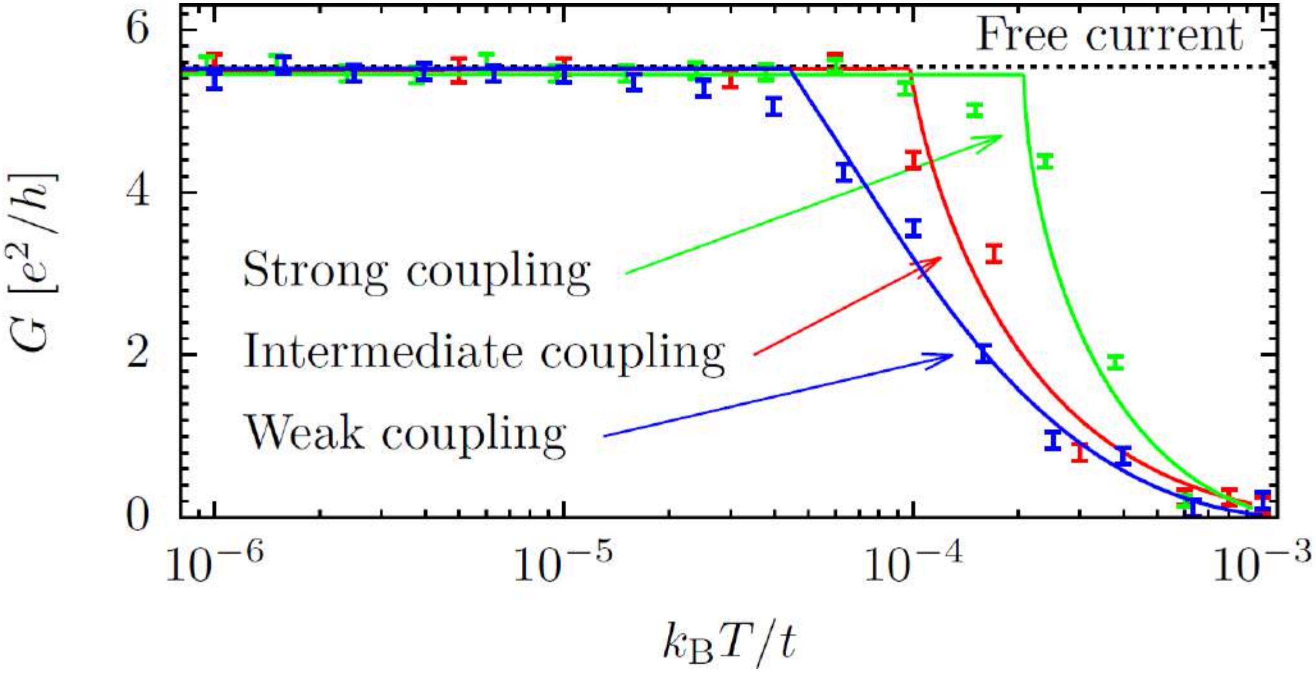

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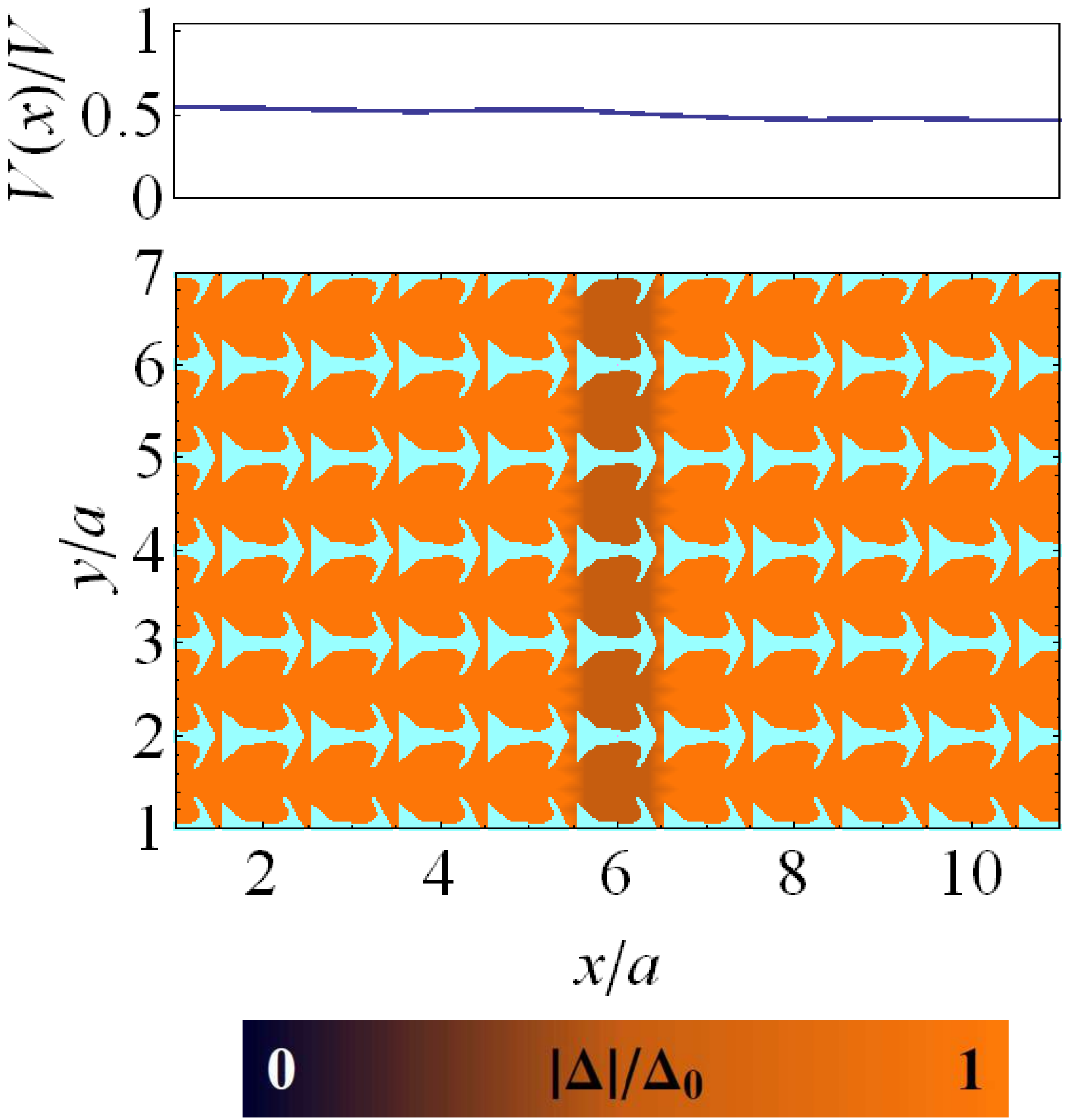

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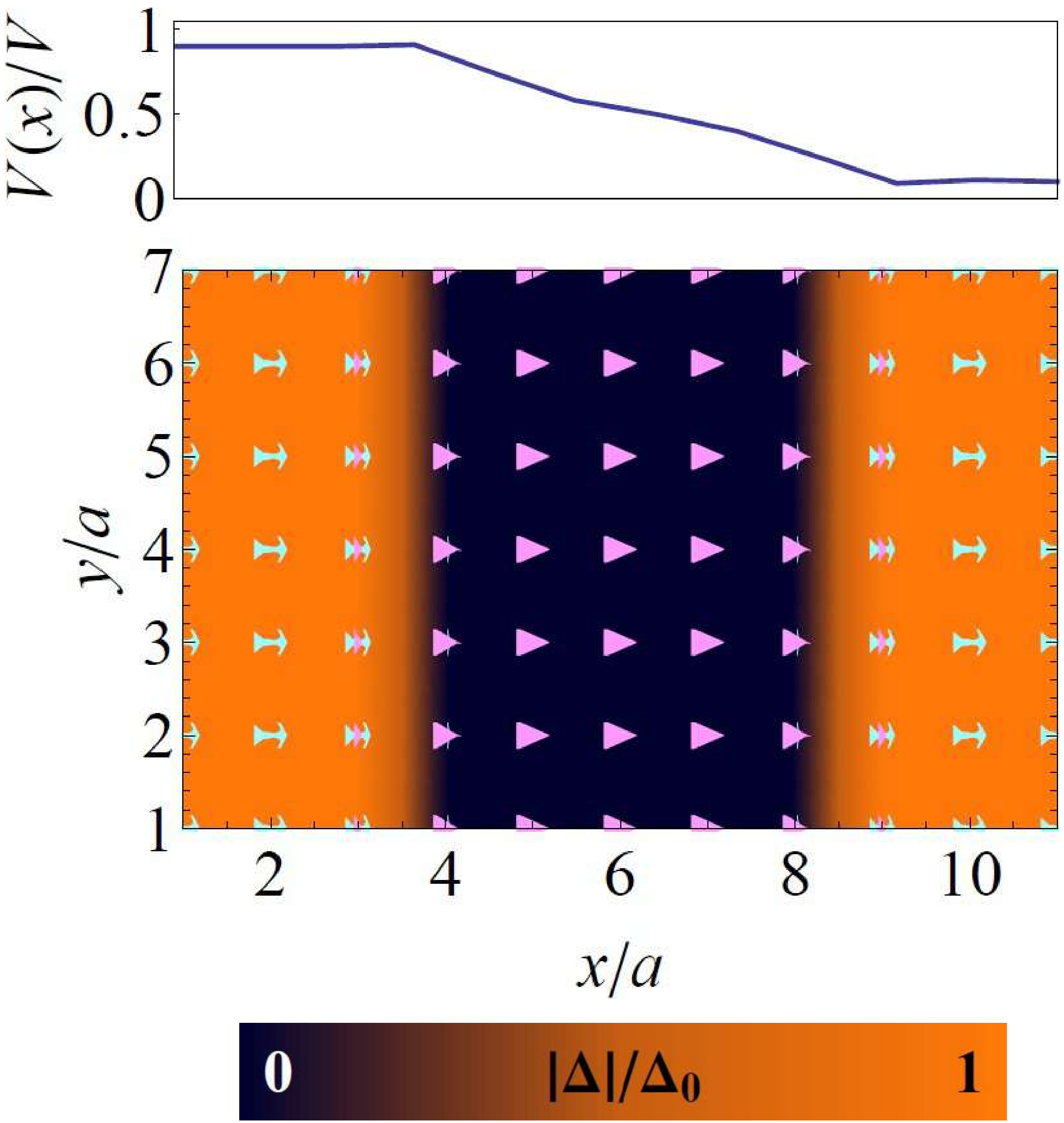

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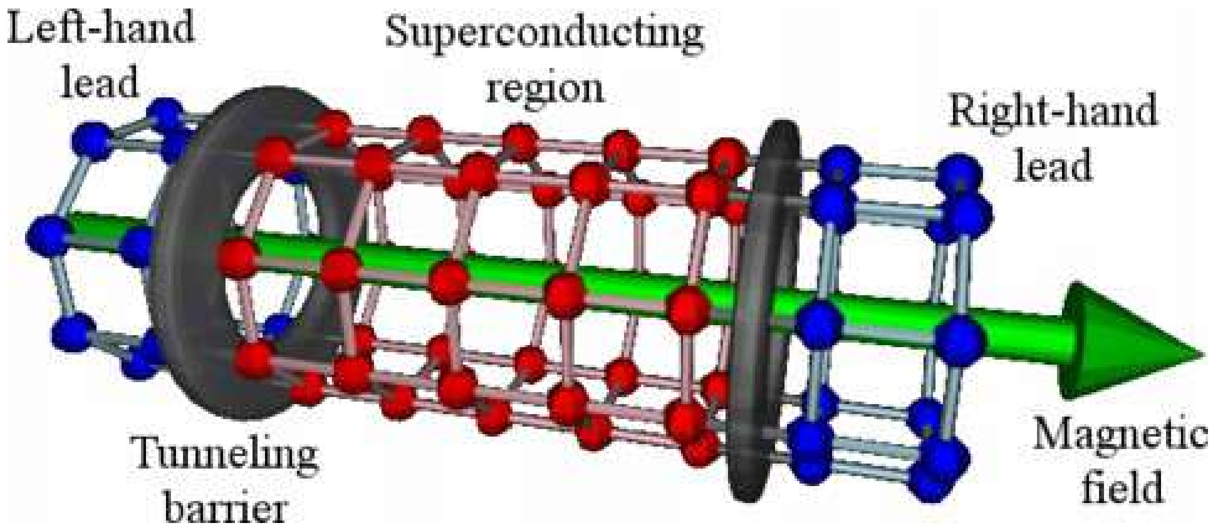
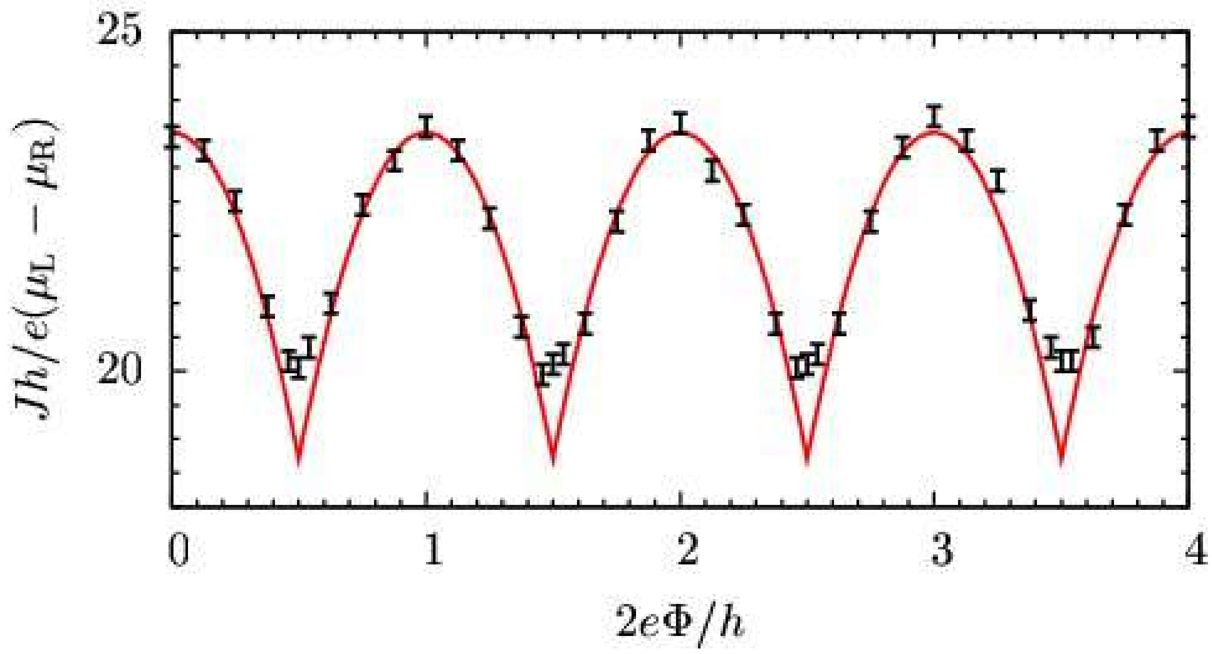
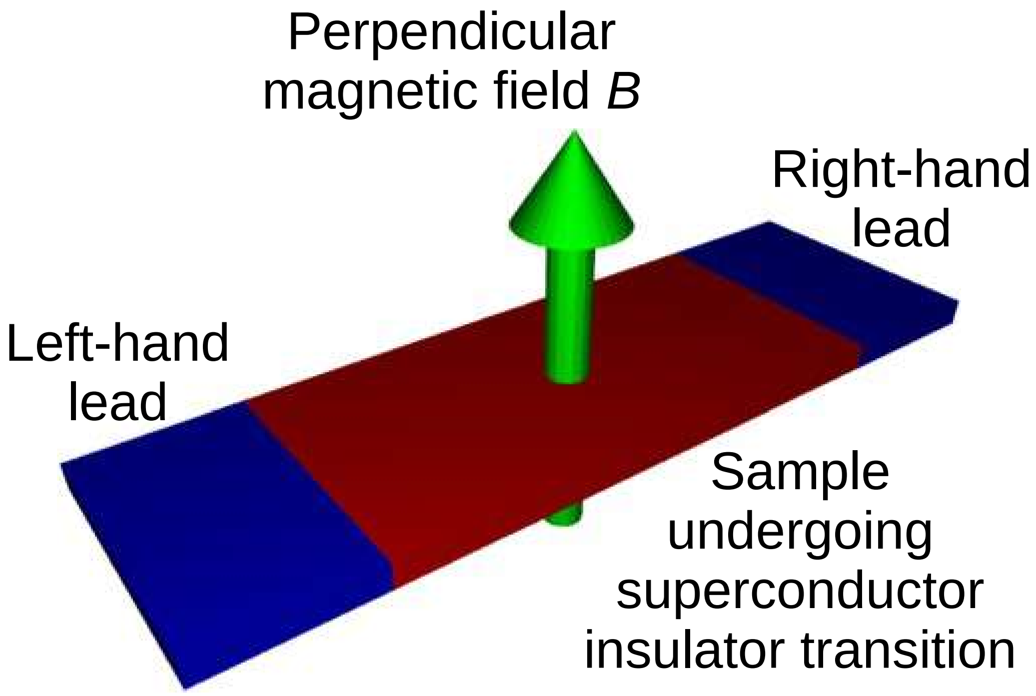
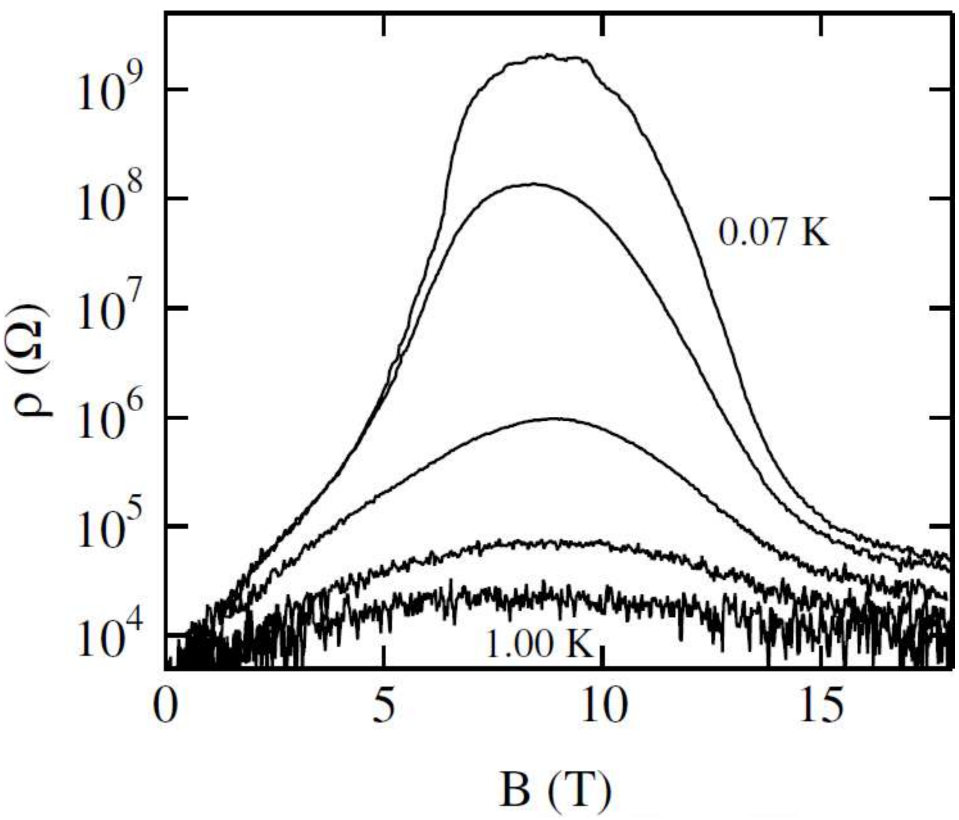




Magnetoresistance peak

- Study superconductor-insulator transition in dirty sample with perpendicular magnetic field





