Pair density wave ferromagnetism

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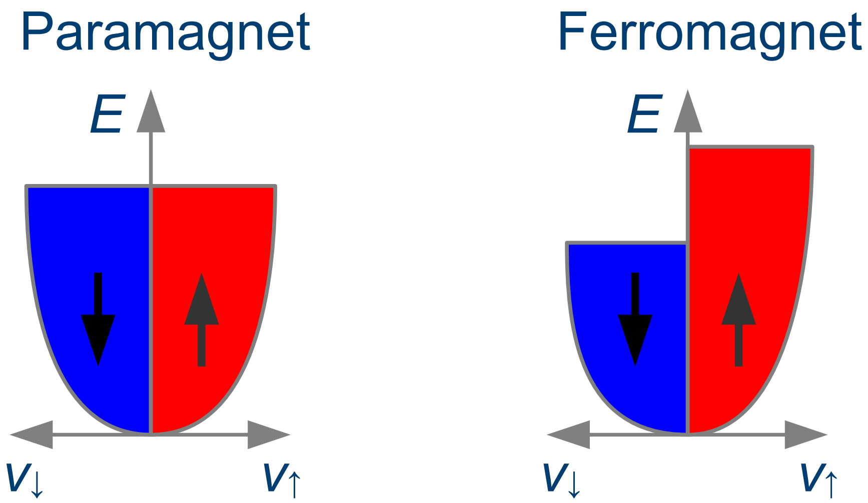
Minimal Hamiltonian

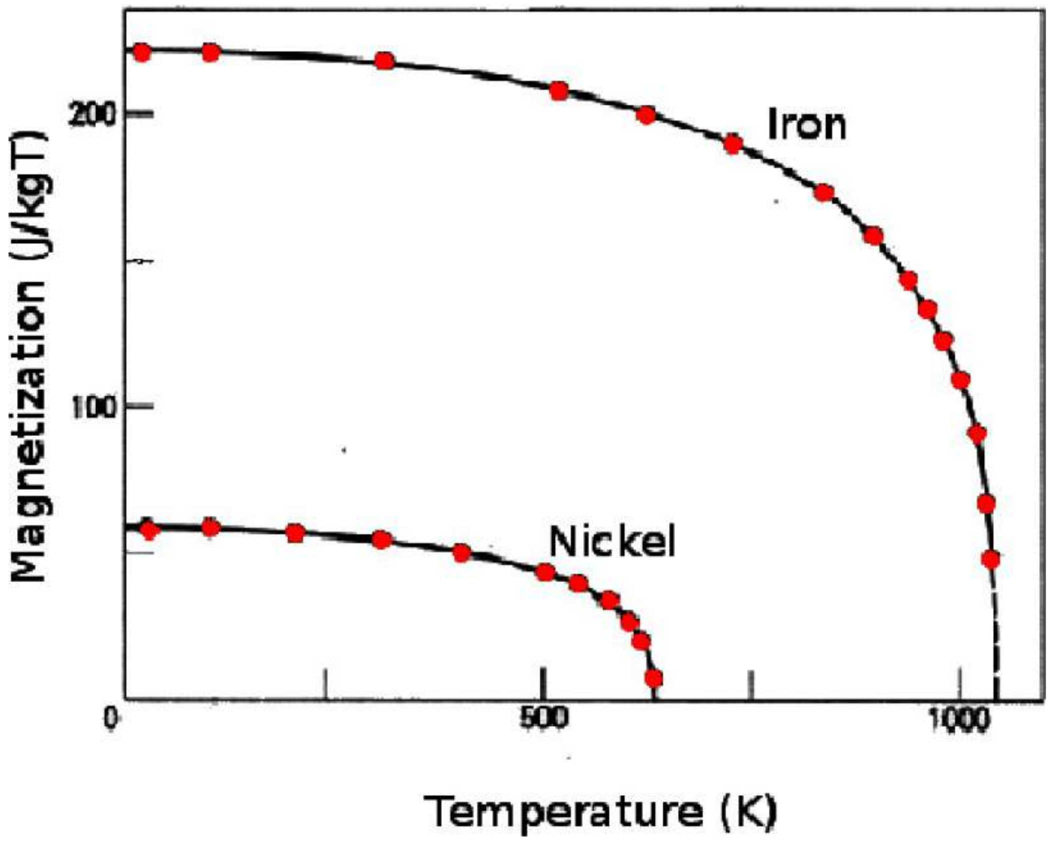
$$H=\sum_{\boldsymbol{k}\sigma}\epsilon_{\boldsymbol{k}}c^{\dagger}_{\boldsymbol{k}\sigma}c_{\boldsymbol{k}\sigma}+g\sum_{\boldsymbol{k}\boldsymbol{p}\boldsymbol{q}}c^{\dagger}_{\boldsymbol{k}+\boldsymbol{q}/2\uparrow}c^{\dagger}_{\boldsymbol{p}-\boldsymbol{q}/2\downarrow}c_{\boldsymbol{p}+\boldsymbol{q}/2\downarrow}c_{\boldsymbol{k}-\boldsymbol{q}/2\uparrow}$$

Stoner model

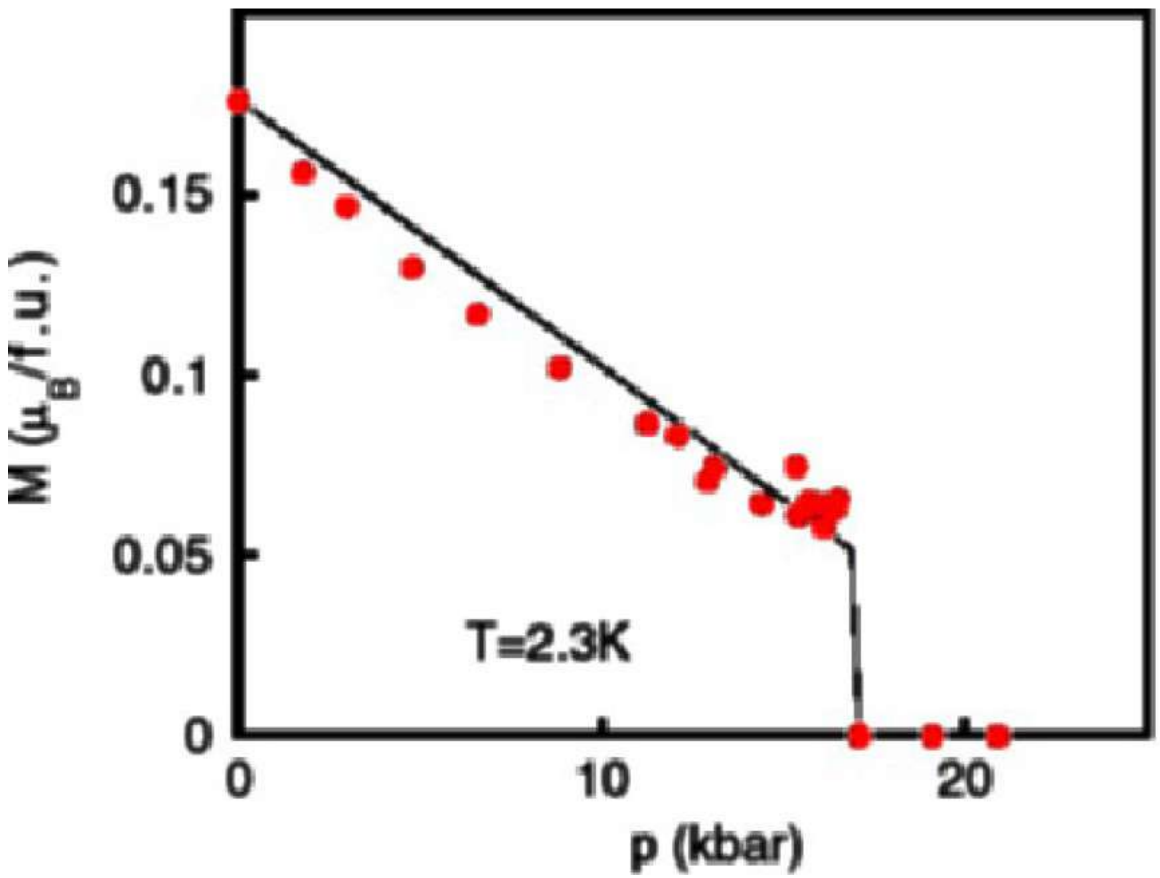
$$E = \sum_{\boldsymbol{k}\sigma} \epsilon_{\boldsymbol{k}} n_\sigma(\epsilon_{\boldsymbol{k}}) + g N_\uparrow N_\downarrow$$



Stoner magnetism: Iron & Nickel



Mysteries in magnetism: $ZrZn_2$



Uhlarz *et al.* PRL (2004)

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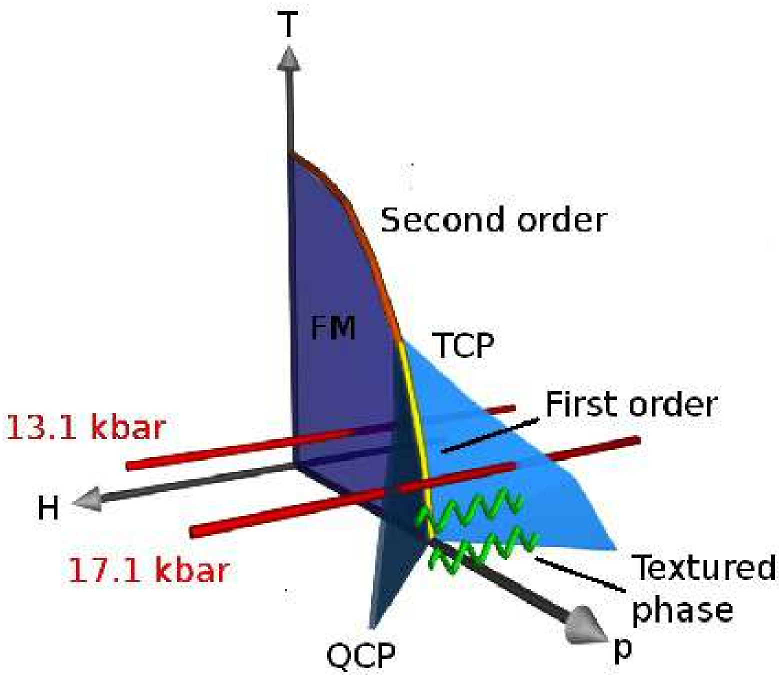
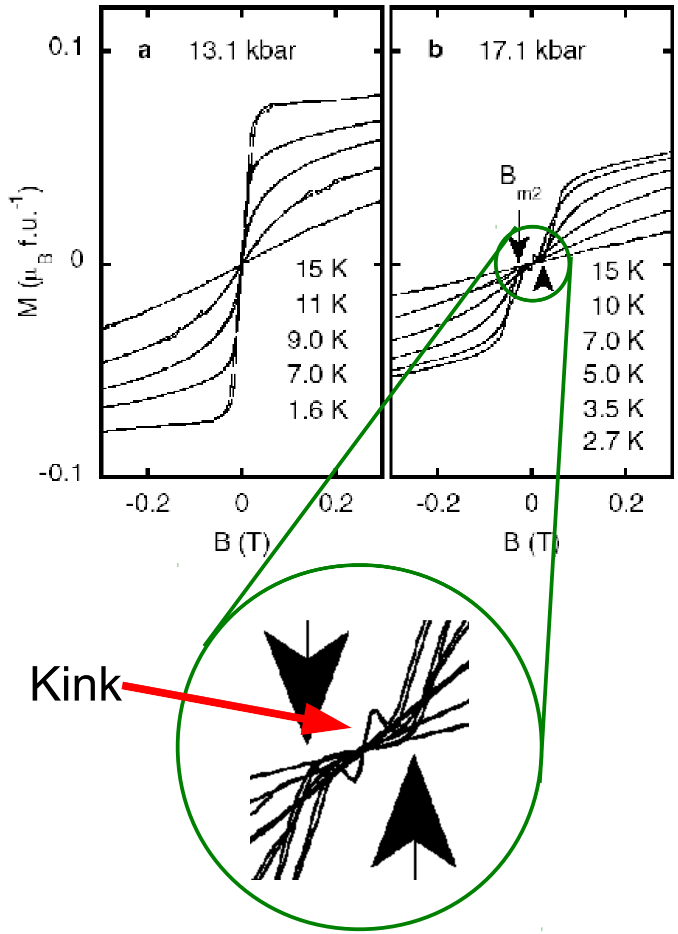
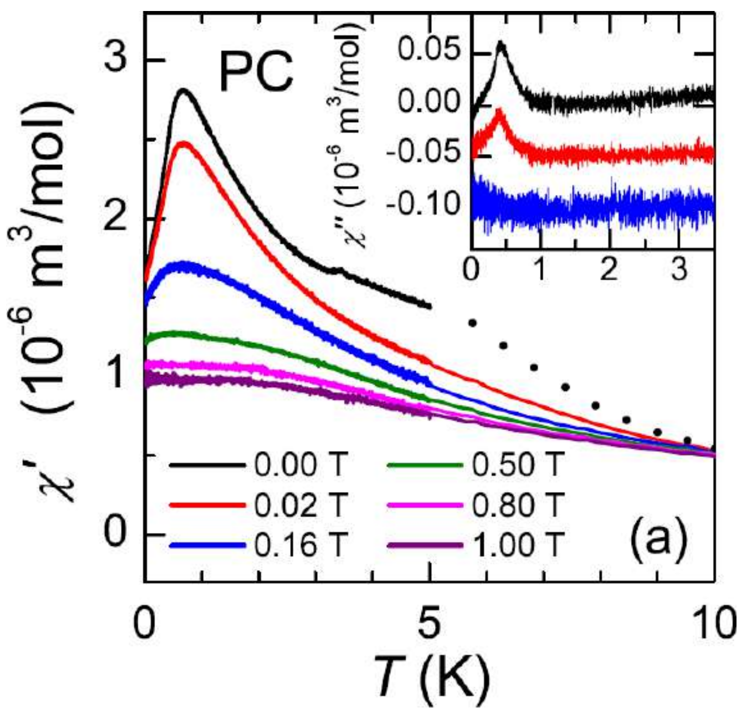




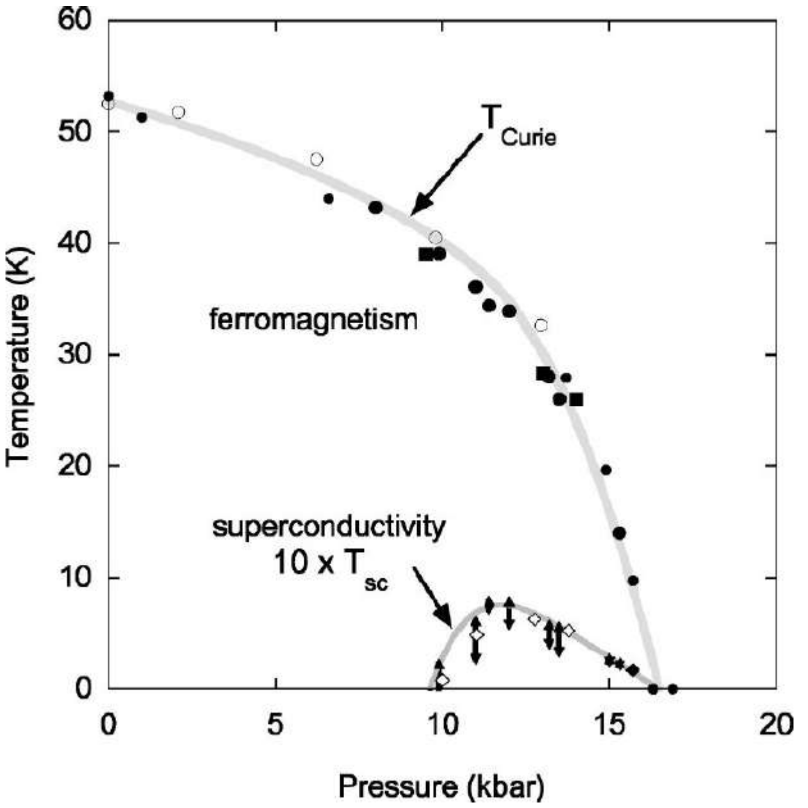
Uhlarz *et al.* PRL (2004)