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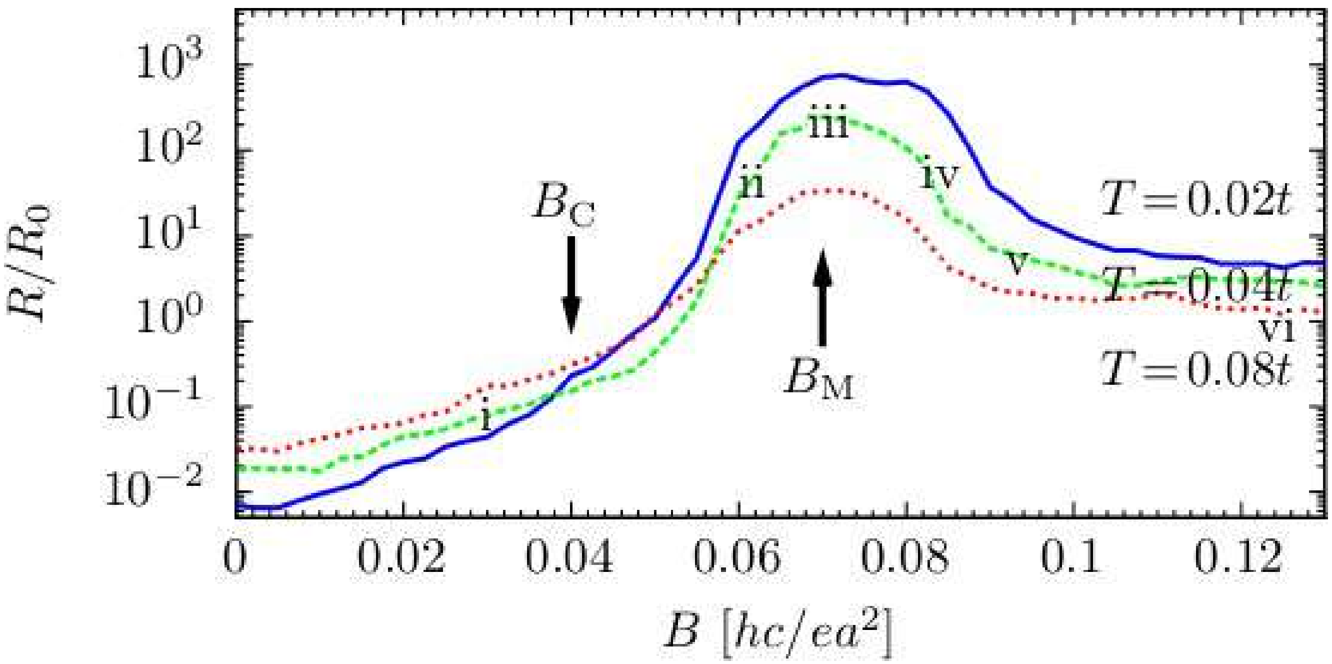
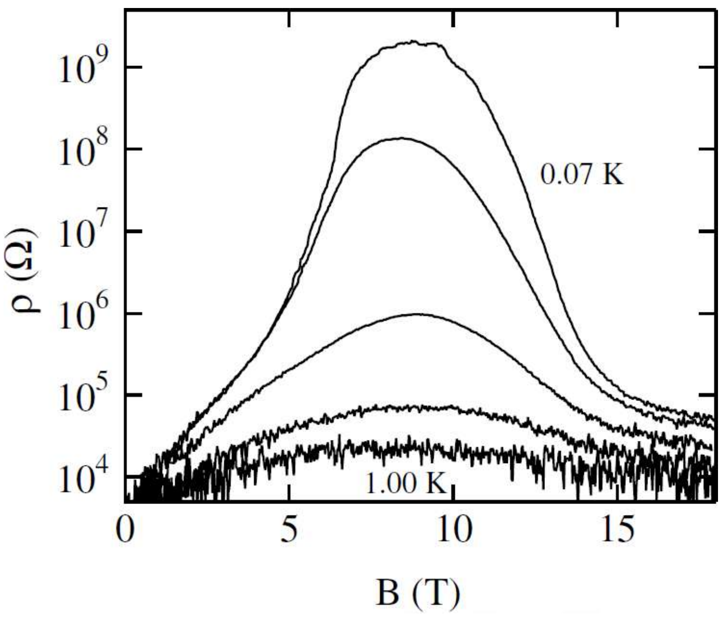
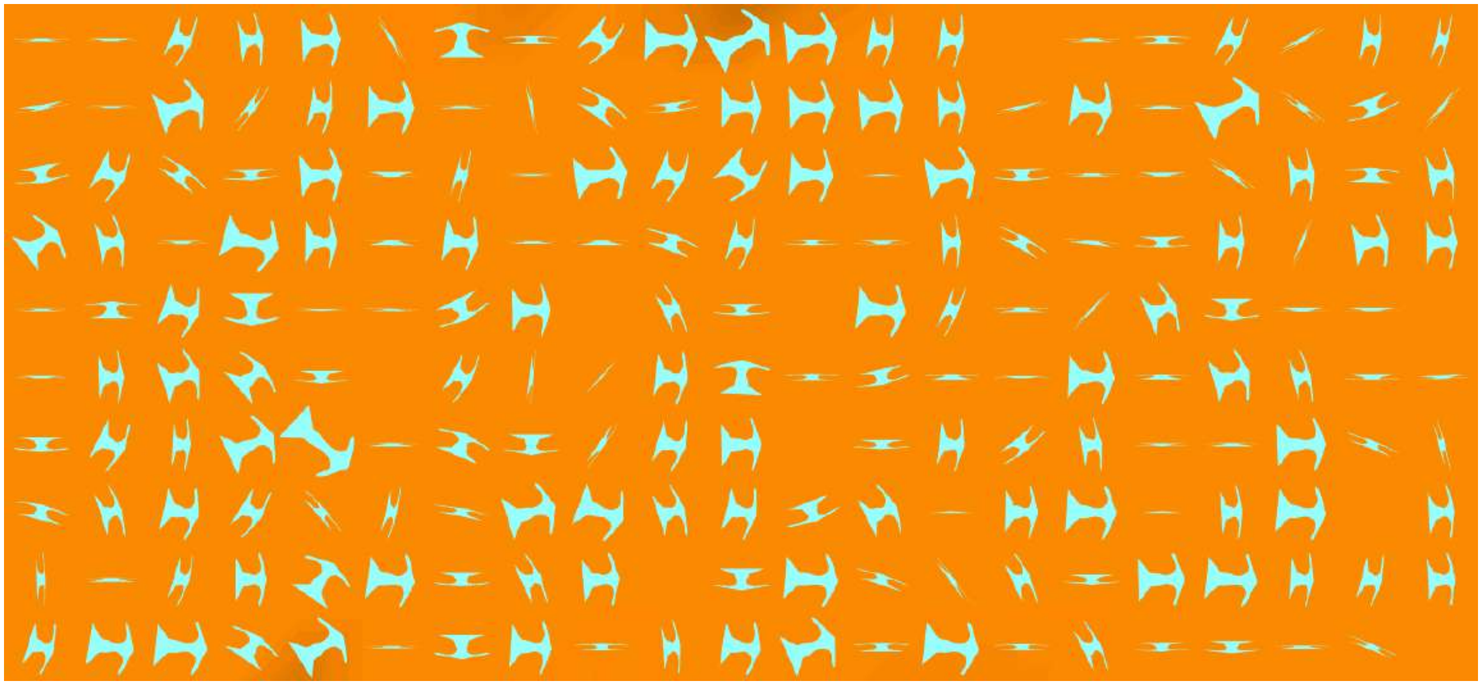
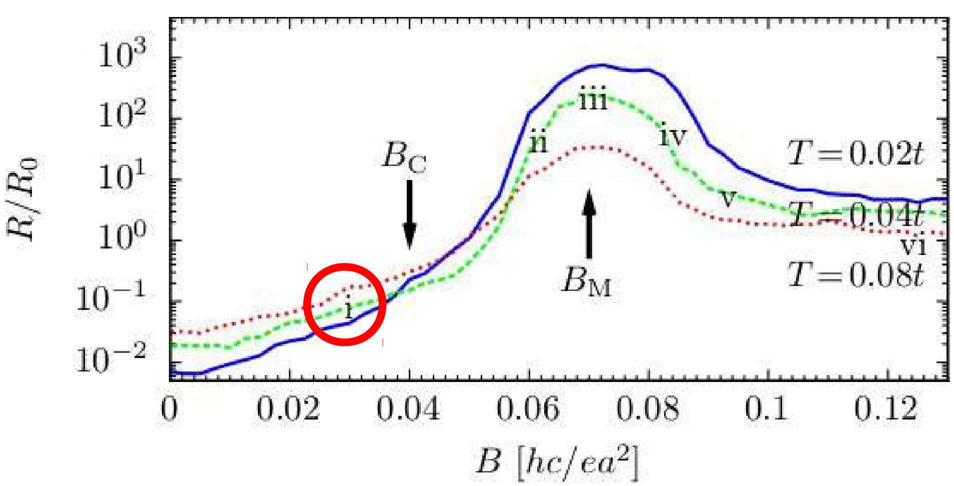
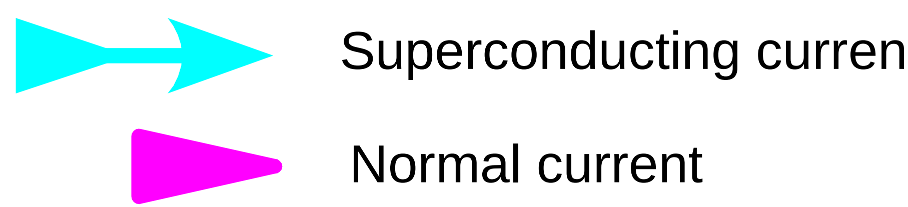




Clues: current maps







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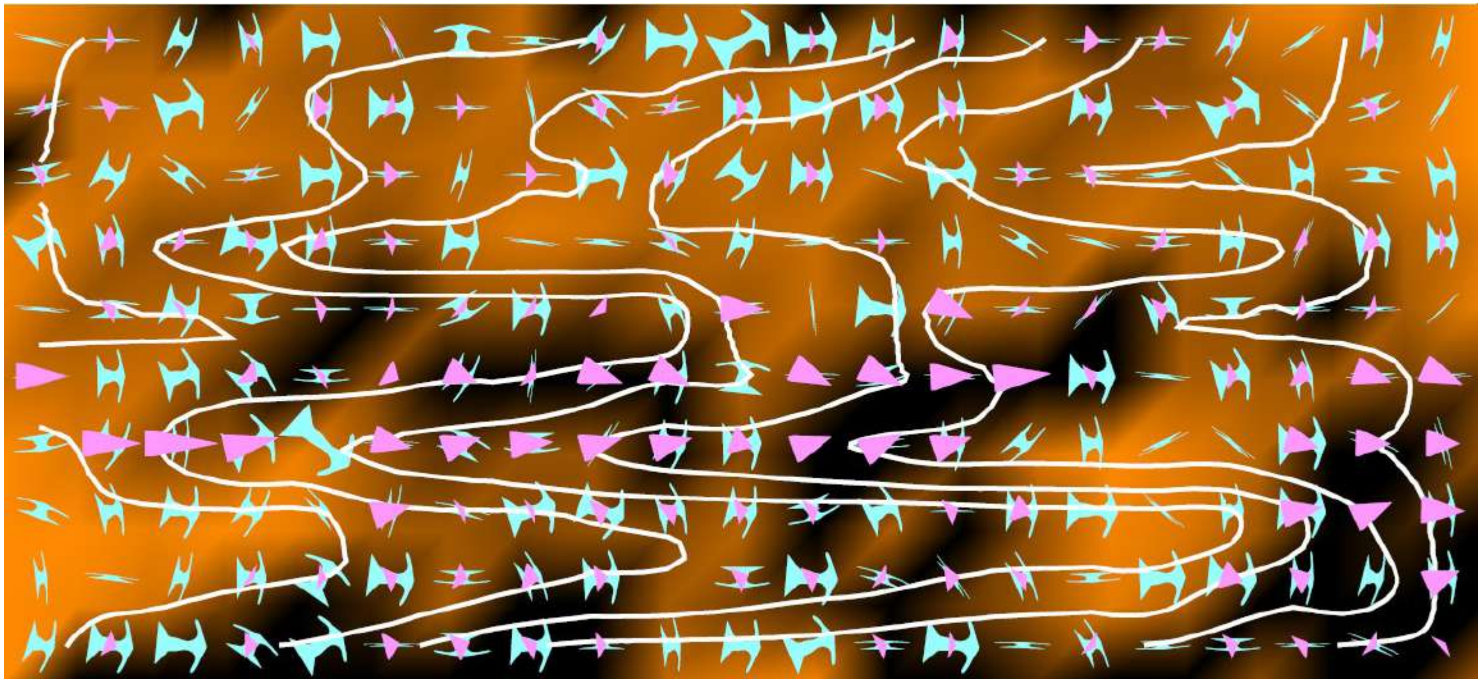
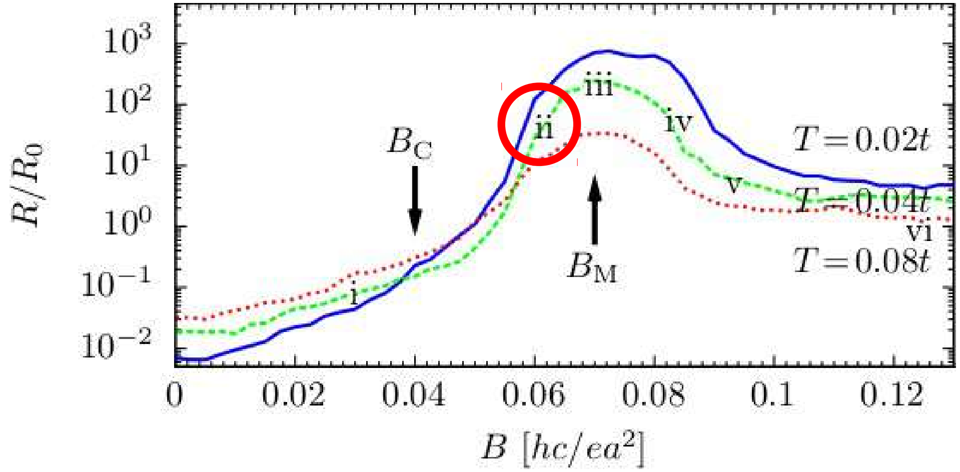
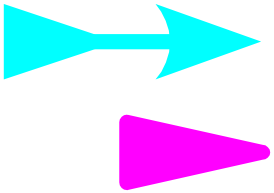


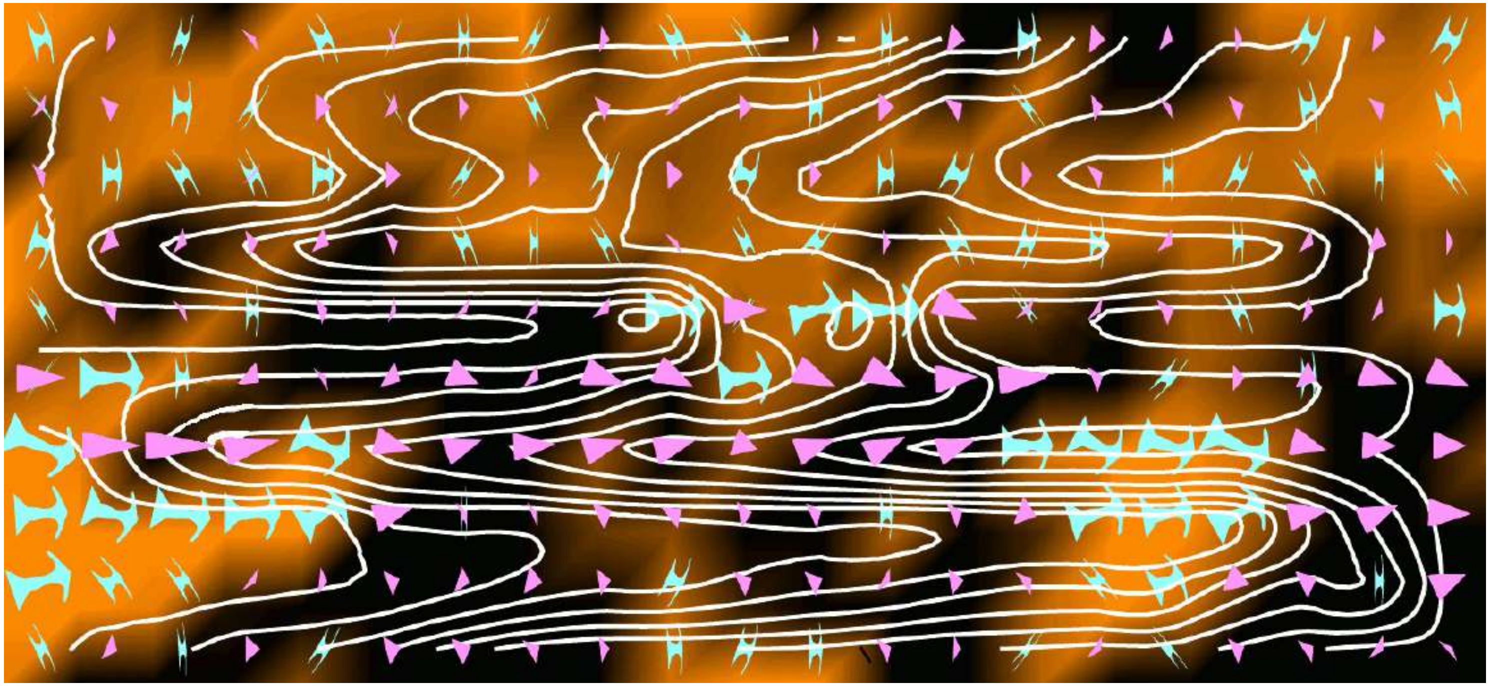


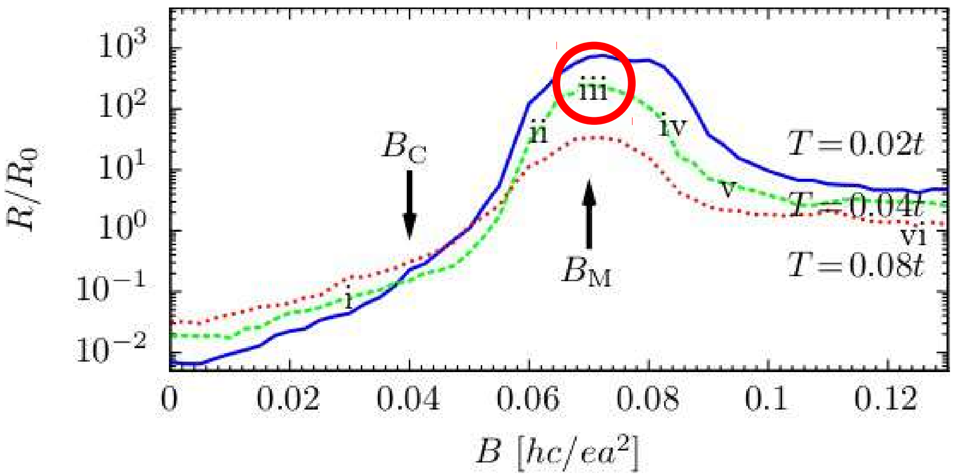


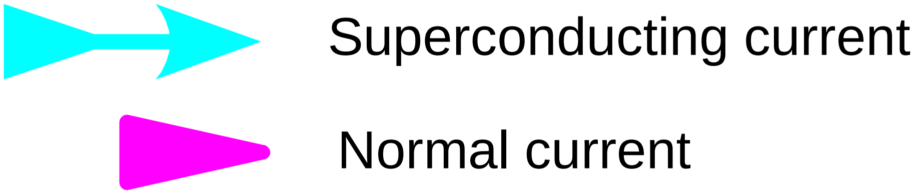
Superconducting current

Normal current

Clues: current maps







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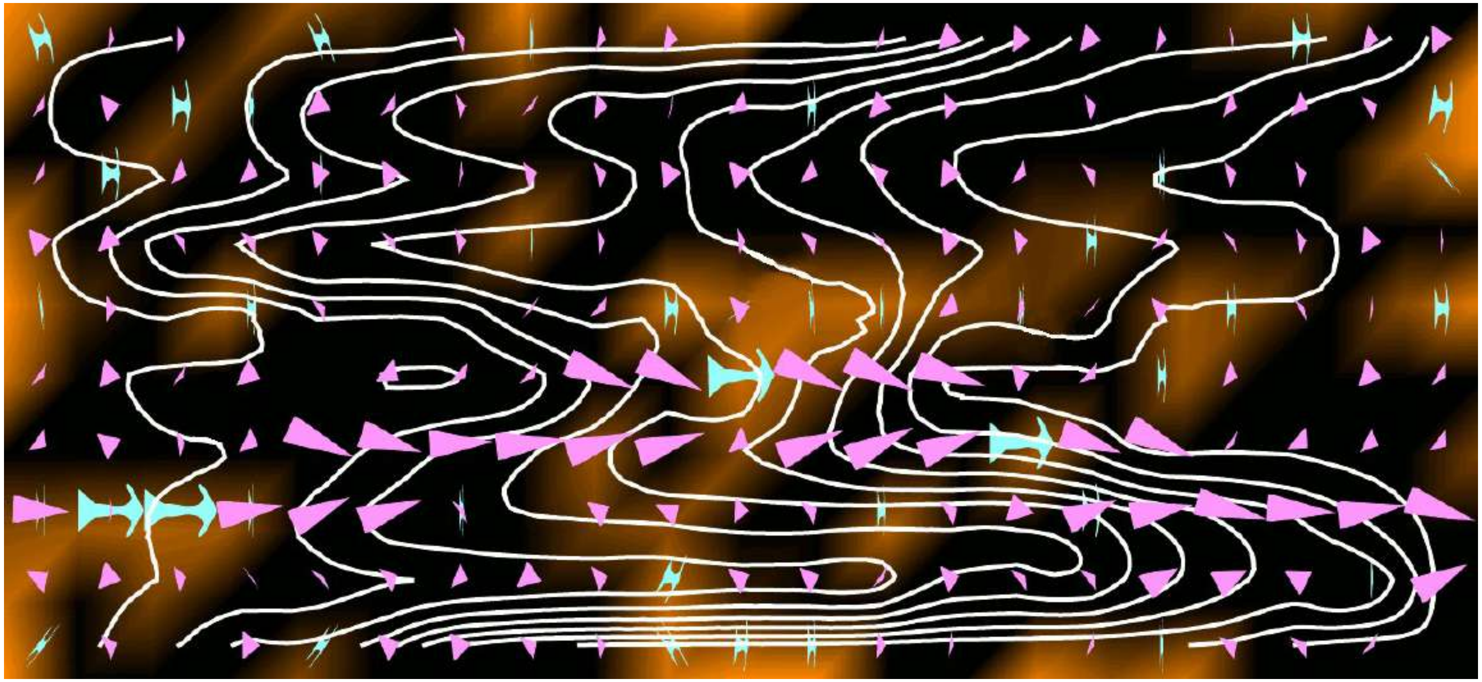

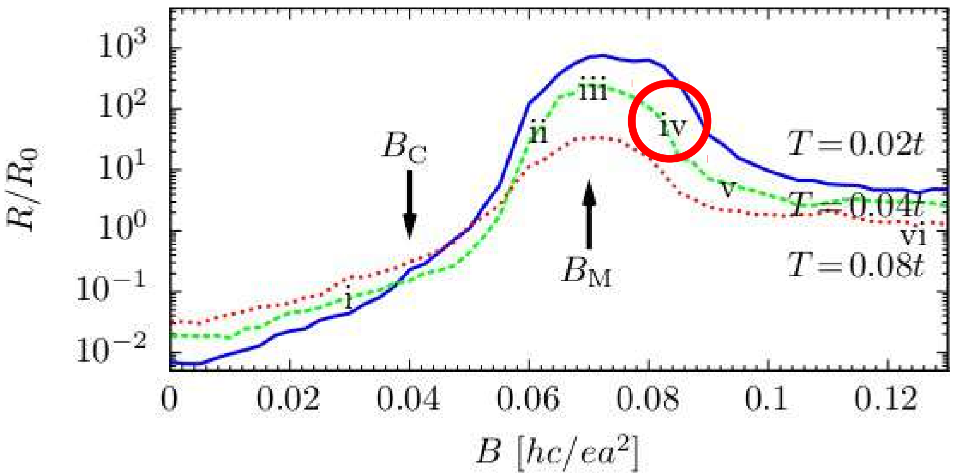

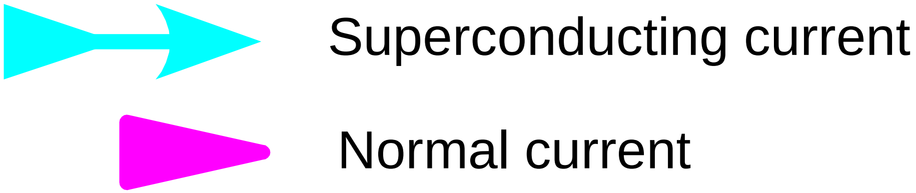

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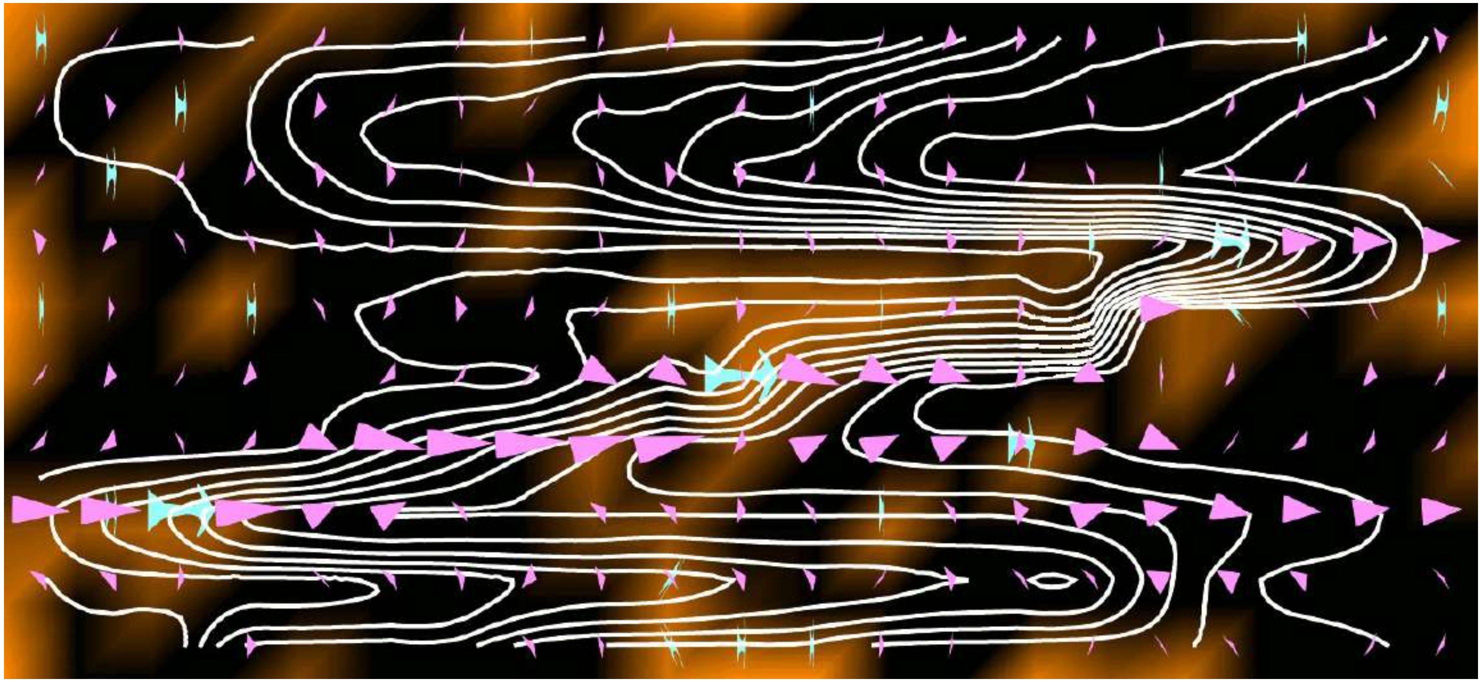
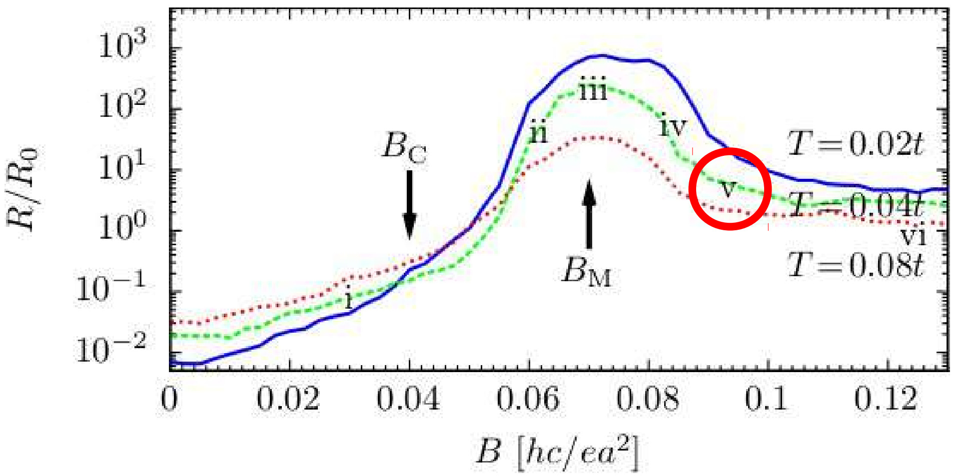
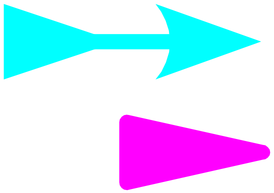


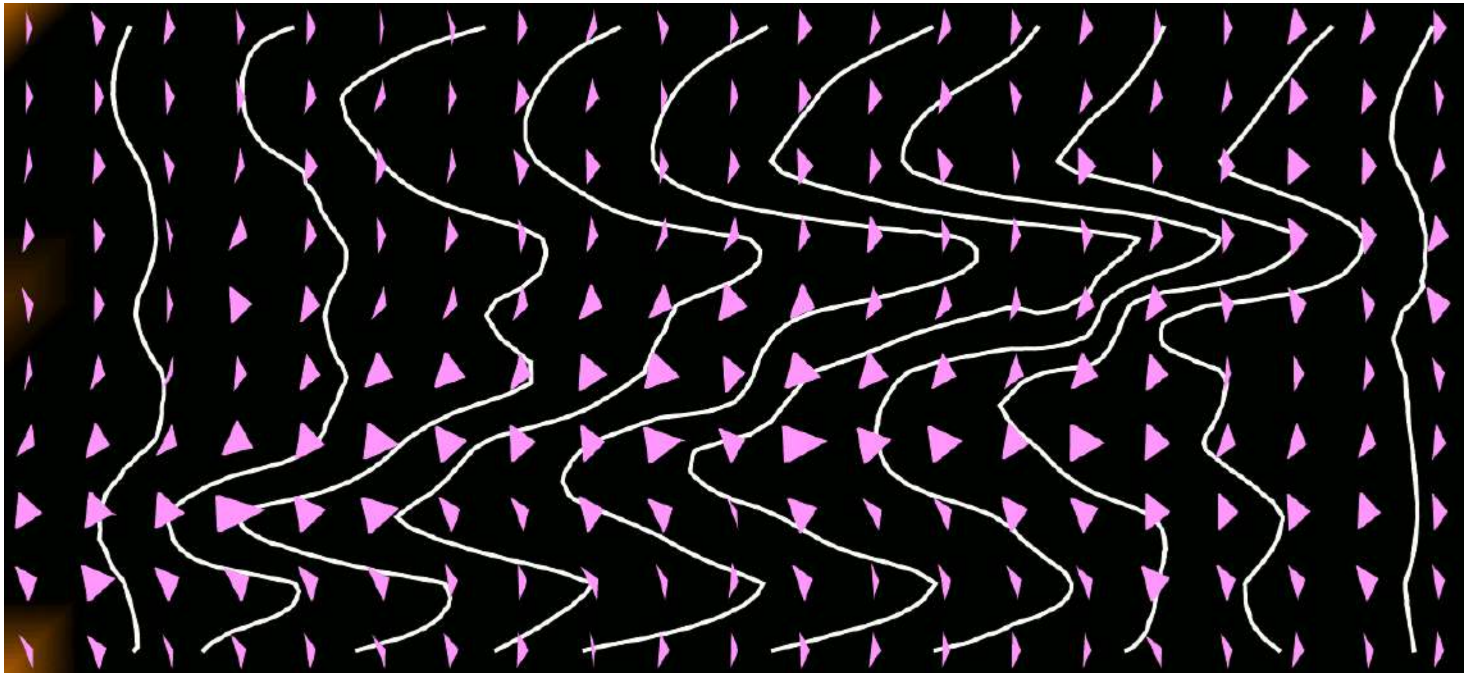


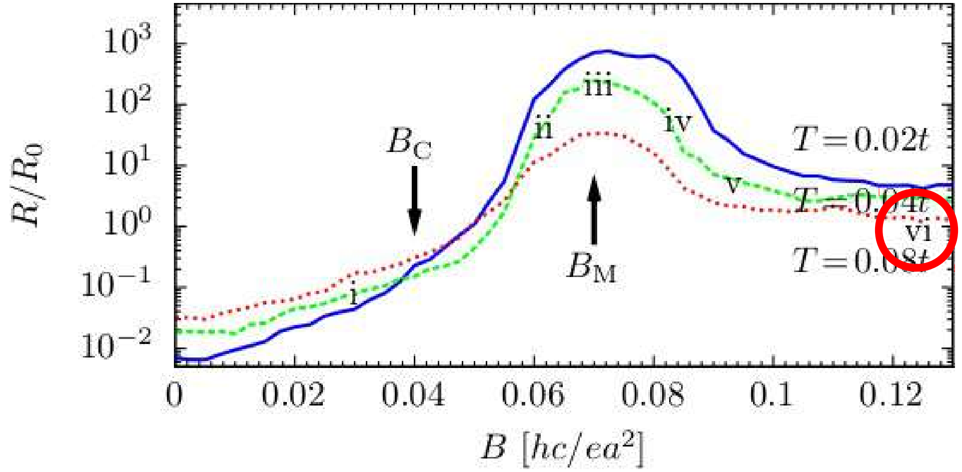


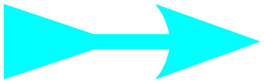
Superconducting current

Normal current

Clues: current maps



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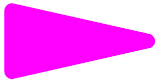
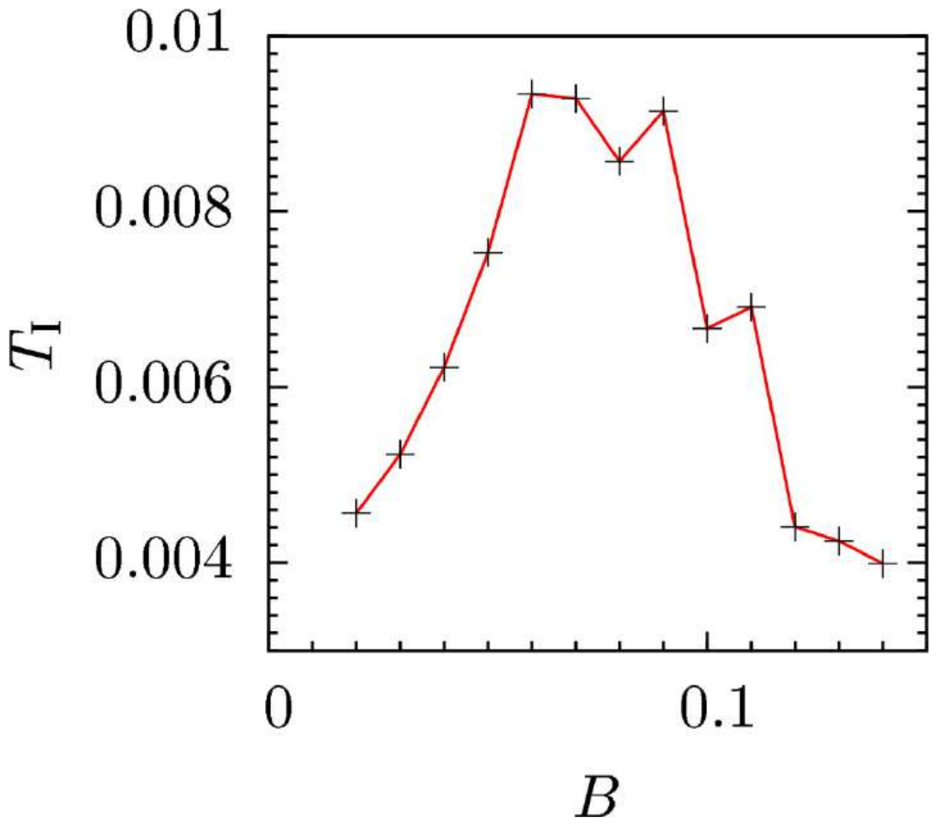
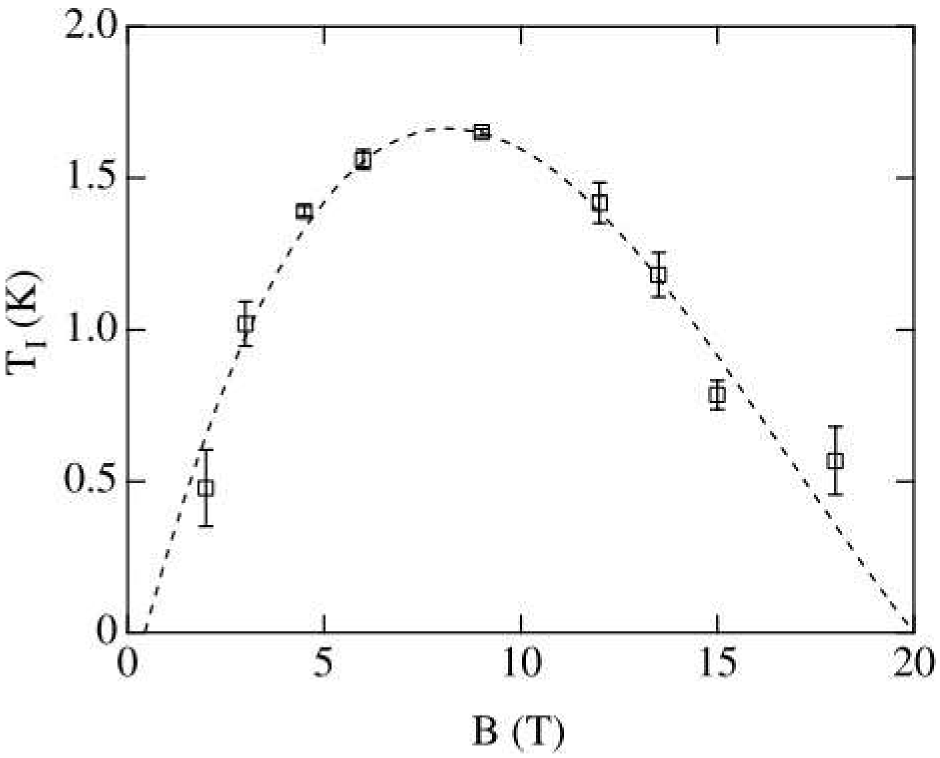