Mysteries in magnetism: CeFePO



Lausberg *et al*. Phys. Rev. Lett. **109**, 216402 (2012)

Mysteries in magnetism: UGe_2



Huxley *et al*. Phys. Rev. B **63**, 144519 (2001)

Motivation: Larkin-Pikin

$$Z = \int_{-\infty}^{\infty} d\varphi \exp\left[-\frac{aM^2 + bM^2\varphi + c\varphi^2}{T} \right]$$

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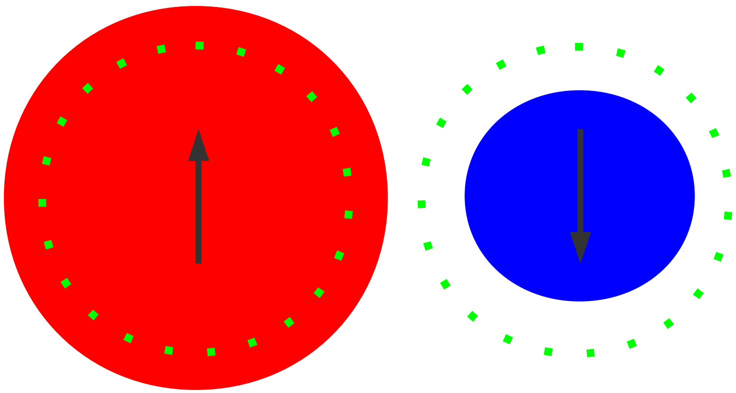
$$F=aM^2-\frac{b^2M^4}{4c}$$

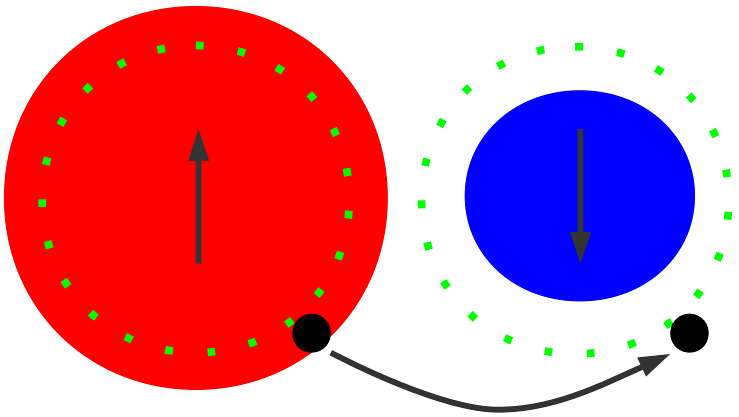
Fluctuation contributions

Ferromagnetic



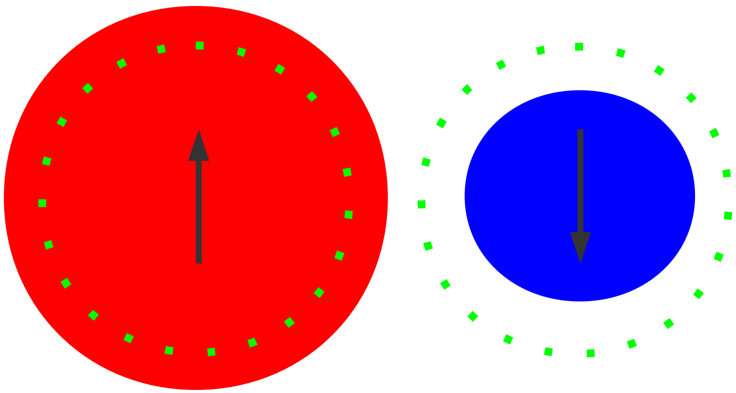
Fluctuation contributions

Ferromagnetic

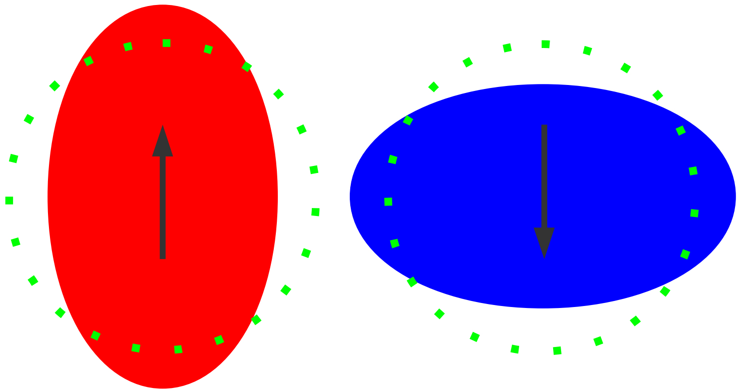


Fluctuation contributions

Ferromagnetic

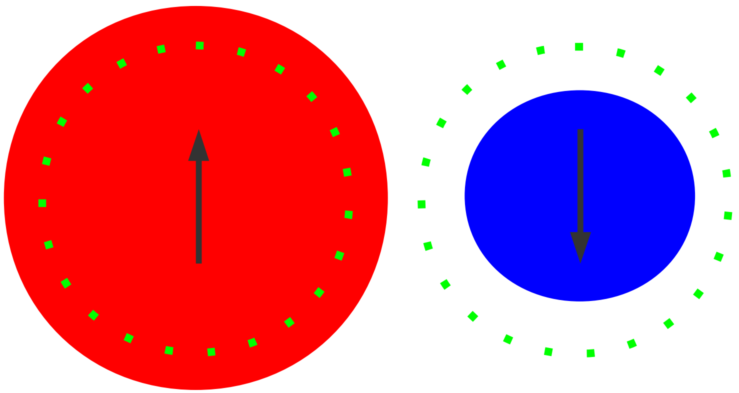


Nematic



Fluctuation contributions

Ferromagnetic



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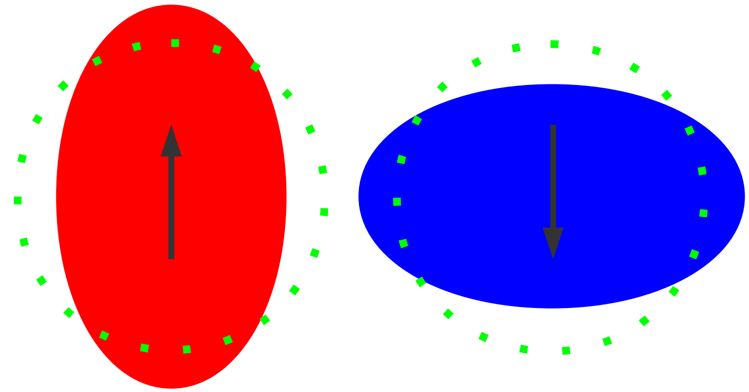
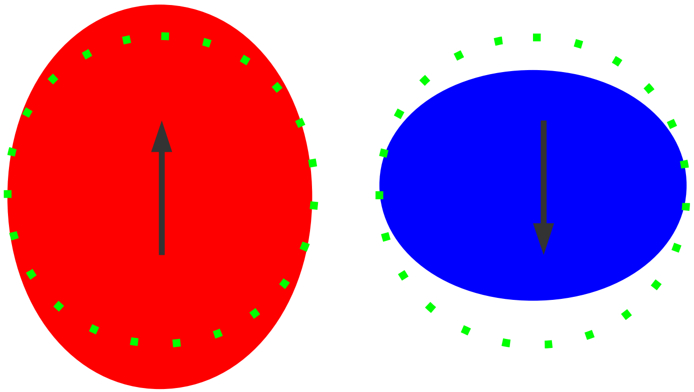


Spin spiral



Fluctuation contributions

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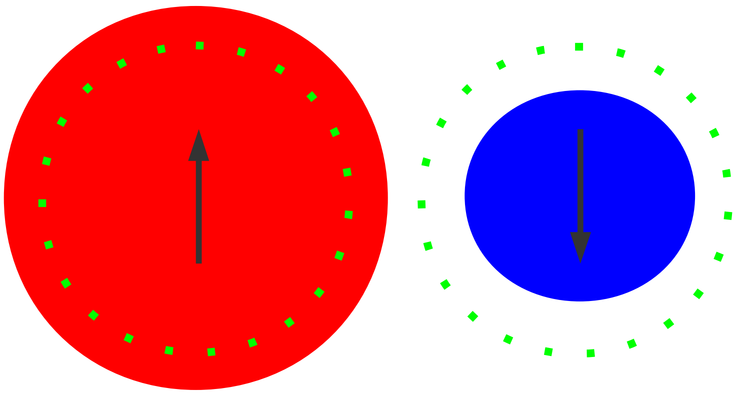

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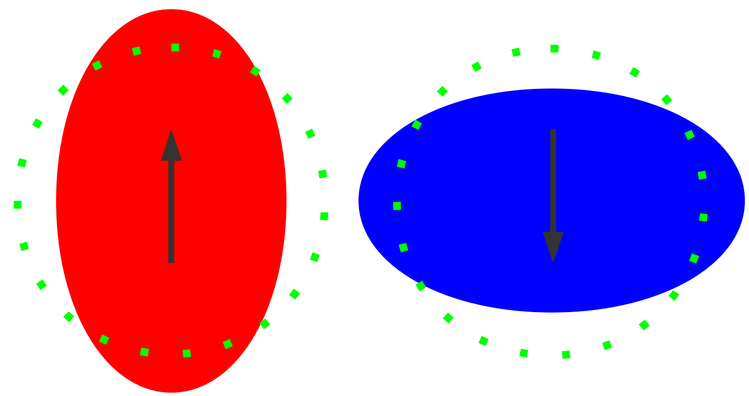

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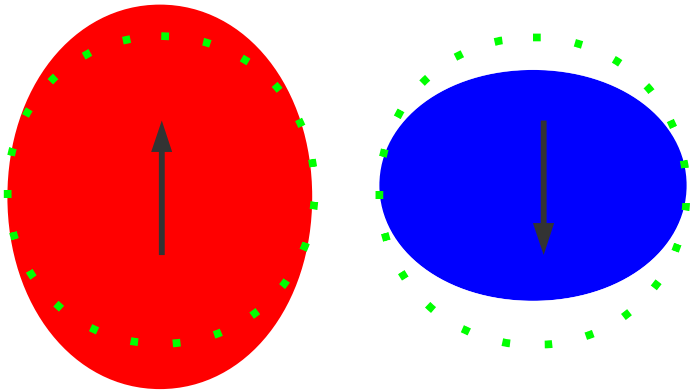
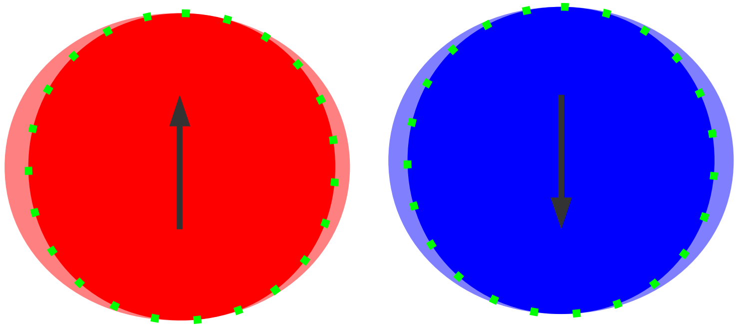
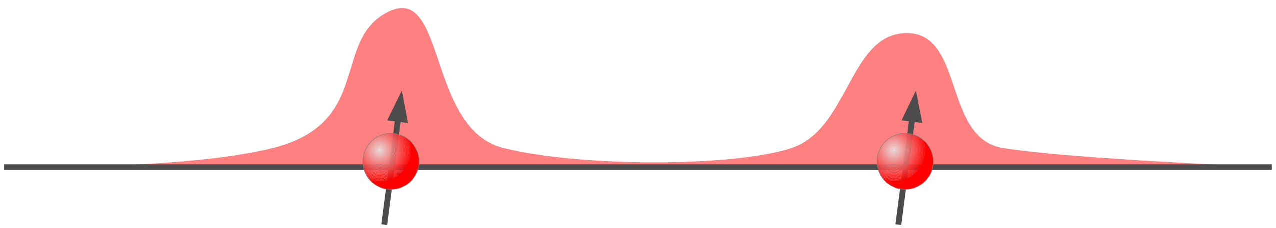


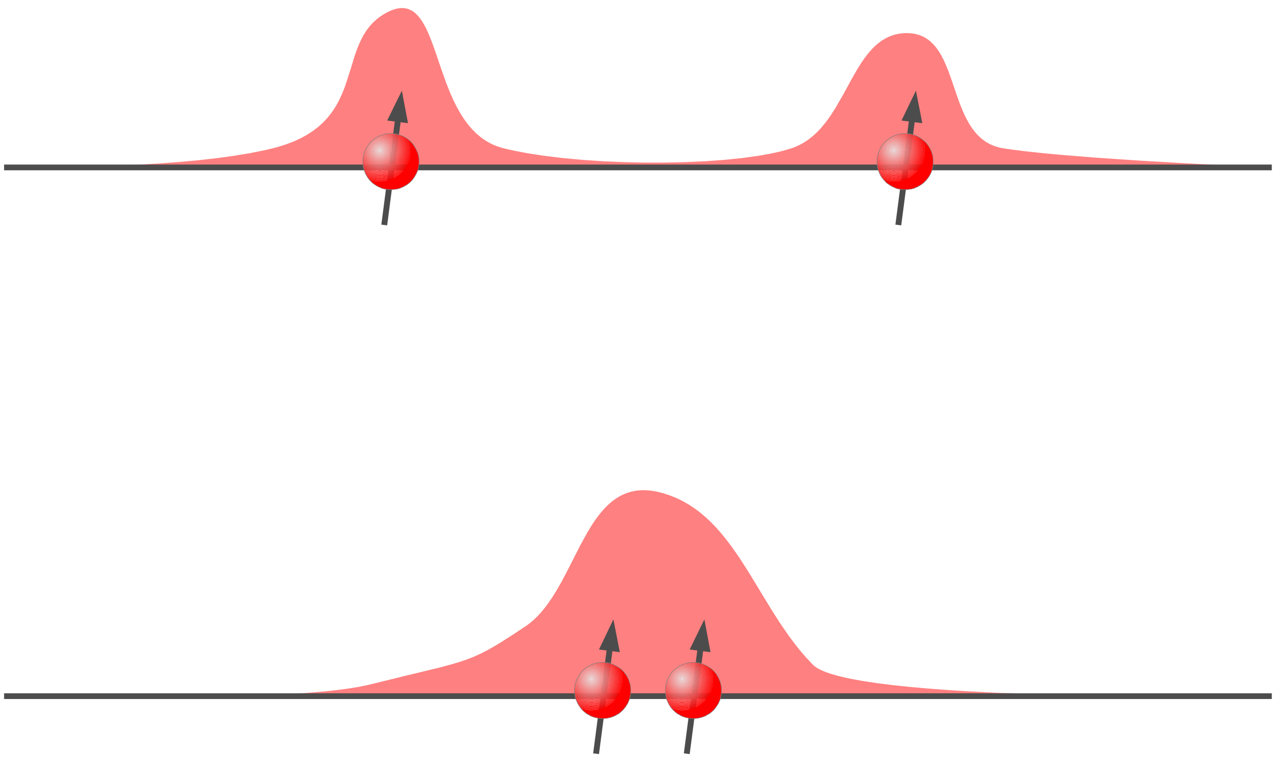
Superconducting



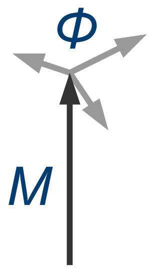
Fluctuation contributions



Fluctuation contributions



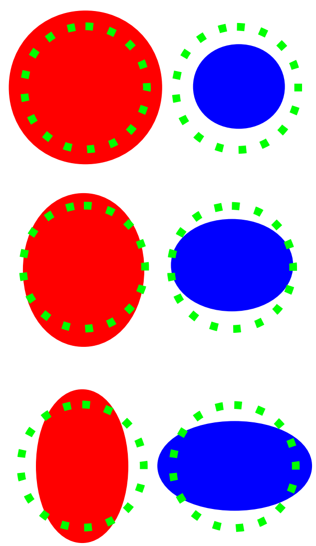
Fluctuation contributions

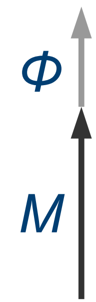


Ferromagnetic

Spin spiral

Nematic





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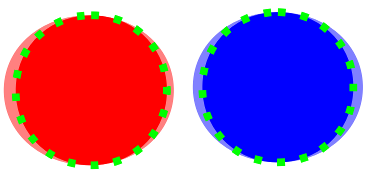