Normal current

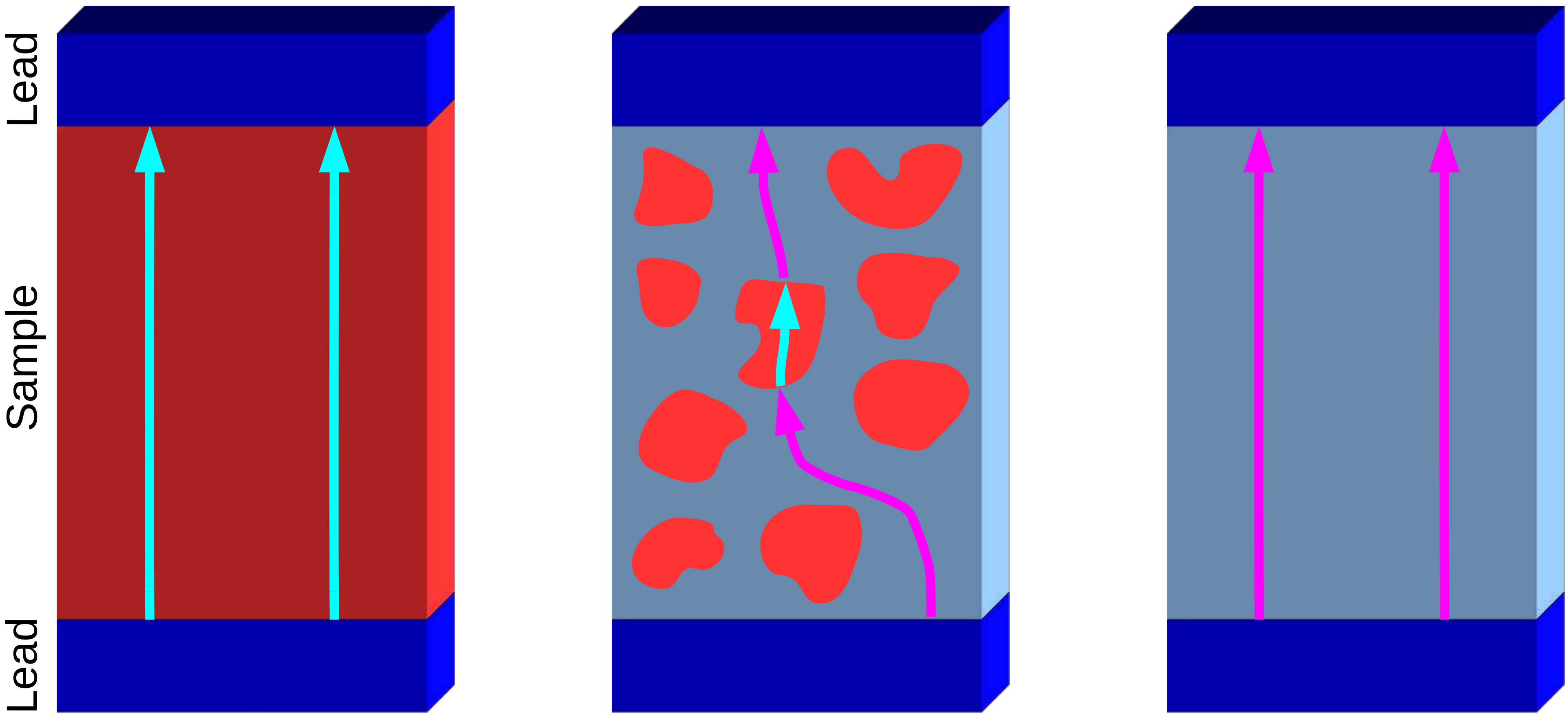
Clues: activated transport

- Activated transport $\rho = \rho_0 e^{T_I/T}$





Proposed mechanism

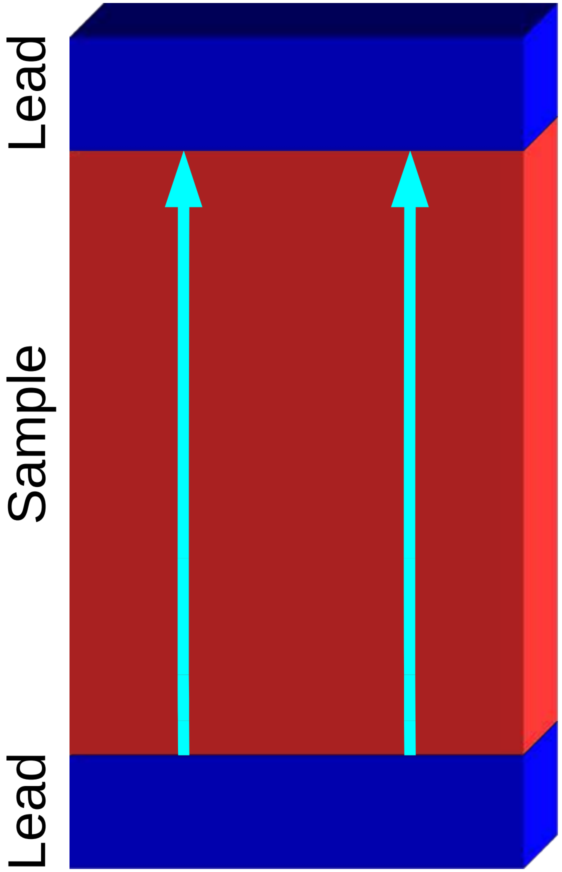


Sample entirely superconducting

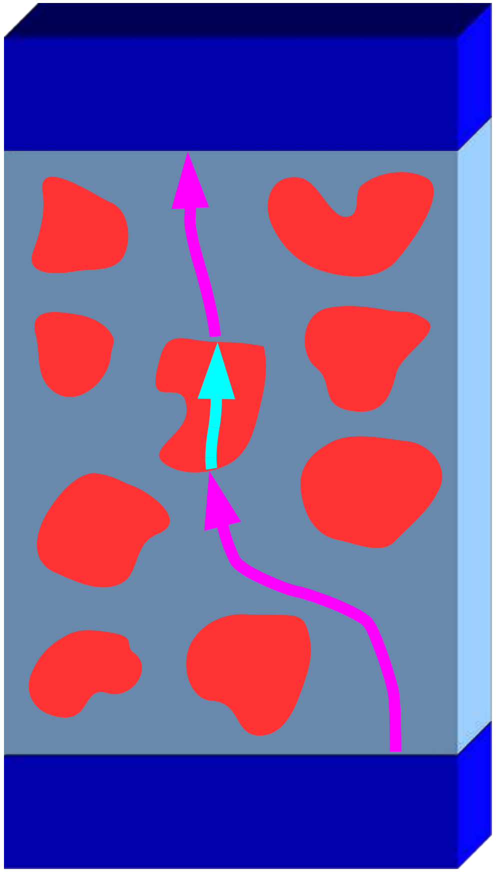
Superconducting puddles have a charging energy and a tunneling barrier

Sample entirely normal

Proposed mechanism



Sample entirely superconducting



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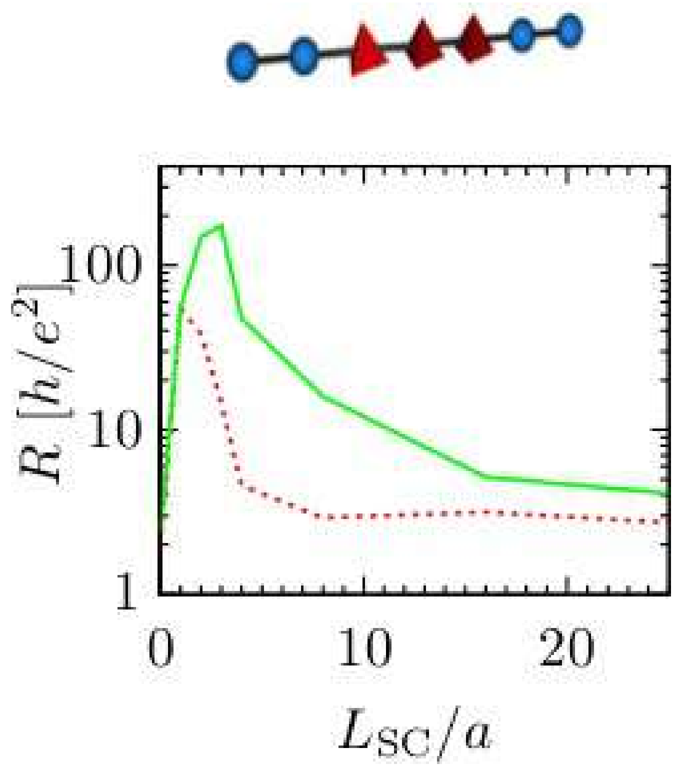
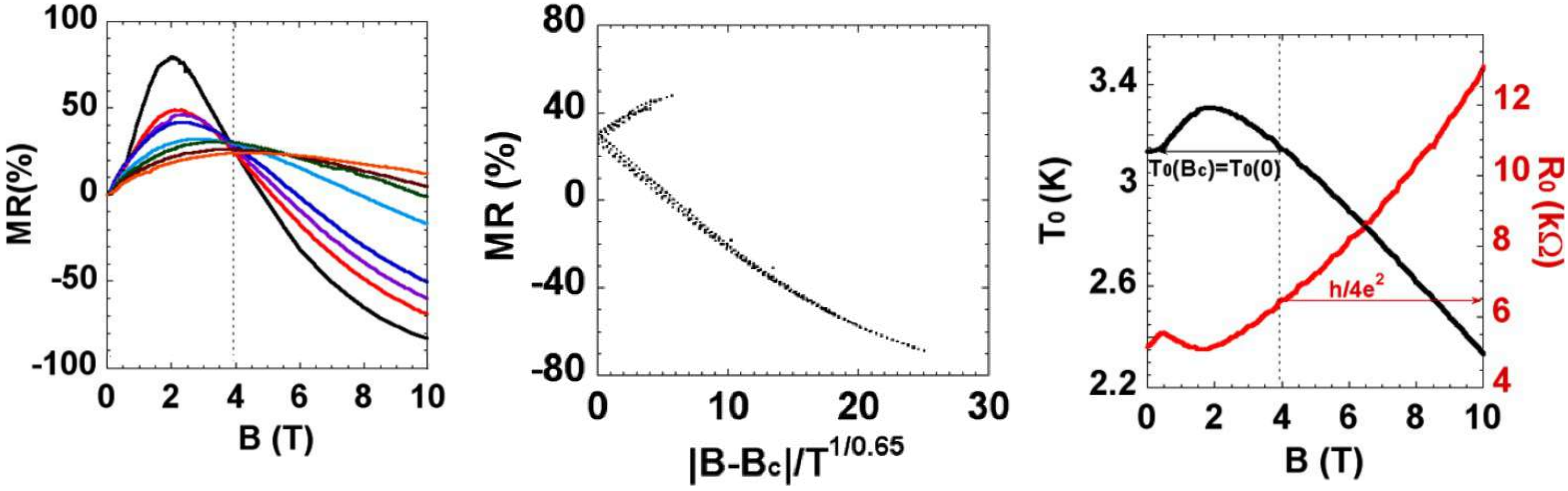


Highly disordered systems

$$MR(B,T)=\frac{R(B,T)-R(0,T)}{R(0,T)}$$

$$R(B,T)=R_0(B)\,e^{T_A/T}$$

$$T_A(0)=T_A(B_C)$$



[Lin & Goldman 11]

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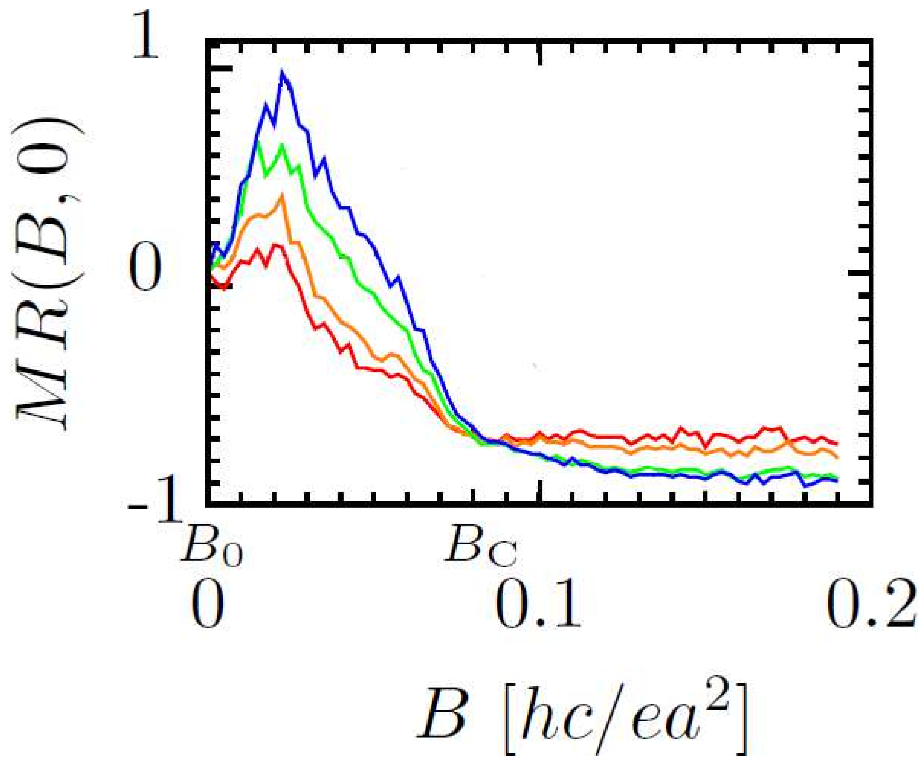
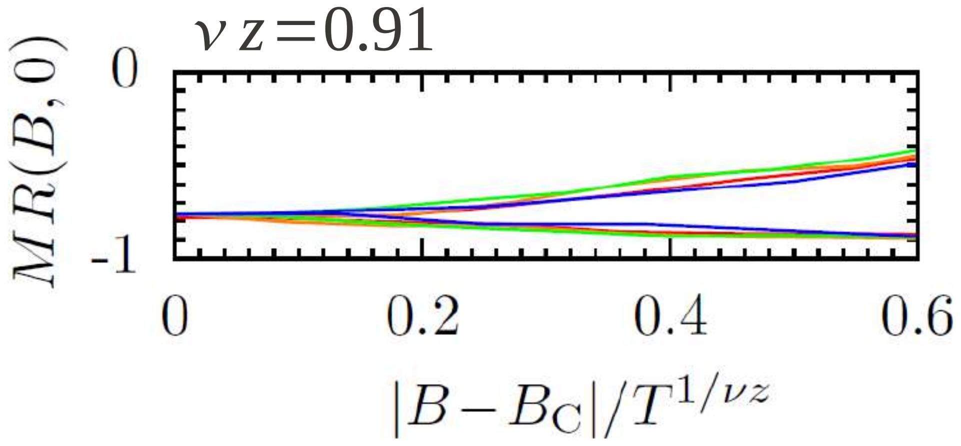




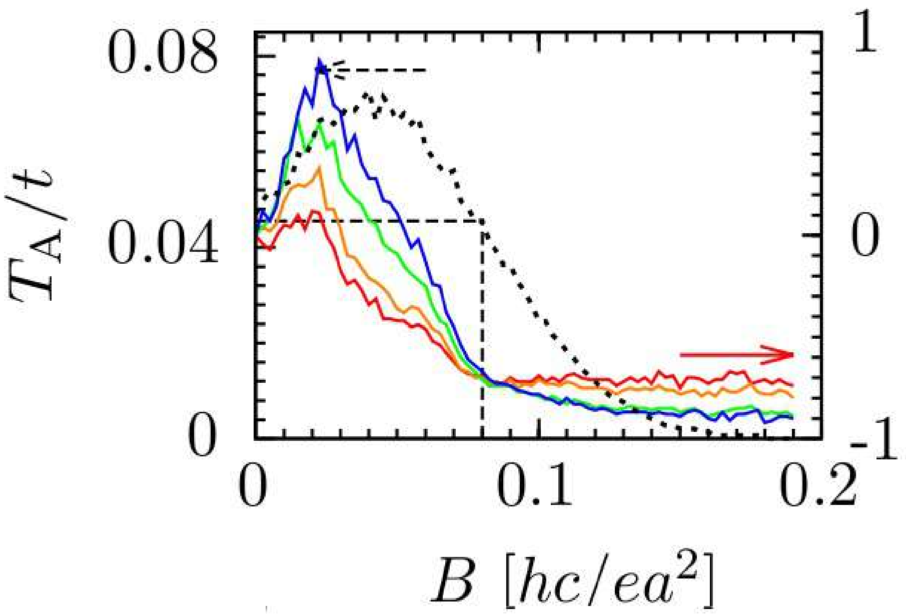
Highly disordered films

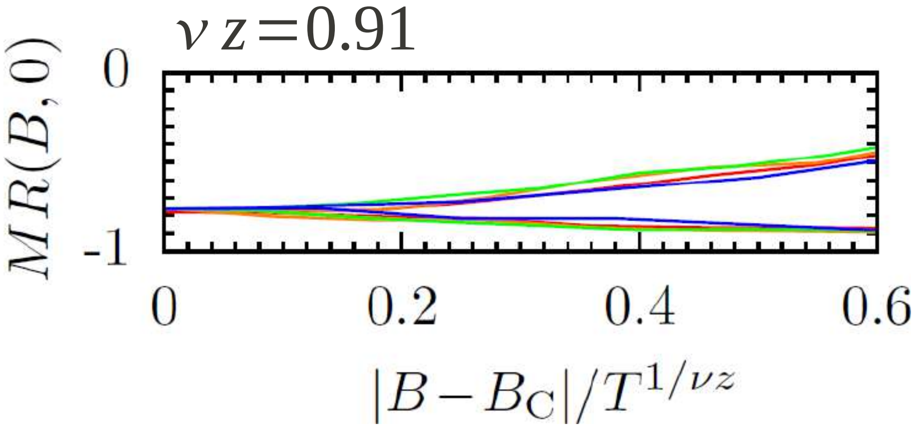
$$MR(B,T)=\frac{R(B,T)-R(0,T)}{R(0,T)}$$

$$R(B,T)=R_0(B)\,e^{T_A/T}$$

$$T_A(0)=T_A(B_C)$$

$$MR(B,T)=\frac{R_0(B)}{R_0(0)}\left(1+\frac{T'_A(B_C)(B-B_C)}{T}\right)-1$$

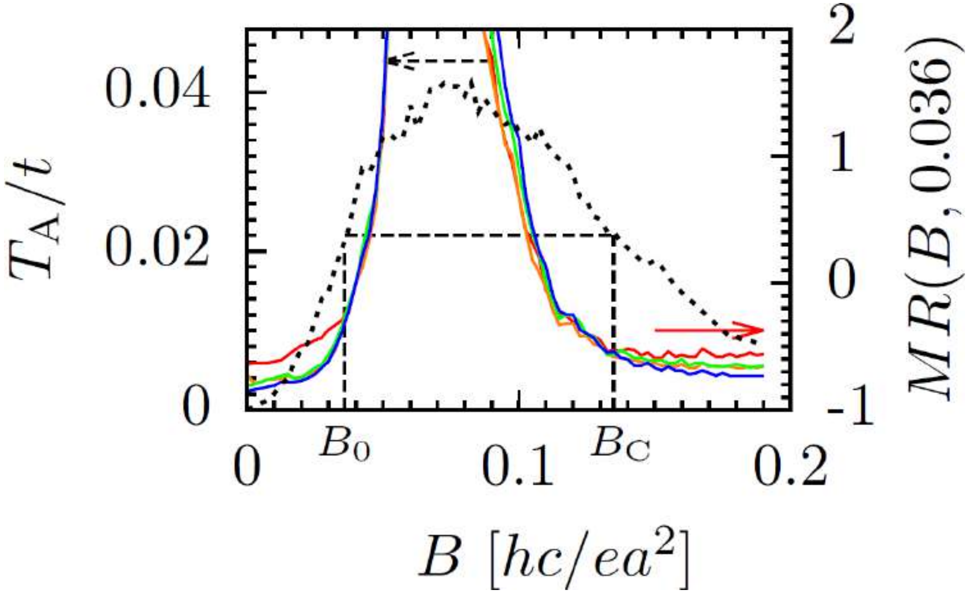




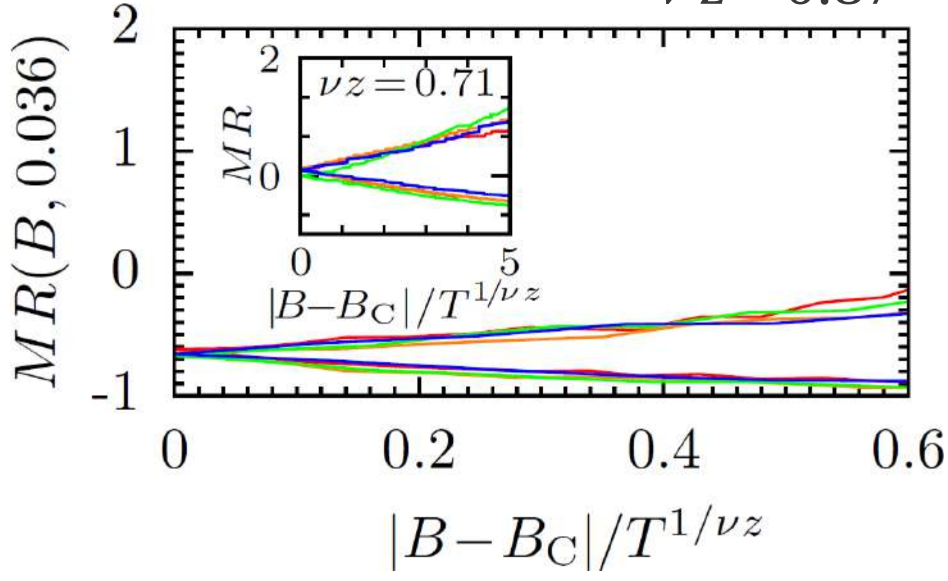
Highly disordered films

$$MR(B,T) = \frac{R(B,T) - R(B_0,T)}{R(B_0,T)}$$

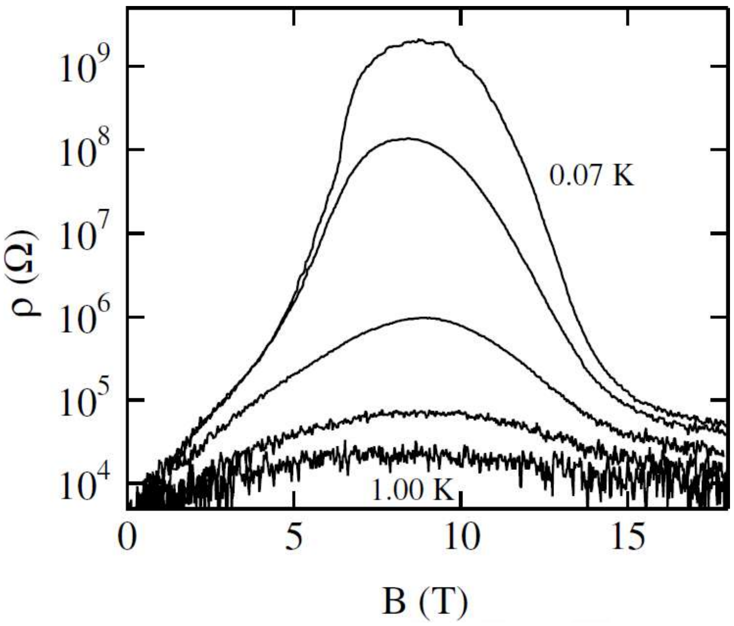
$$MR(B,T) = \frac{R_0(B)}{R_0(B_0)}\left(1 + \frac{T'_A(B_C)(B - B_C)}{T}\right) - 1$$