Minimal Hamiltonian

$$H=\sum_{\boldsymbol{k}\sigma}\epsilon_{\boldsymbol{k}}c^{\dagger}_{\boldsymbol{k}\sigma}c_{\boldsymbol{k}\sigma}+g\sum_{\boldsymbol{k}\boldsymbol{p}\boldsymbol{q}}c^{\dagger}_{\boldsymbol{k}+\boldsymbol{q}/2\uparrow}c^{\dagger}_{\boldsymbol{p}-\boldsymbol{q}/2\downarrow}c_{\boldsymbol{p}+\boldsymbol{q}/2\downarrow}c_{\boldsymbol{k}-\boldsymbol{q}/2\uparrow}$$

Taking into account fluctuations

$$F_{\text{fluct}} = g^2 \sum_{\boldsymbol{k}\,\boldsymbol{p}\,s} \frac{\langle \Omega | c^\dagger_{\boldsymbol{k}_1\uparrow} c^\dagger_{\boldsymbol{k}_2\downarrow} c_{\boldsymbol{k}_3\downarrow} c_{\boldsymbol{k}_4\uparrow} | s \rangle \langle s | c^\dagger_{\boldsymbol{p}_1\uparrow} c^\dagger_{\boldsymbol{p}_2\downarrow} c_{\boldsymbol{p}_3\downarrow} c_{\boldsymbol{p}_4\uparrow} | \Omega \rangle}{\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4}}$$

Integral approach

$$F = \sum_{\boldsymbol{k}\,\sigma} \epsilon_{\boldsymbol{k}}\, n_{\sigma}\left(\epsilon_{\boldsymbol{k}}\right)$$

Integral approach

$$F = \sum_{\boldsymbol{k}\sigma} \epsilon_{\boldsymbol{k}} n_\sigma(\epsilon_{\boldsymbol{k}}) + g \sum_{\boldsymbol{k}} n_\uparrow(\epsilon_{\boldsymbol{k}}) \sum_{\boldsymbol{p}} n_\downarrow(\epsilon_{\boldsymbol{p}})$$

Integral approach

$$F=\sum_{\boldsymbol{k}\sigma}\epsilon_{\boldsymbol{k}}n_\sigma(\epsilon_{\boldsymbol{k}})+g\sum_{\boldsymbol{k}}n_\uparrow(\epsilon_{\boldsymbol{k}})\sum_{\boldsymbol{p}}n_\downarrow(\epsilon_{\boldsymbol{p}})$$

$$+g^2\sum_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3\boldsymbol{k}_4}\frac{n_\uparrow(\epsilon_{\boldsymbol{k}_1})n_\downarrow(\epsilon_{\boldsymbol{k}_2})[n_\uparrow(\epsilon_{\boldsymbol{k}_3})+n_\downarrow(\epsilon_{\boldsymbol{k}_4})]}{\epsilon_{\boldsymbol{k}_1}+\epsilon_{\boldsymbol{k}_2}-\epsilon_{\boldsymbol{k}_3}-\epsilon_{\boldsymbol{k}_4}}$$

Landau expansion

$$F = \alpha M^2 + \beta M^4 + \gamma M^6$$

$$\alpha = g - \frac{g^2 \mu^{1/2}}{\sqrt{2}\pi^2}$$

$$\beta = \frac{g^4 \mu^{-3/2}}{48\sqrt{2}\pi^2}$$

$$\gamma = \frac{15 g^6 \mu^{-7/2}}{5760\sqrt{2}\pi^2}$$

Landau expansion

$$F = \alpha M^2 + \beta M^4 + \gamma M^6$$

$$\alpha = g - \frac{g^2 \mu^{1/2}}{\sqrt{2}\pi^2}\left[1 - \frac{27}{2\pi^3}\left(\frac{T}{\mu}\right)^2\right] - \frac{16\sqrt{2}(1+2\ln 2)g^4}{3(2\pi)^6}$$

$$\beta = \frac{g^4 \mu^{-3/2}}{48\sqrt{2}\pi^2}\left[1 + \frac{405}{\pi^3}\left(\frac{T}{\mu}\right)^2\right] + \frac{16\sqrt{2} g^6}{3(2\pi)^6}\left[1 + \ln\left(\frac{(gM)^2 + (\pi T/4 e^C)^2}{4\mu^2}\right)\right]$$

$$\gamma = \frac{15 g^6 \mu^{-7/2}}{5760\sqrt{2}\pi^2}\left[1 + \frac{1701}{2\pi^3}\left(\frac{T}{\mu}\right)^2\right]$$

Landau expansion

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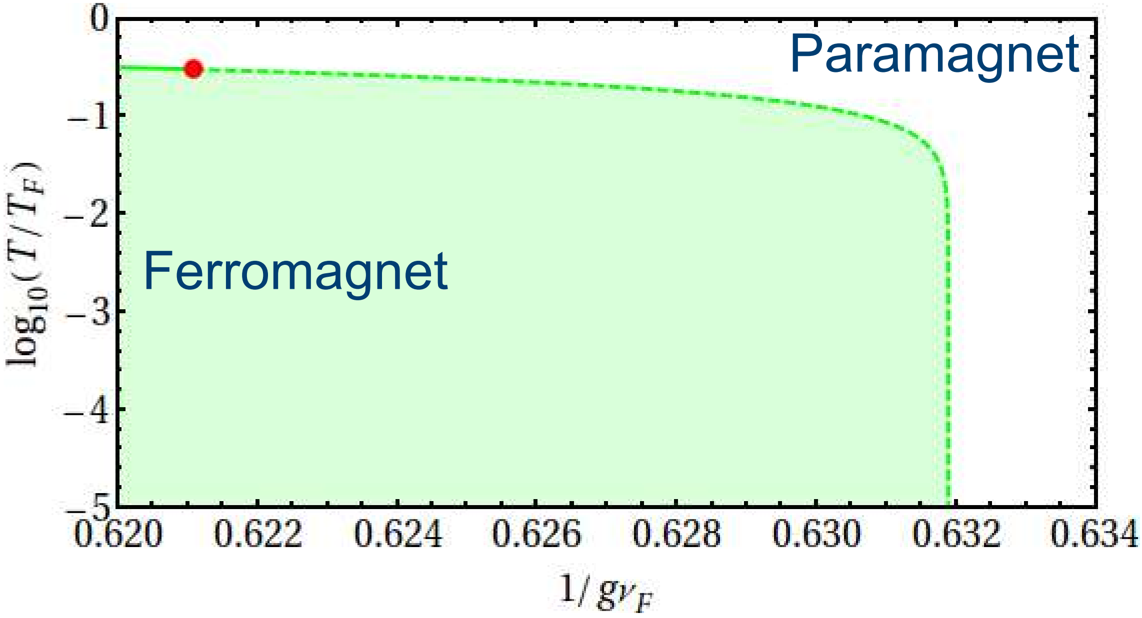
$$F_\beta \sim M^4 \ln M$$

Phase diagram



Spin spiral

$$\epsilon_{\boldsymbol{k}-\boldsymbol{Q}} - gM - \mu = \frac{k^2}{2} - \boldsymbol{k}\cdot\boldsymbol{Q} + \frac{Q^2}{2} - gM - \mu$$

Spin spiral

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$$F[M^2] \rightarrow \langle F[M^2 + \theta^2 Q^2] \rangle - \langle F[\theta^2 Q^2] \rangle$$

$$\theta = \frac{\boldsymbol{k}\cdot\boldsymbol{Q}}{|\boldsymbol{k}||\boldsymbol{Q}|}$$

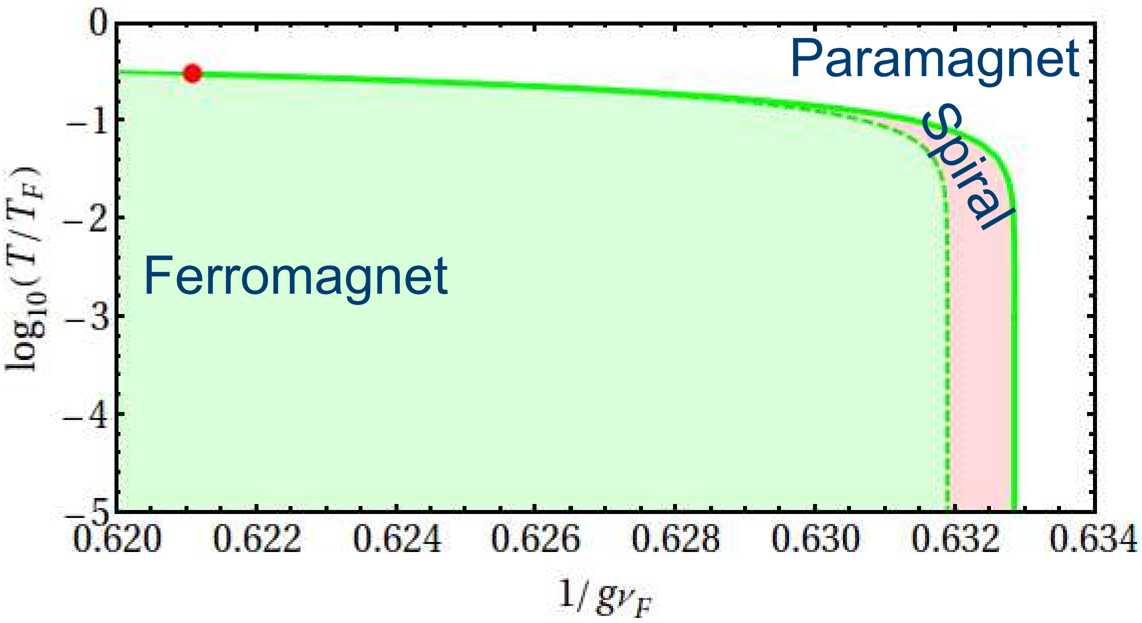
Spin spiral

$$F = \alpha M^2 + \beta M^4 + \gamma M^6$$

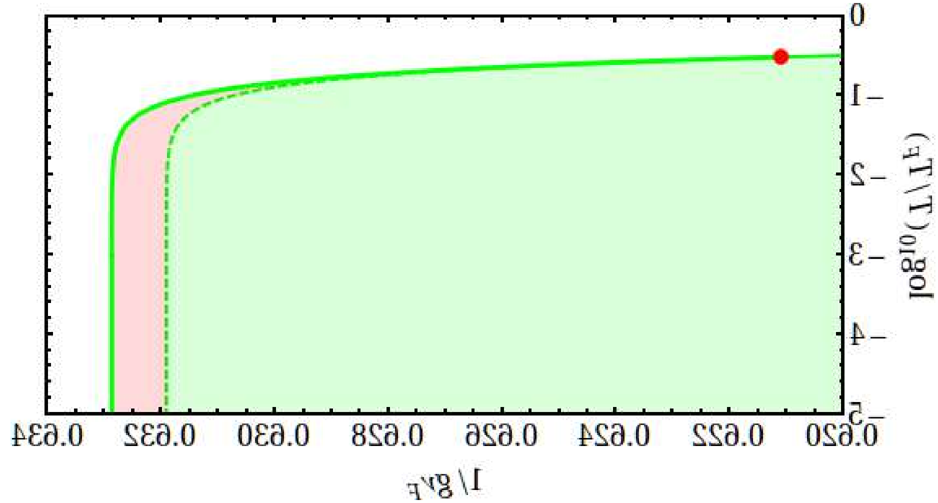
$$\Big\downarrow \quad M^2 \rightarrow \langle M^2 + \theta^2 Q^2 \rangle$$

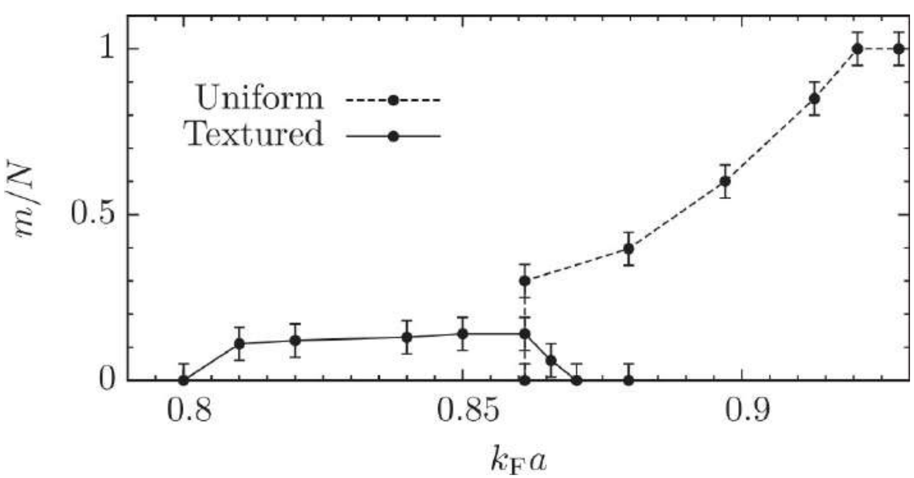
$$F = \left(\alpha + \frac{2}{3} \beta Q^2 + \frac{2}{5} \gamma Q^4 \right) M^2 + (\beta + \gamma Q^2) M^4 + \gamma M^6$$

Phase diagram



Comparison to Monte Carlo





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Superconductivity expansion

Diagonalize with
Bogoliubov transform

$$H = H_0 - \sum_{\boldsymbol{k}} \left(\Delta_{\boldsymbol{k}} c^{\dagger}_{-\boldsymbol{k}\uparrow} c^{\dagger}_{\boldsymbol{k}\uparrow} + \bar{\Delta}_{\boldsymbol{k}} c_{-\boldsymbol{k}\uparrow} c_{\boldsymbol{k}\uparrow} \right)$$

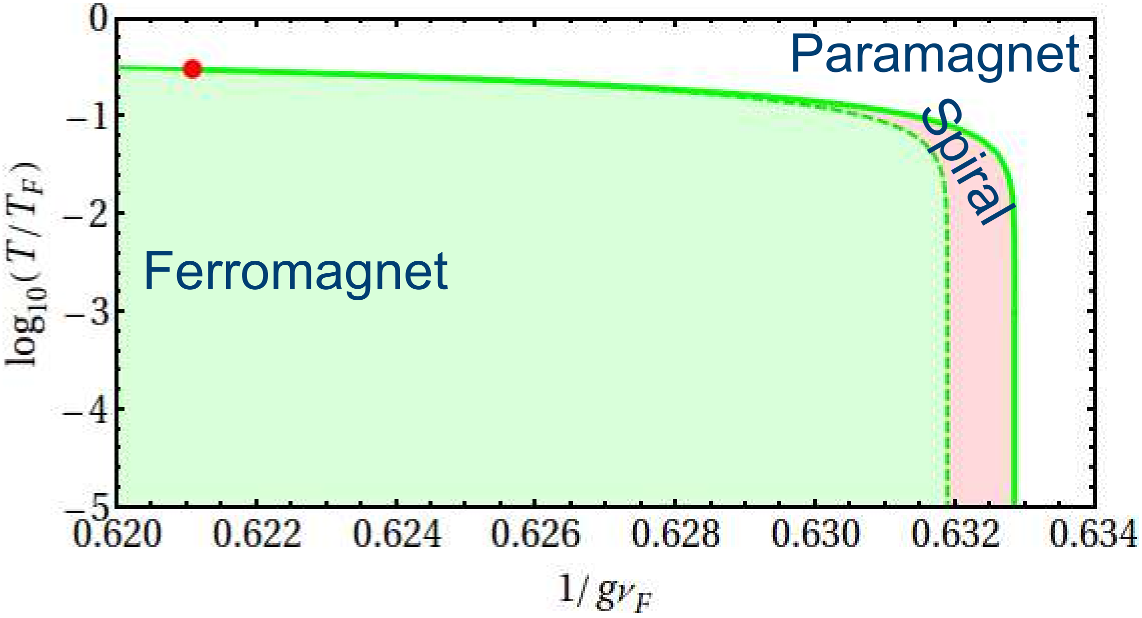
$$+ H_{\text{int}} + \sum_{\boldsymbol{k}} \left(\Delta_{\boldsymbol{k}} c^{\dagger}_{-\boldsymbol{k}\uparrow} c^{\dagger}_{\boldsymbol{k}\uparrow} + \bar{\Delta}_{\boldsymbol{k}} c_{-\boldsymbol{k}\uparrow} c_{\boldsymbol{k}\uparrow} \right)$$