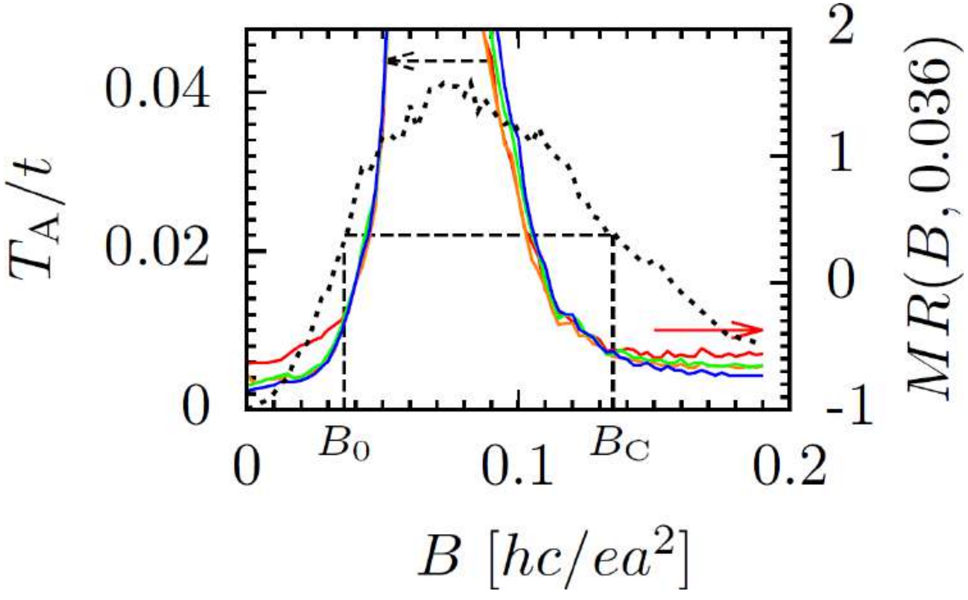
$\nu z = 0.87$

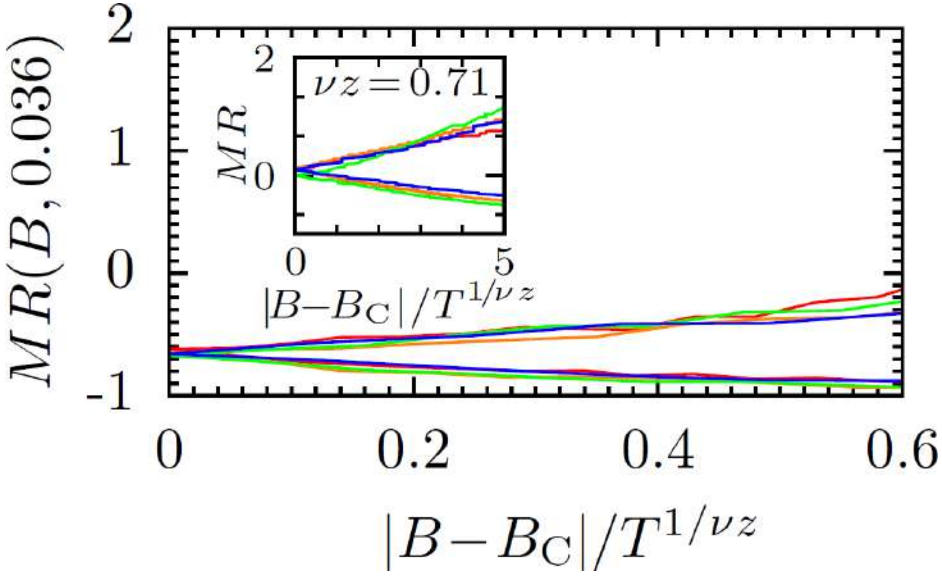


Highly disordered films



[Sambandamurthy 04]



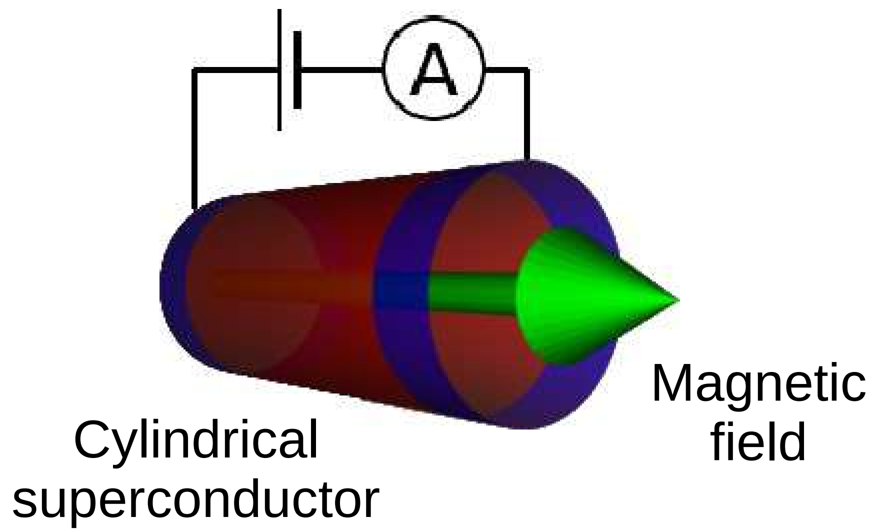


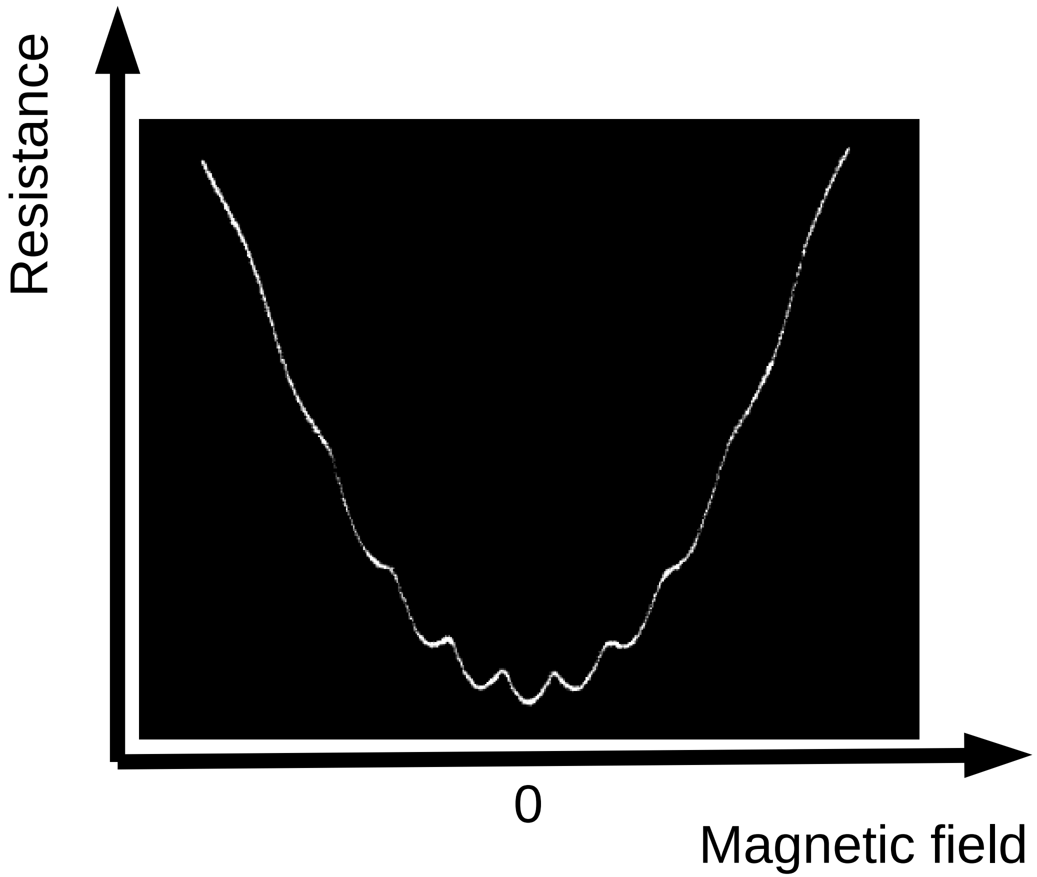
Summary & future prospects

- Developed new formalism that includes thermal phase fluctuations to calculate and probe transport in superconductors
- Magnetoresistance peak could be driven by activated transport through superconducting islands
- Universal scaling of MR curves could be consequence of activated transport
- Superconductor-insulator transition in small diameter cylinders is driven by phase fluctuations
- Flexibility allows us to study wide range of unexplained effects

Little-Parks in a large diameter cylinder

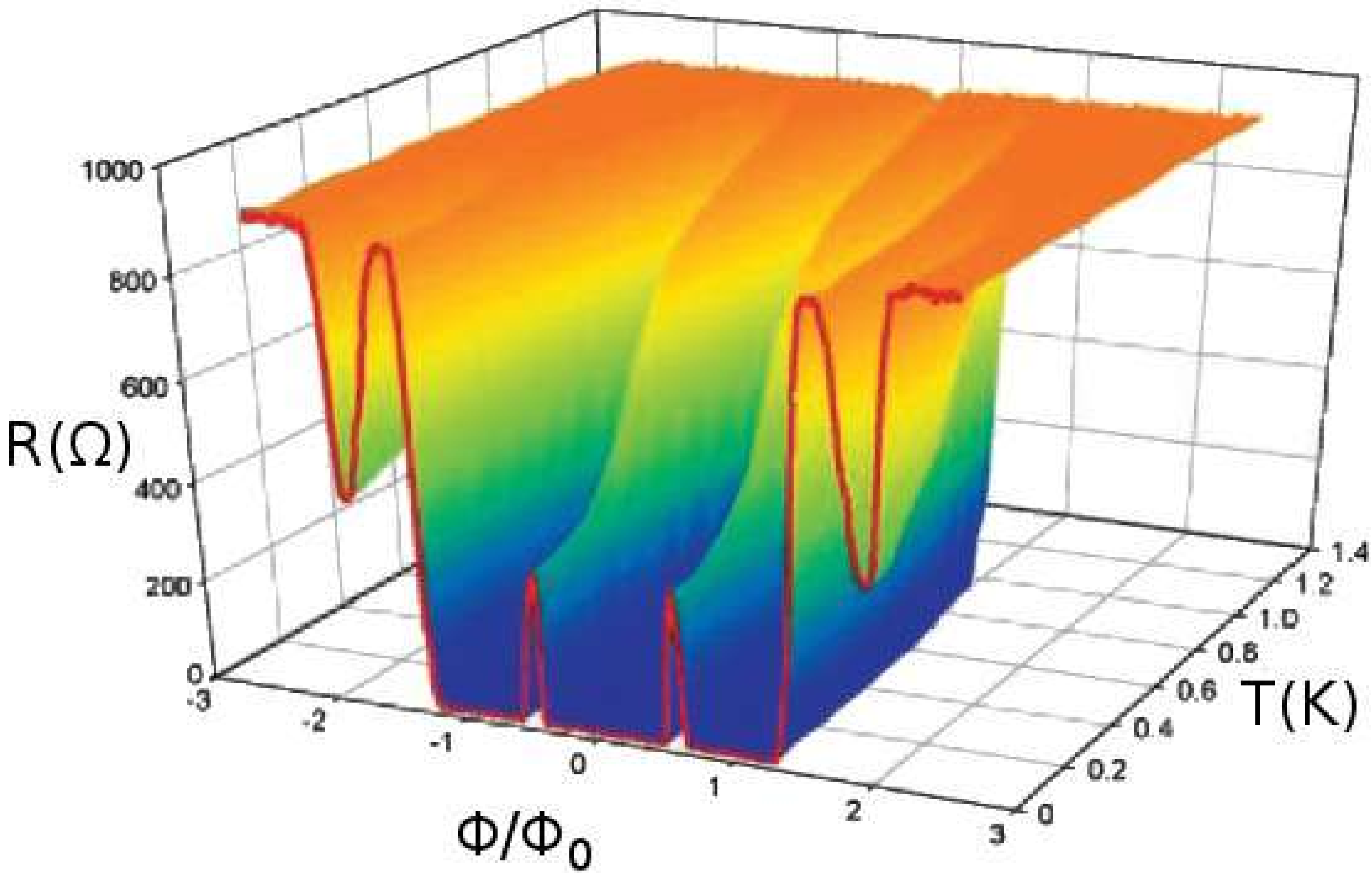
- Cylindrical superconductor held at transition temperature and zero threading flux [Little & Parks, PRL 1962]



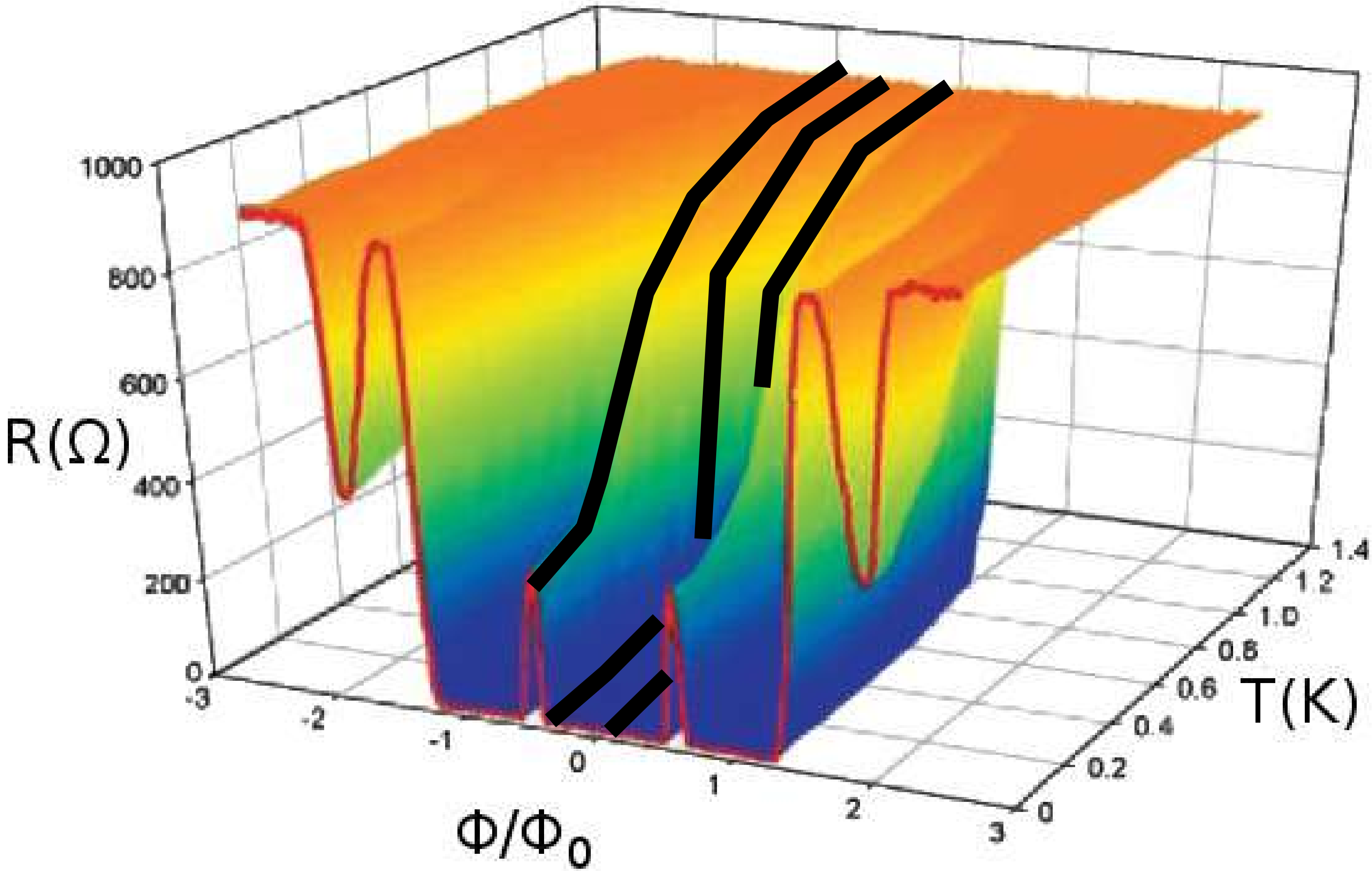


Little-Parks in a small diameter cylinder

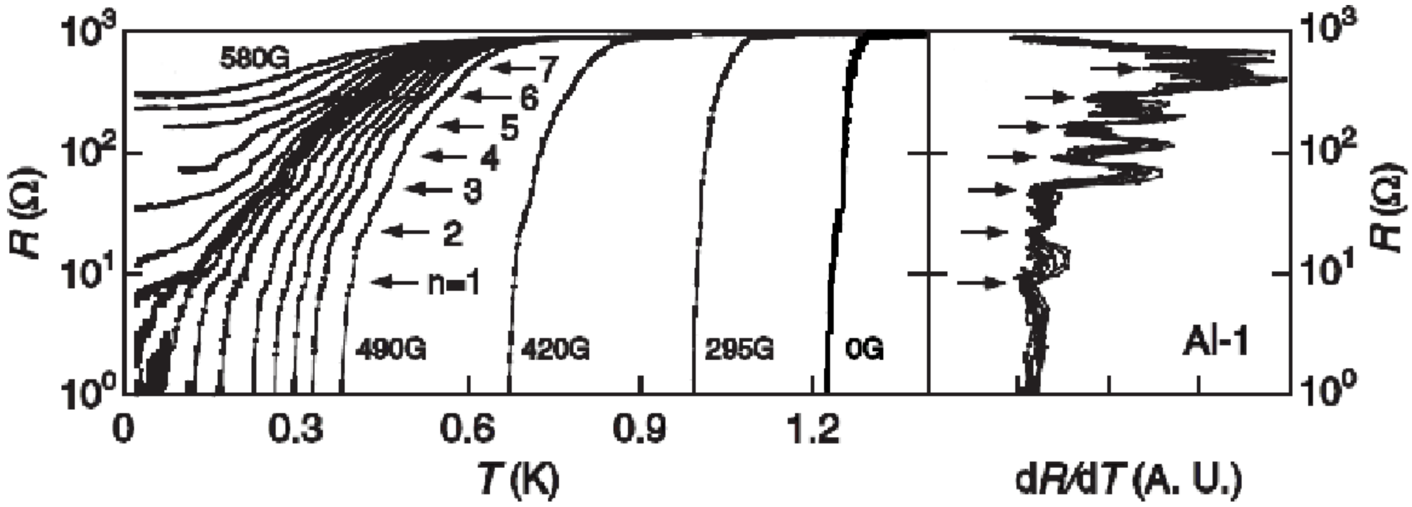
- Reduce cylinder diameter to superconducting correlation length [Liu *et al*., Science 2001; Wang *et al*., PRL 2005]