calculate perturbatively

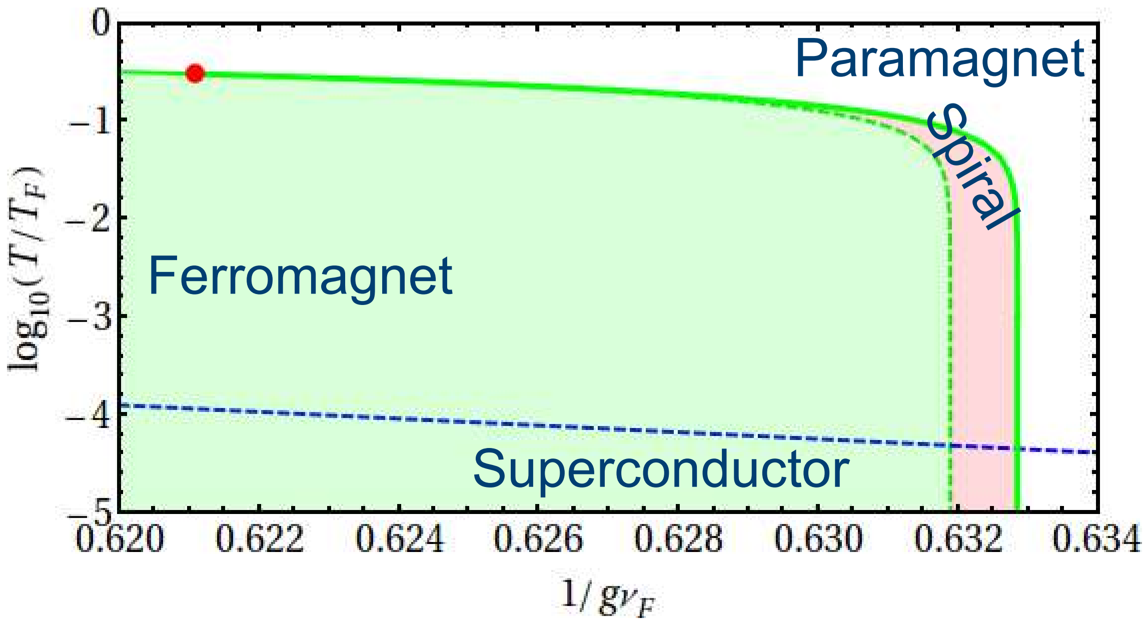
Tricritical point temperature

$$T_c = \frac{2\mu_\uparrow e^C}{\pi} \exp\left(- \frac{\left(1 - \partial_\epsilon \Re \Sigma_\uparrow(\boldsymbol{k}, \epsilon_{\boldsymbol{k}})\right) \langle \theta_{\boldsymbol{k}}^2 \rangle}{\langle \theta_{\boldsymbol{k}+\boldsymbol{Q}} \theta_{\boldsymbol{k}} \Re \chi_{\downarrow\downarrow}(\boldsymbol{Q}, \epsilon_{\boldsymbol{k}+\boldsymbol{Q}} - \epsilon_{\boldsymbol{k}}) \rangle} \right)$$