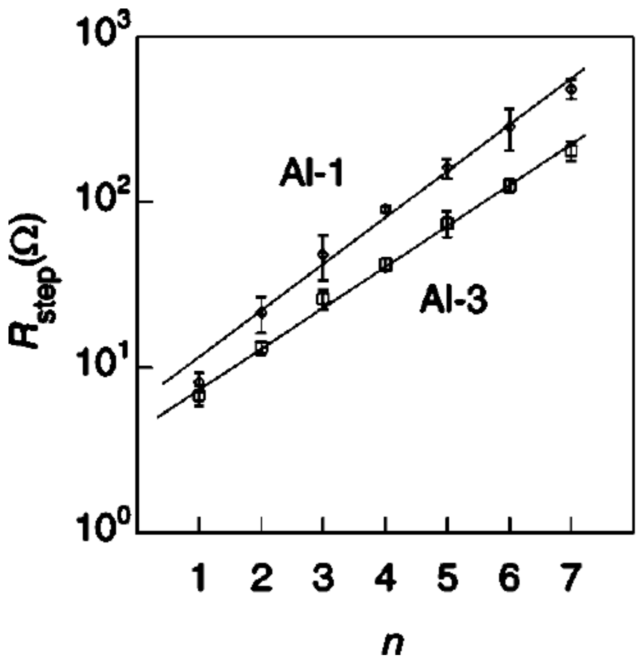


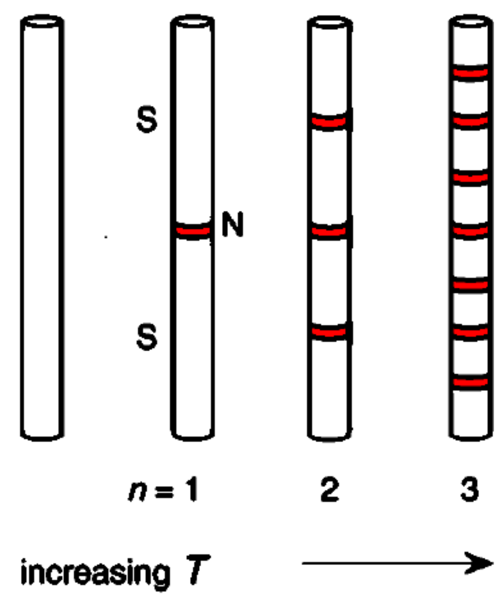
Little-Parks in a small diameter cylinder



Quantum phase transition hypothesis

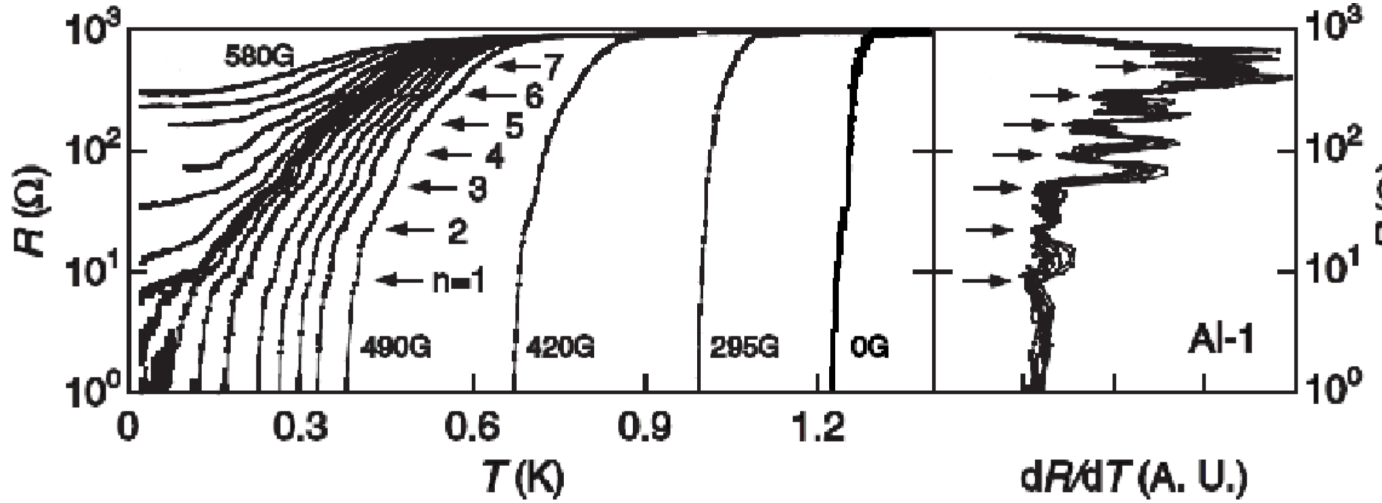


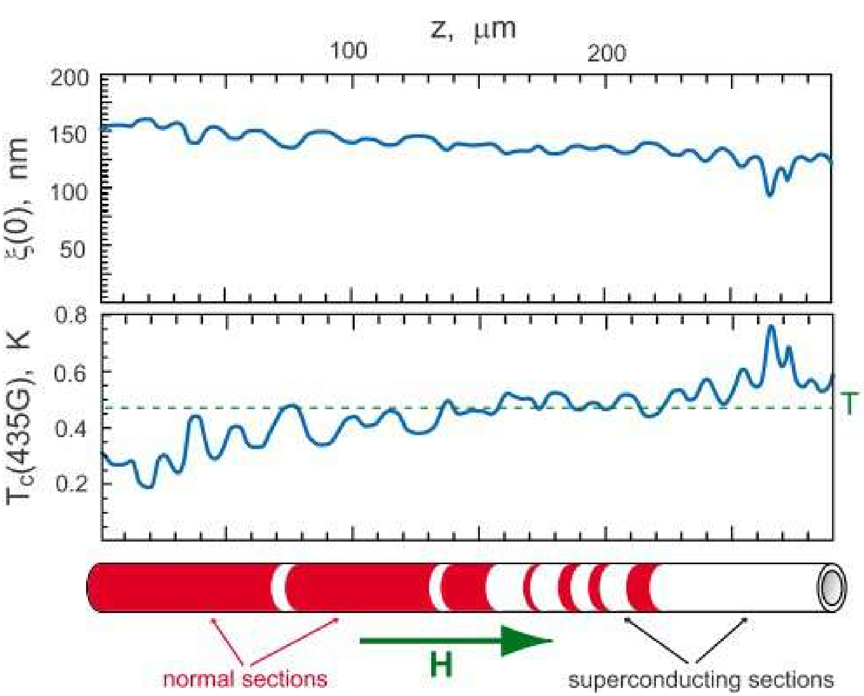


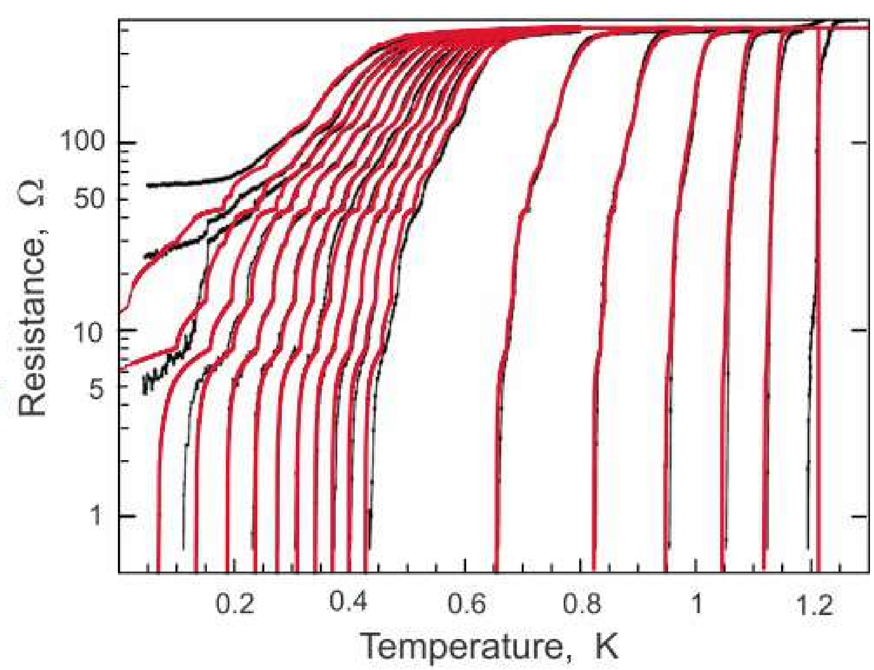


Wang et al. PRL (2005)

Mean-field BCS transition hypothesis



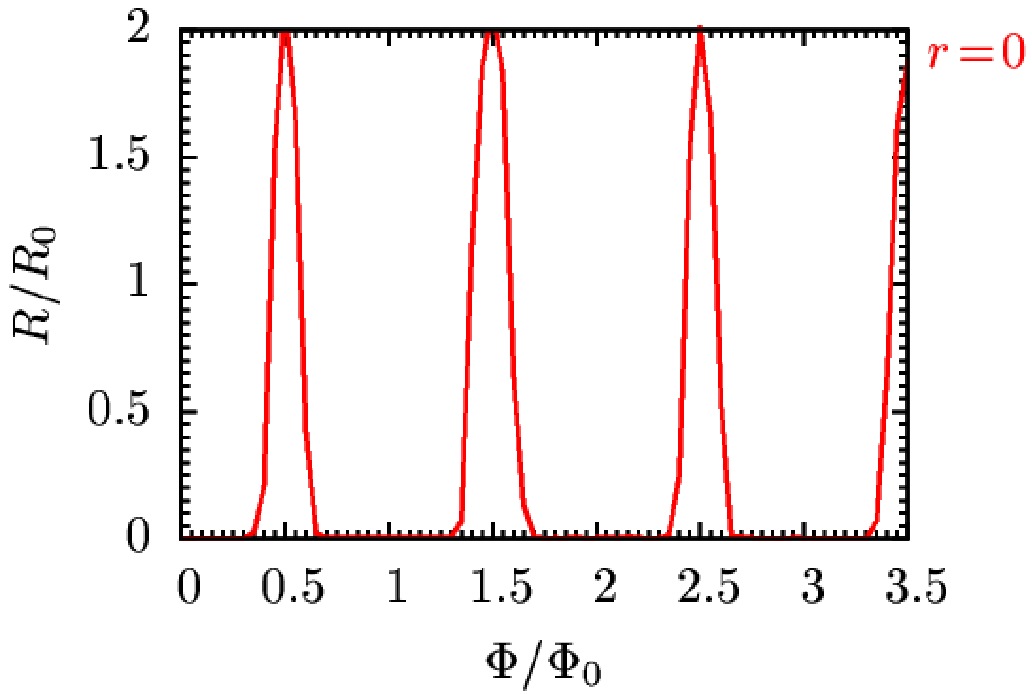


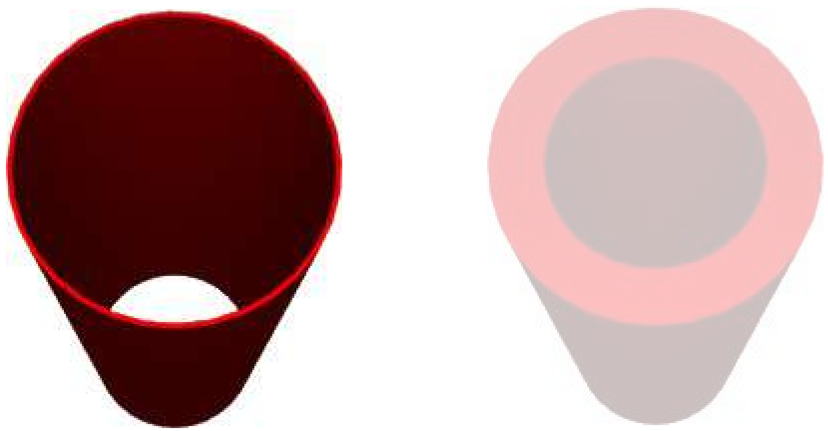


Dao & Chibotaru, PRB (2009)

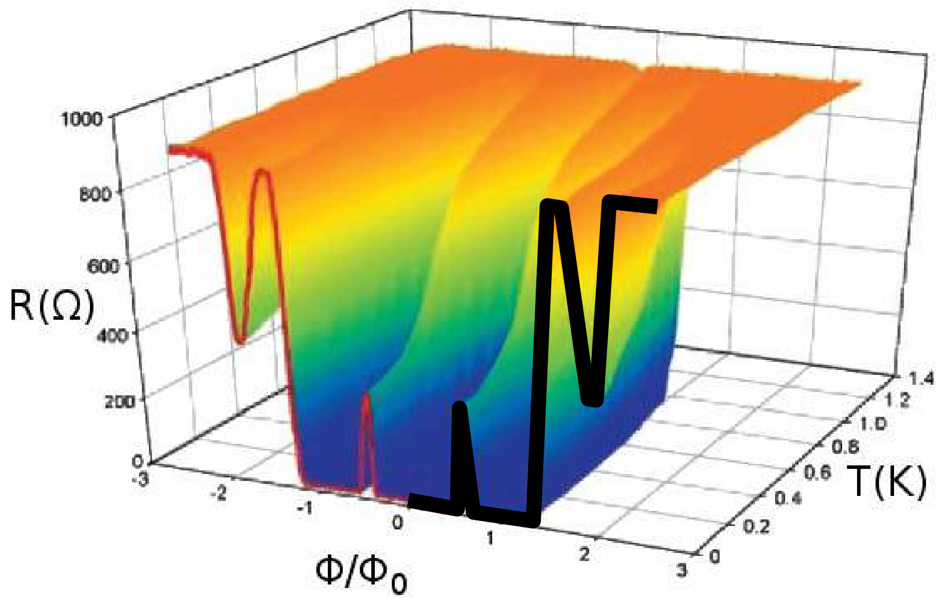
Little-Parks in a small diameter cylinder

Theory:



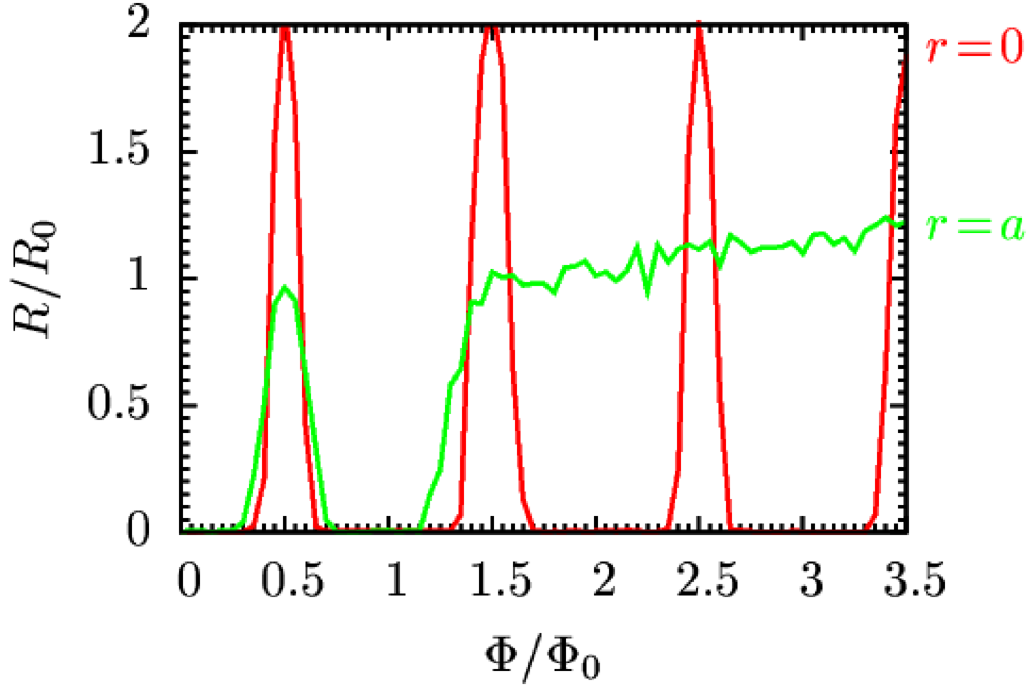


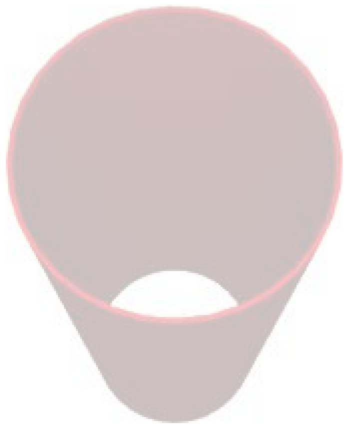
Experiment:



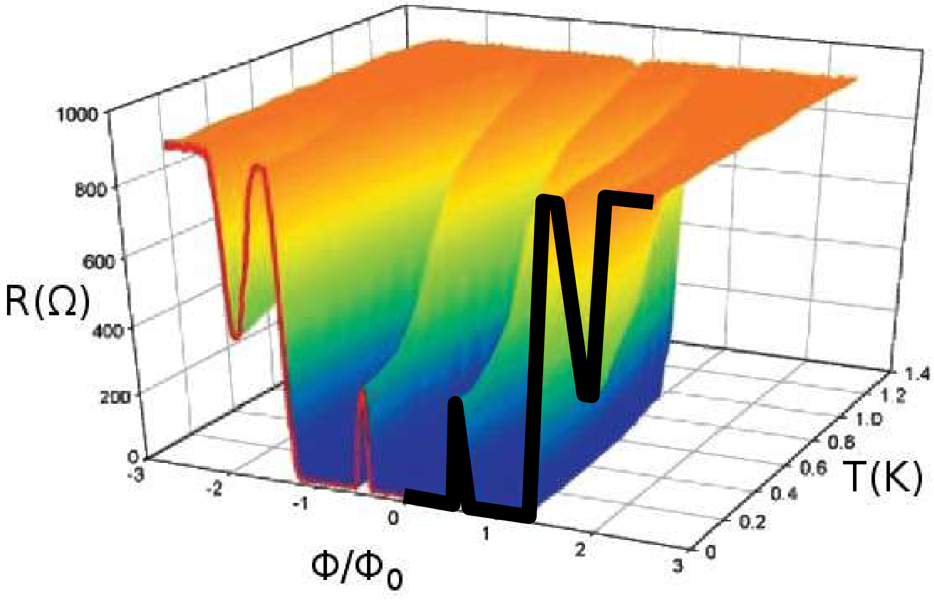
Little-Parks in a small diameter cylinder

Theory:



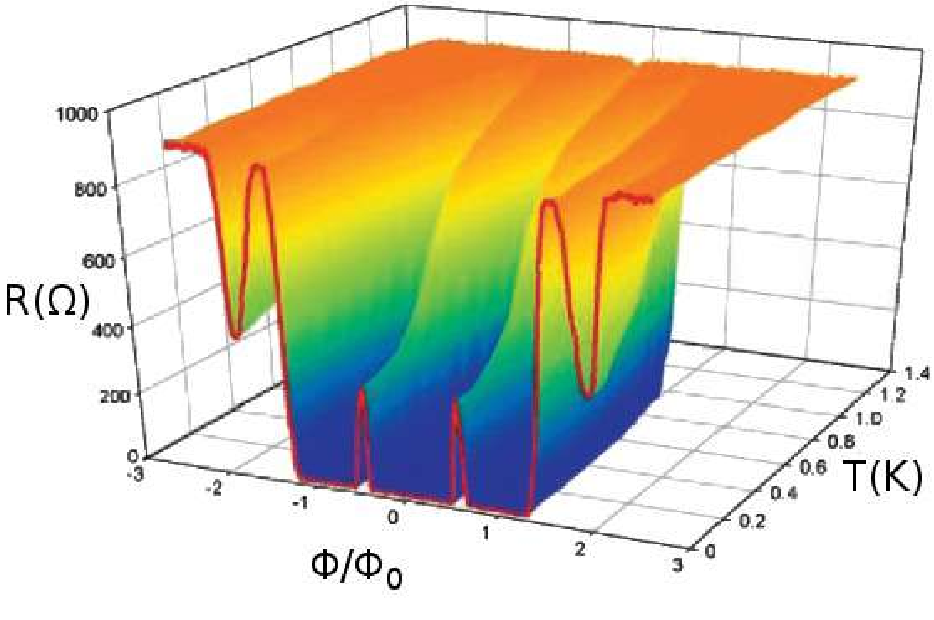


Experiment:



Little-Parks in a small diameter cylinder

Experiment:



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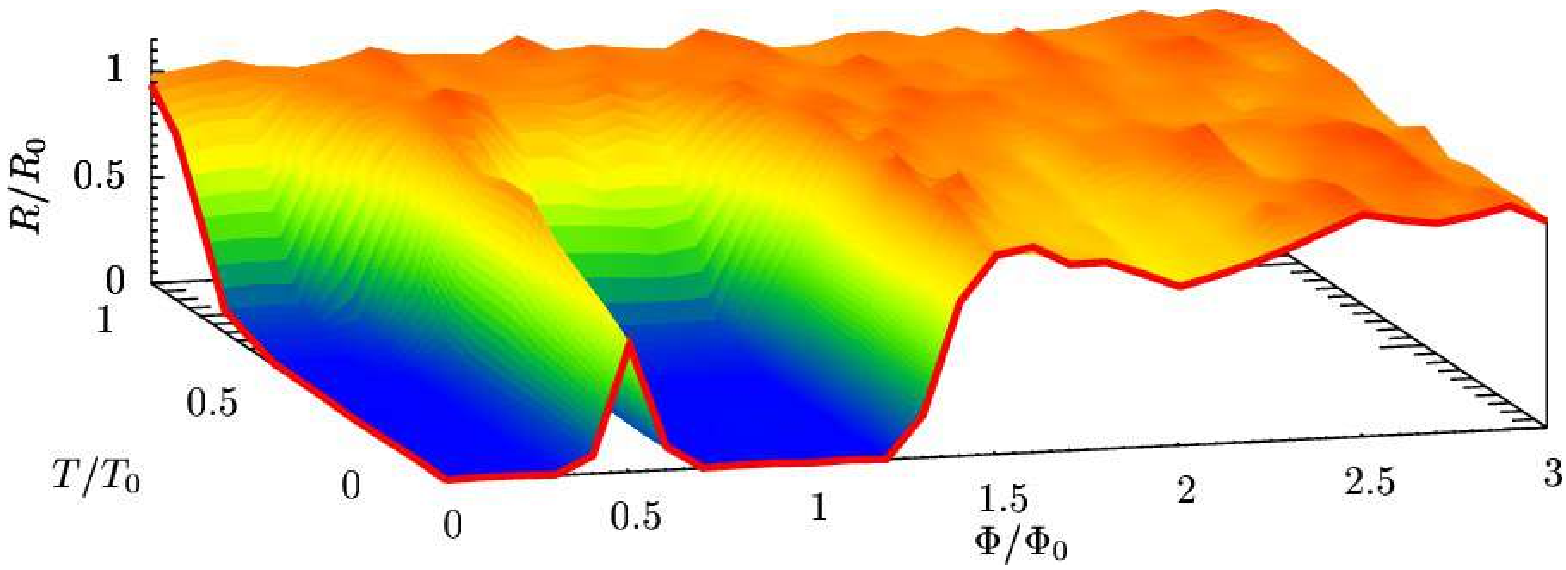
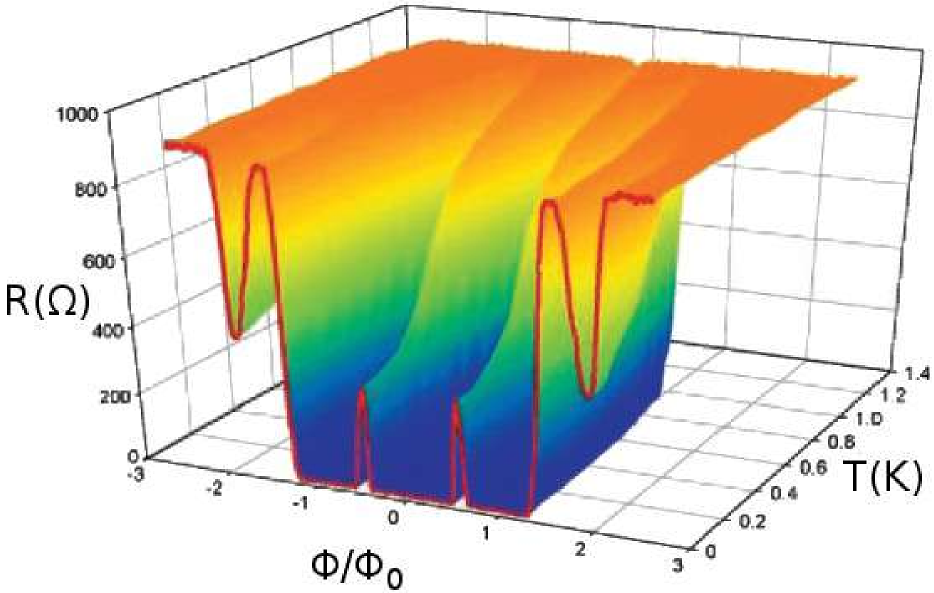
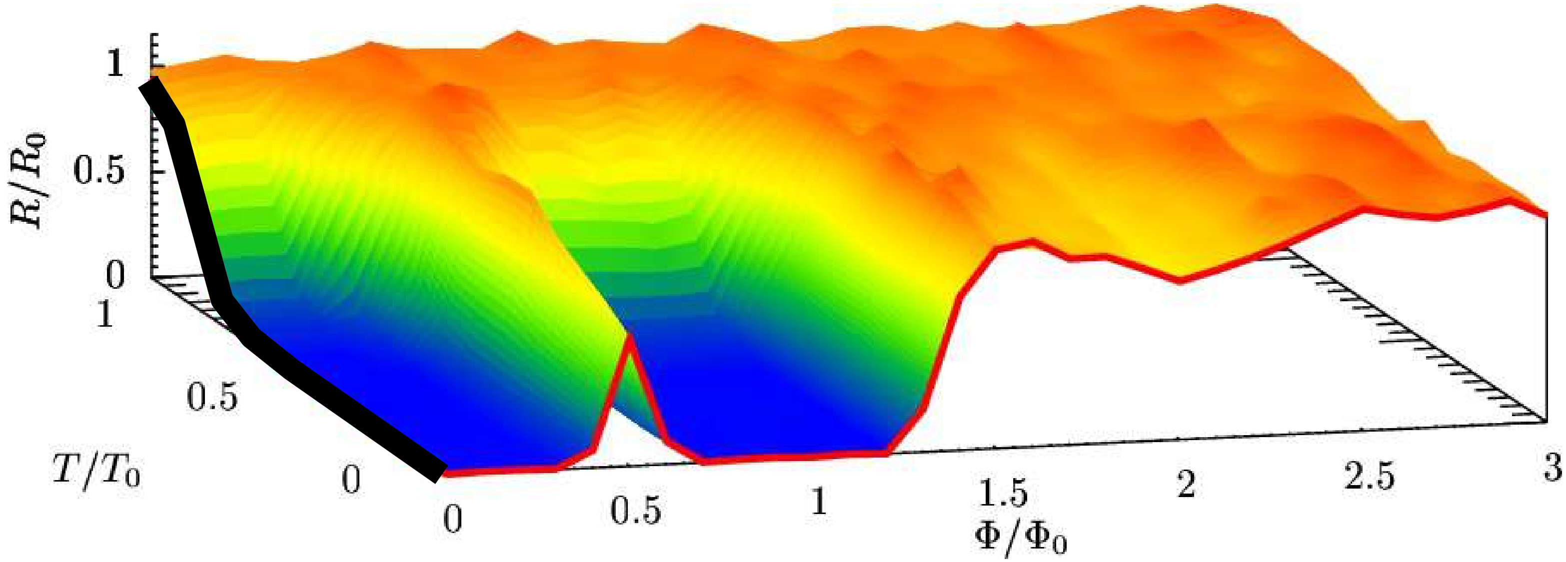


Little-Parks in a small diameter cylinder

Experiment:

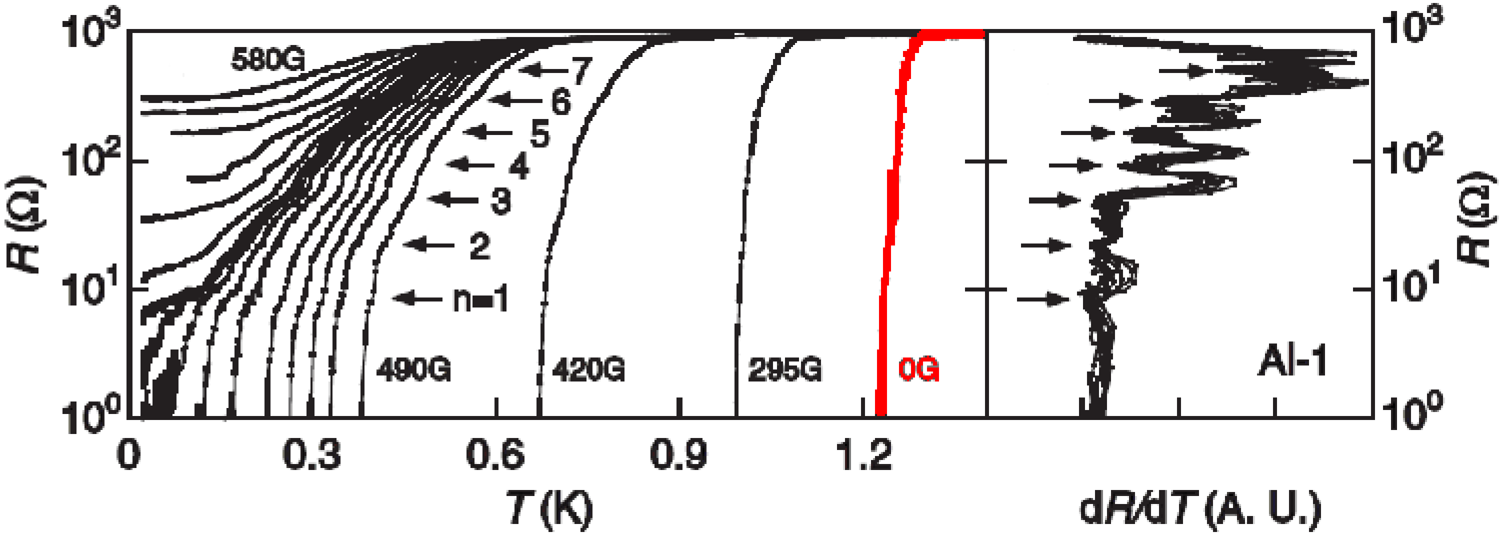


Theory:

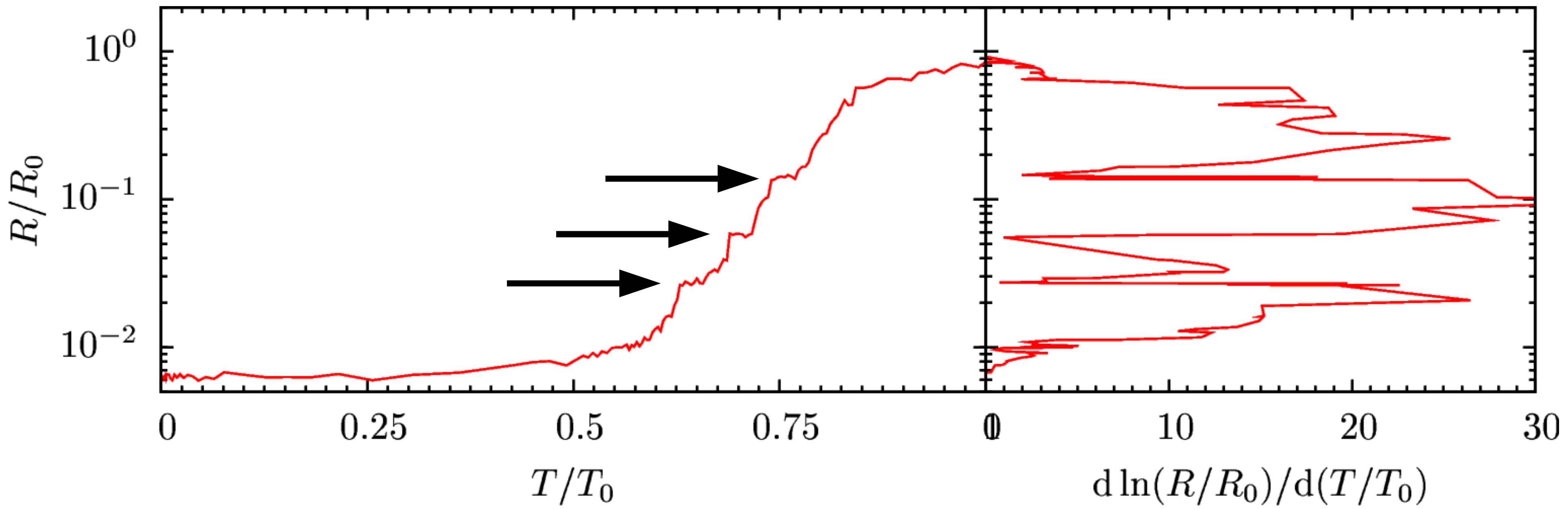


Evidence of phase reconstruction

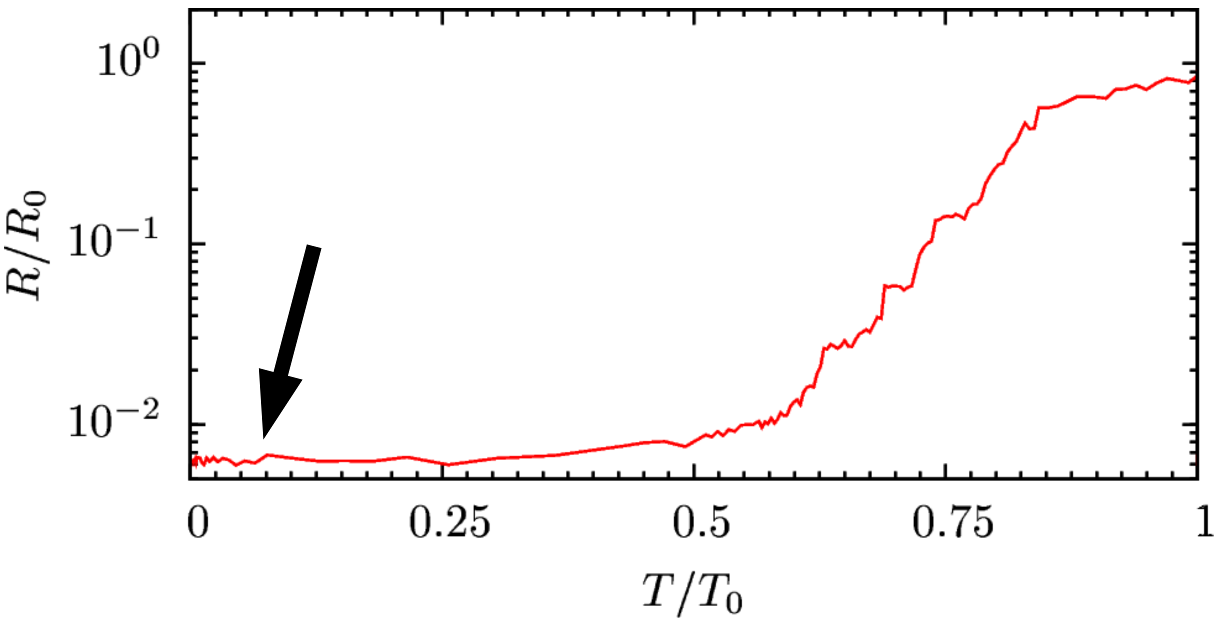
- Experiment:

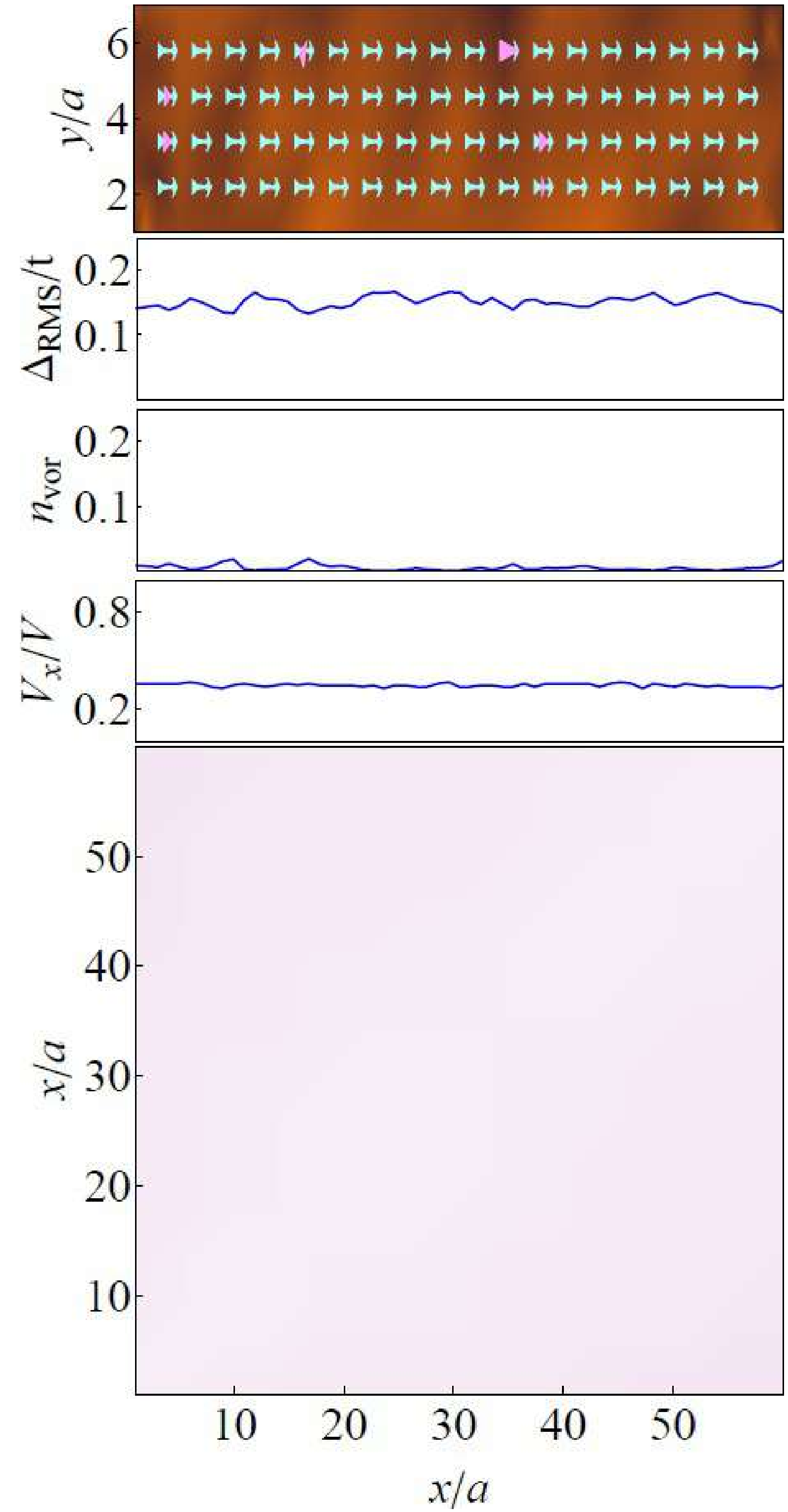


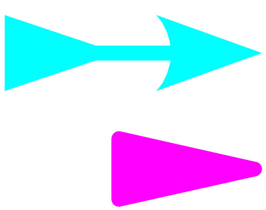
- Theory:



Completely superconducting

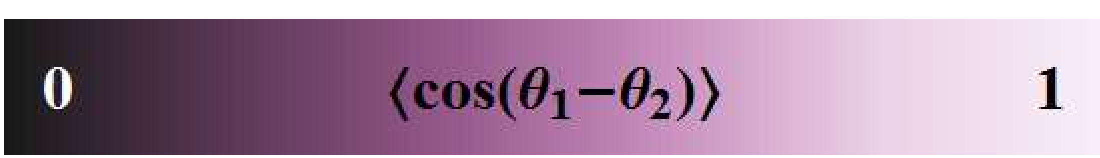




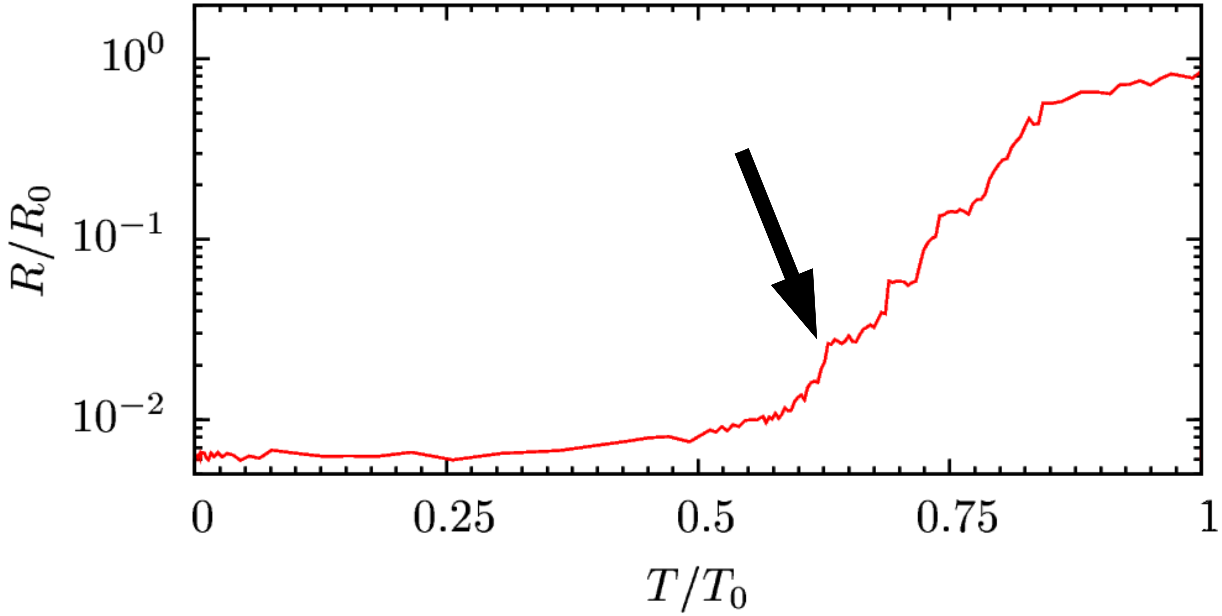


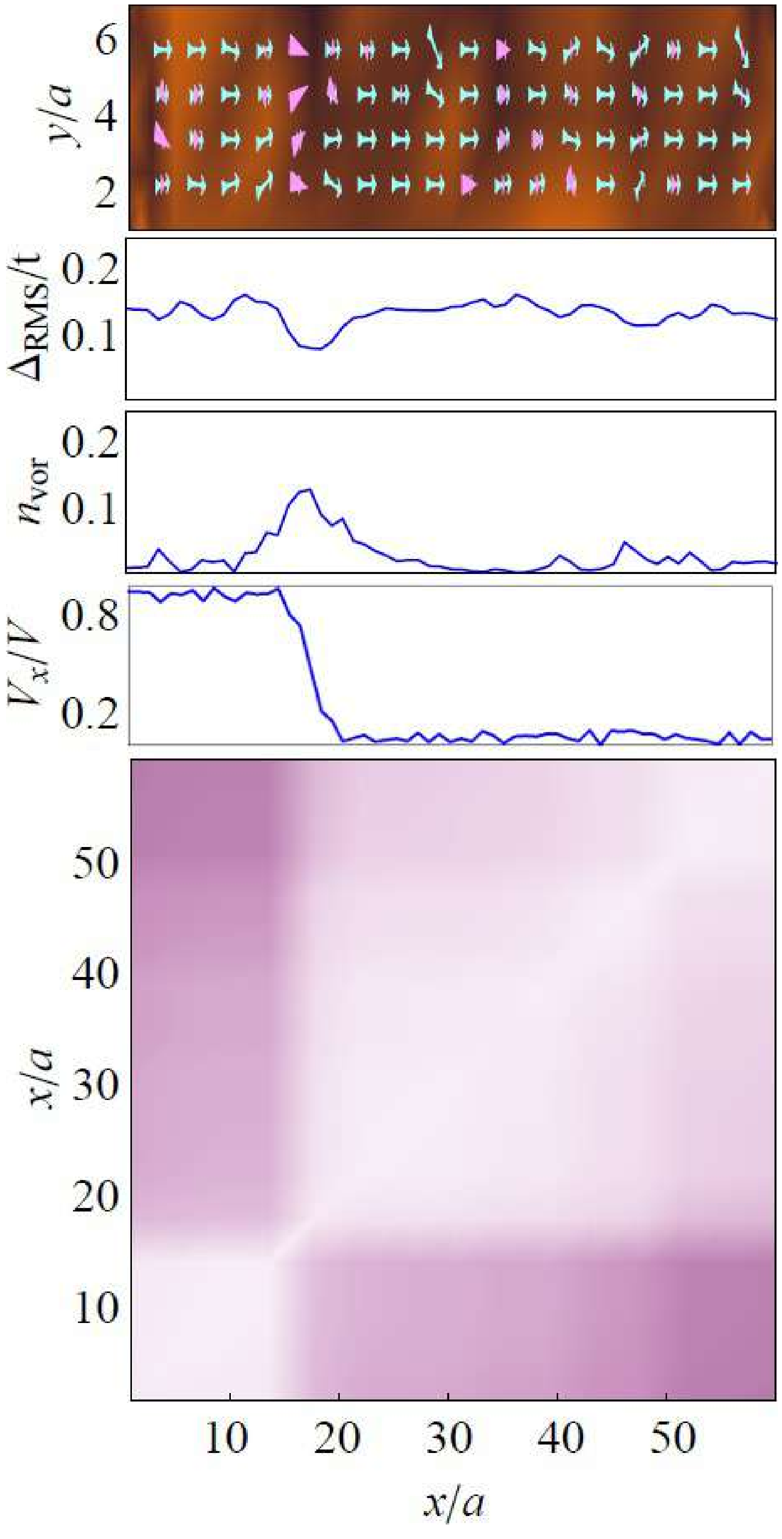
Superconducting current

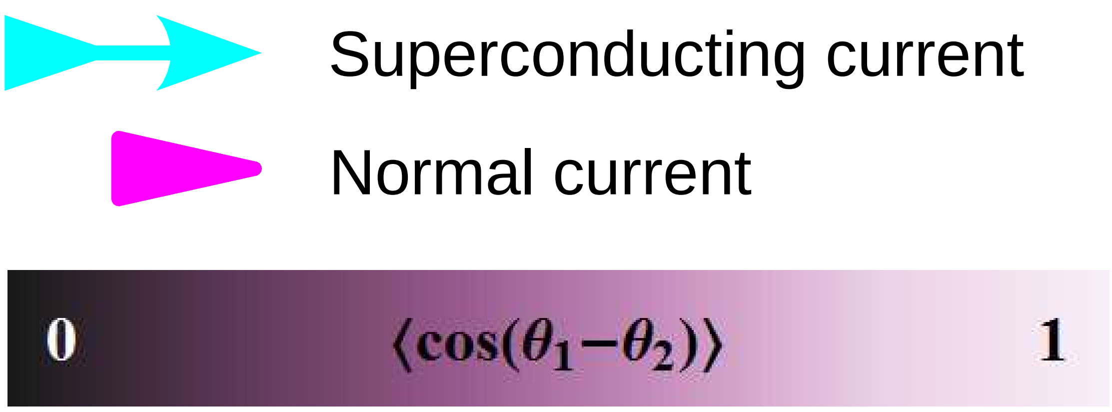
Normal current



Two superconducting regions







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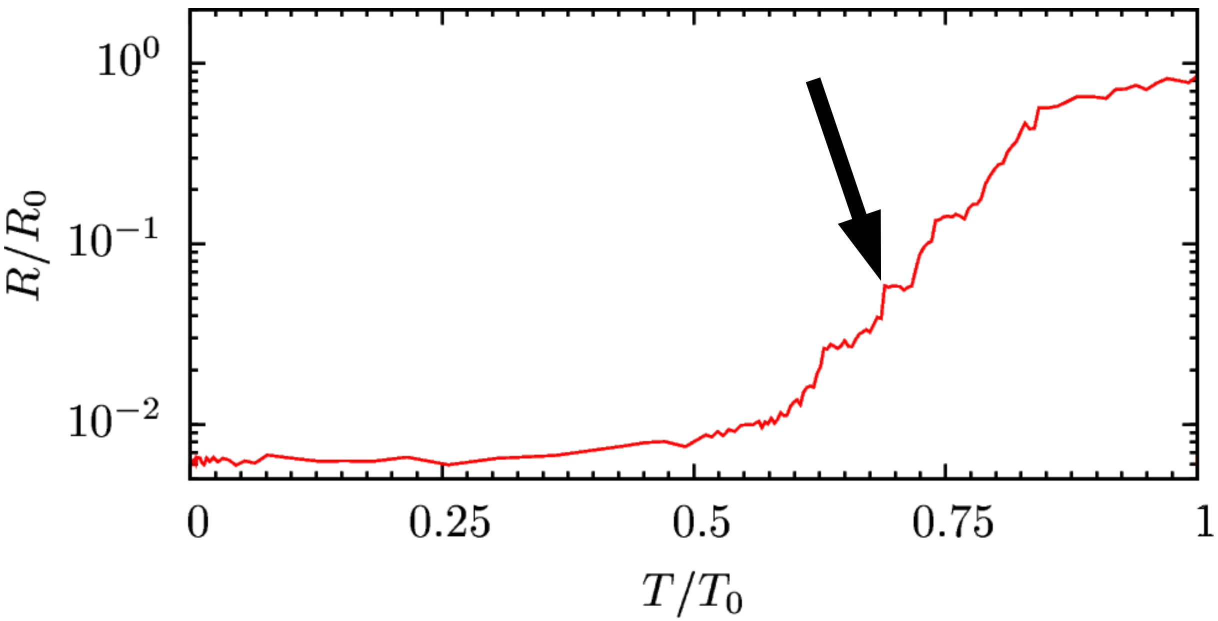
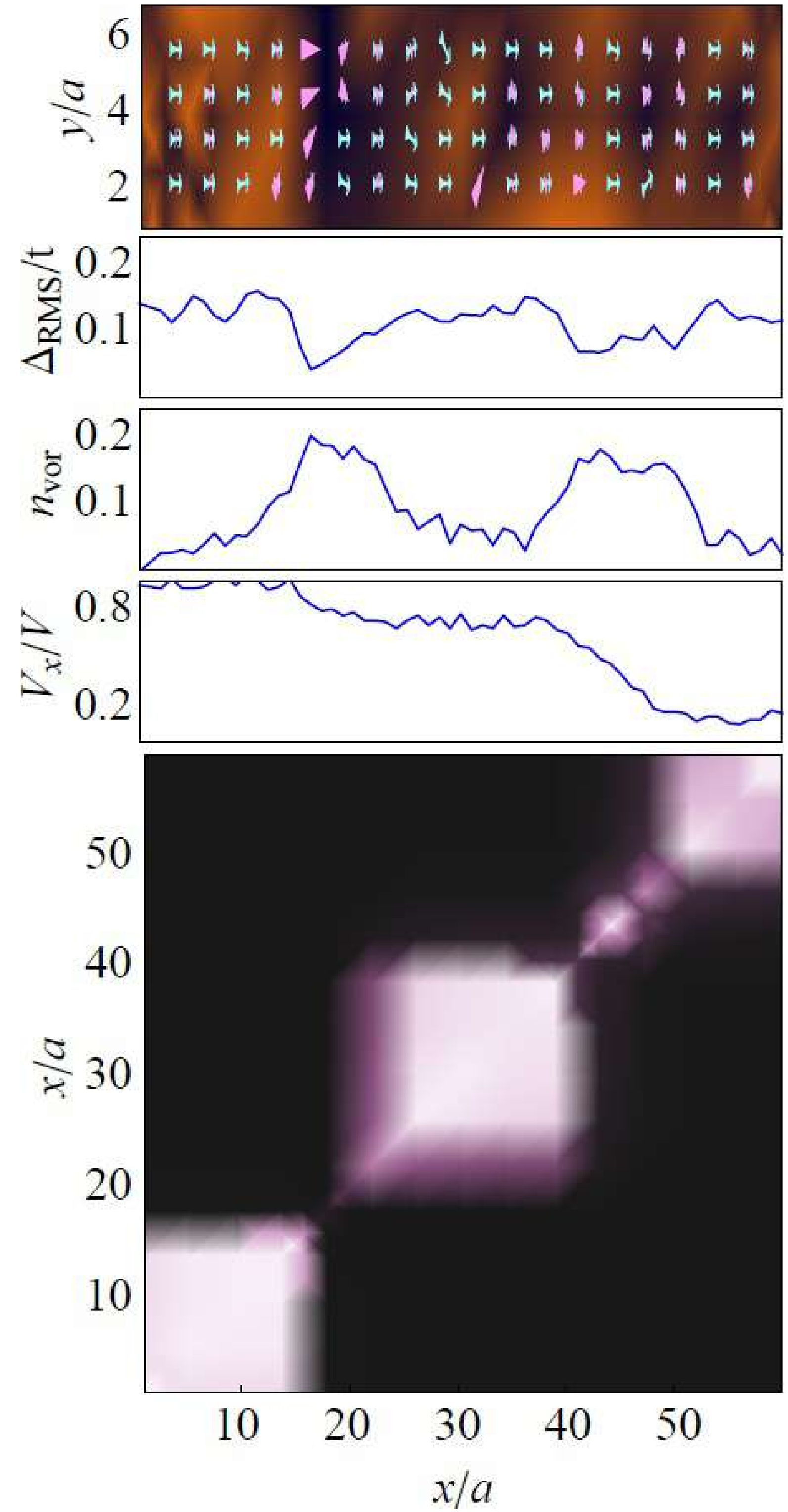
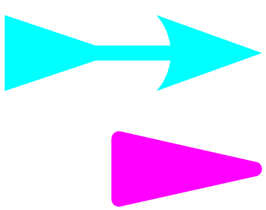
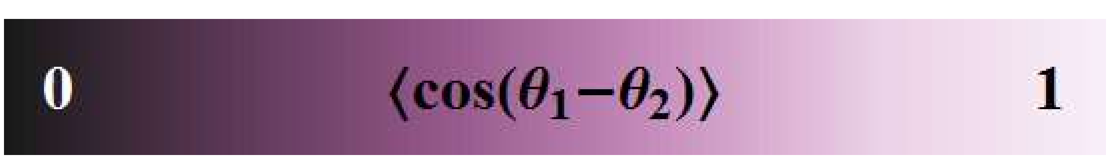




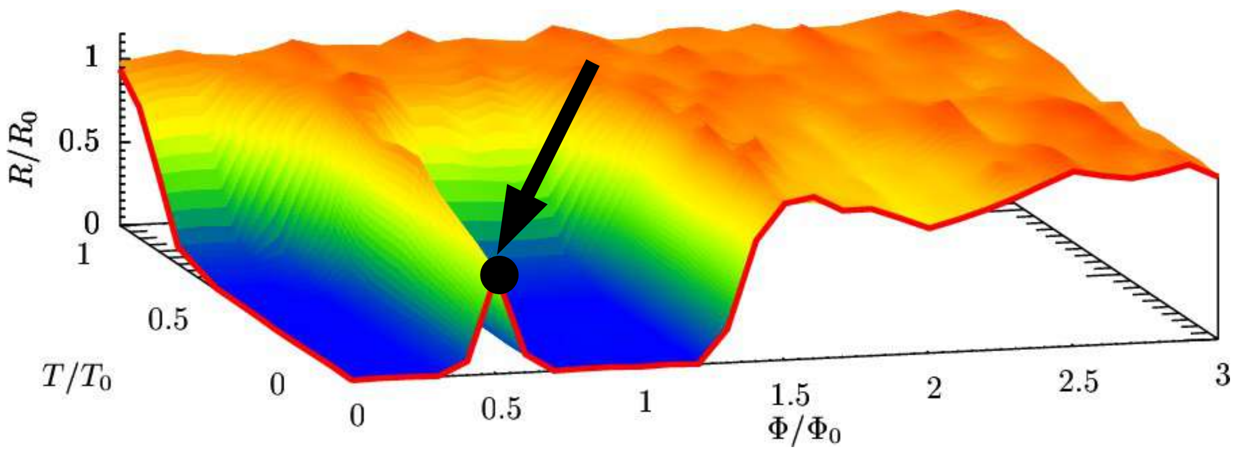


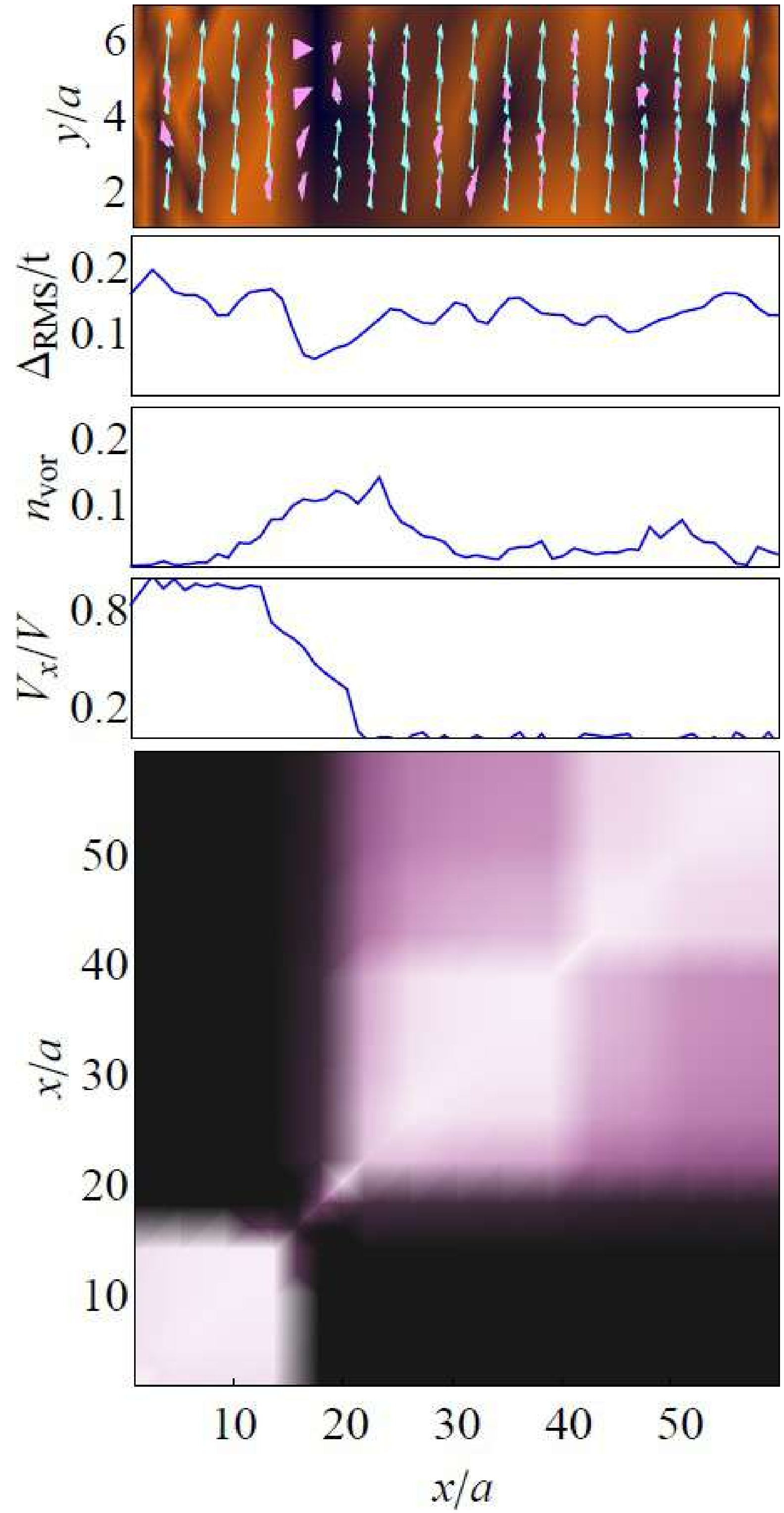
Superconducting current

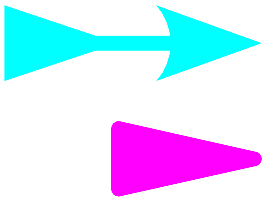
Normal current



Half flux quantum normal state

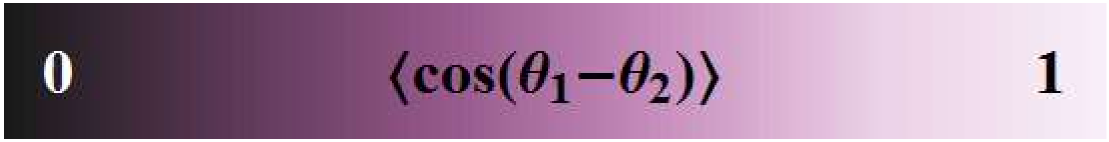






Superconducting current

Normal current



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