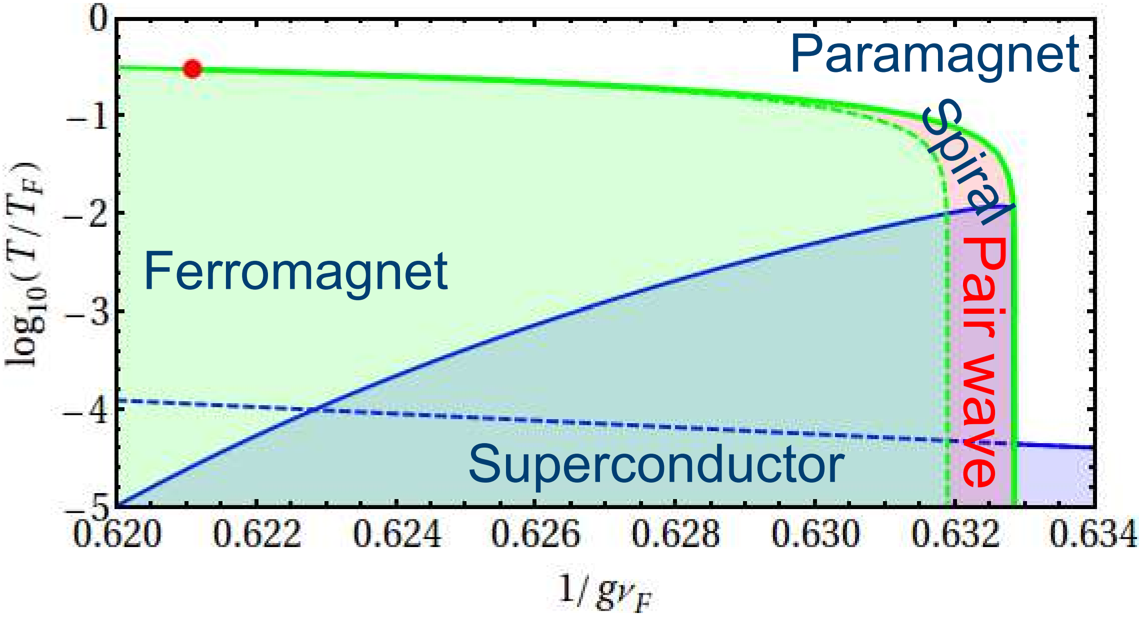
Phase diagram



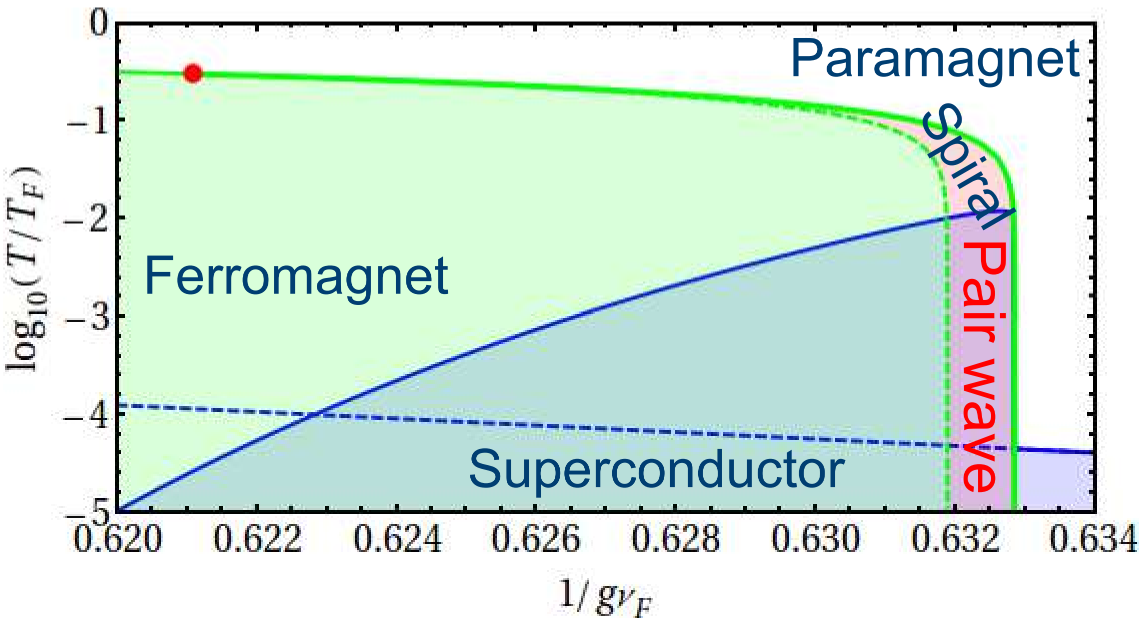
Phase diagram

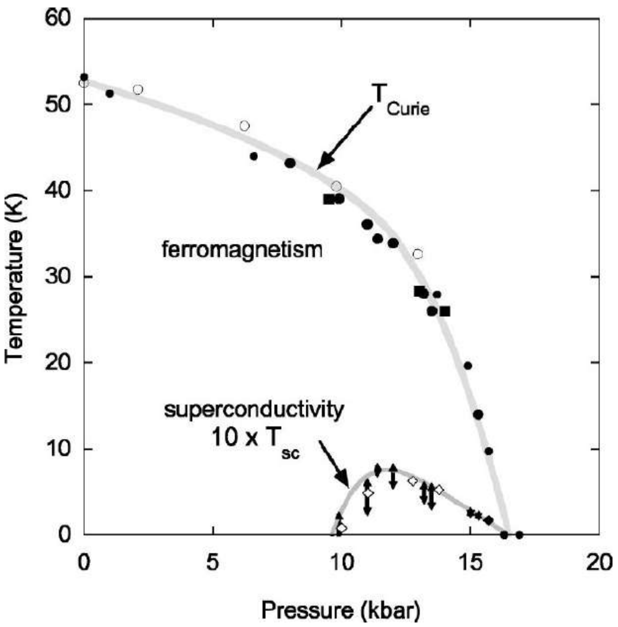


Phase diagram

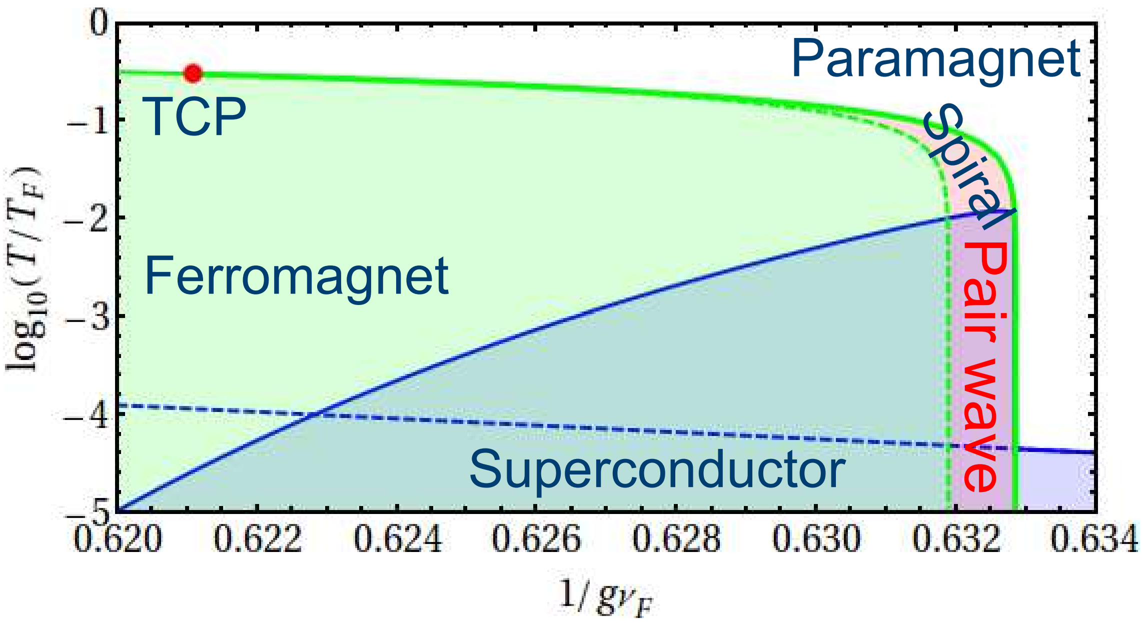


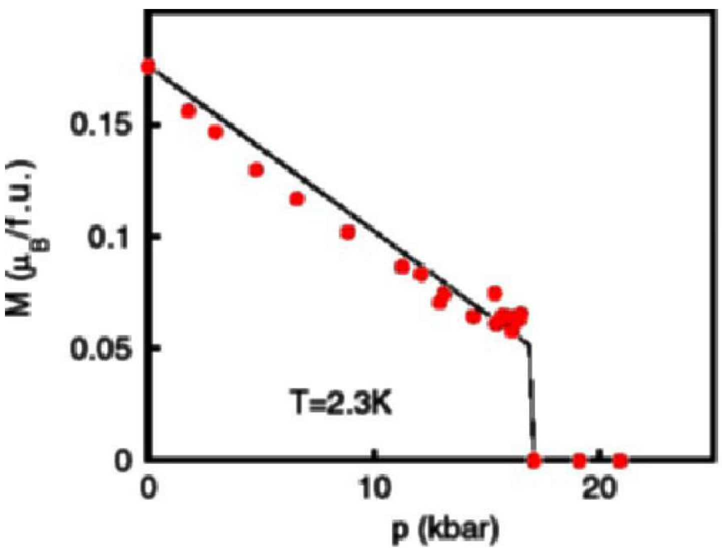
Phase diagram





Tricritical point temperature





Finite ranged interactions

$$H=\sum_{\boldsymbol{k}\sigma}\epsilon_{\boldsymbol{k}}c^{\dagger}_{\boldsymbol{k}\sigma}c_{\boldsymbol{k}\sigma}+g\sum_{\boldsymbol{k}\boldsymbol{p}\boldsymbol{q}}c^{\dagger}_{\boldsymbol{k}+\boldsymbol{q}/2\uparrow}c^{\dagger}_{\boldsymbol{p}-\boldsymbol{q}/2\downarrow}c_{\boldsymbol{p}+\boldsymbol{q}/2\downarrow}c_{\boldsymbol{k}-\boldsymbol{q}/2\uparrow}$$

↓

$$H=\sum_{\boldsymbol{k}\sigma}\epsilon_{\boldsymbol{k}}c^{\dagger}_{\boldsymbol{k}\sigma}c_{\boldsymbol{k}\sigma}+\sum_{\boldsymbol{k}\boldsymbol{p}\boldsymbol{q}}g(\boldsymbol{q})c^{\dagger}_{\boldsymbol{k}+\boldsymbol{q}/2\uparrow}c^{\dagger}_{\boldsymbol{p}-\boldsymbol{q}/2\downarrow}c_{\boldsymbol{p}+\boldsymbol{q}/2\downarrow}c_{\boldsymbol{k}-\boldsymbol{q}/2\uparrow}$$

$$g(\boldsymbol{q})=\frac{g}{1+b^2q^2}$$

Finite ranged interactions

$$F = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} n_\sigma(\epsilon_{\mathbf{k}}) + g \sum_{\mathbf{k}} n_\uparrow(\epsilon_{\mathbf{k}}) \sum_{\mathbf{p}} n_\downarrow(\epsilon_{\mathbf{p}})$$

$$\Big\downarrow \quad g(\mathbf{q}) = \frac{g}{1+b^2 q^2}$$

$$F = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} n_\sigma(\epsilon_{\mathbf{k}}) + g(0) \sum_{\mathbf{k}} n_\uparrow(\epsilon_{\mathbf{k}}) \sum_{\mathbf{p}} n_\downarrow(\epsilon_{\mathbf{p}})$$

Finite ranged interactions

$$F=\sum_{\boldsymbol{k}\sigma}\epsilon_{\boldsymbol{k}}n_{\sigma}(\epsilon_{\boldsymbol{k}})+g\sum_{\boldsymbol{k}}n_{\uparrow}(\epsilon_{\boldsymbol{k}})\sum_{\boldsymbol{p}}n_{\downarrow}(\epsilon_{\boldsymbol{p}})$$

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$$+\sum_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3\boldsymbol{k}_4}g^{2}(\boldsymbol{k}_1-\boldsymbol{k}_3)\frac{n_{\uparrow}(\epsilon_{\boldsymbol{k}_1})n_{\downarrow}(\epsilon_{\boldsymbol{k}_2})[n_{\uparrow}(\epsilon_{\boldsymbol{k}_3})+n_{\downarrow}(\epsilon_{\boldsymbol{k}_4})]}{\epsilon_{\boldsymbol{k}_1}+\epsilon_{\boldsymbol{k}_2}-\epsilon_{\boldsymbol{k}_3}-\epsilon_{\boldsymbol{k}_4}}$$

Finite ranged interactions

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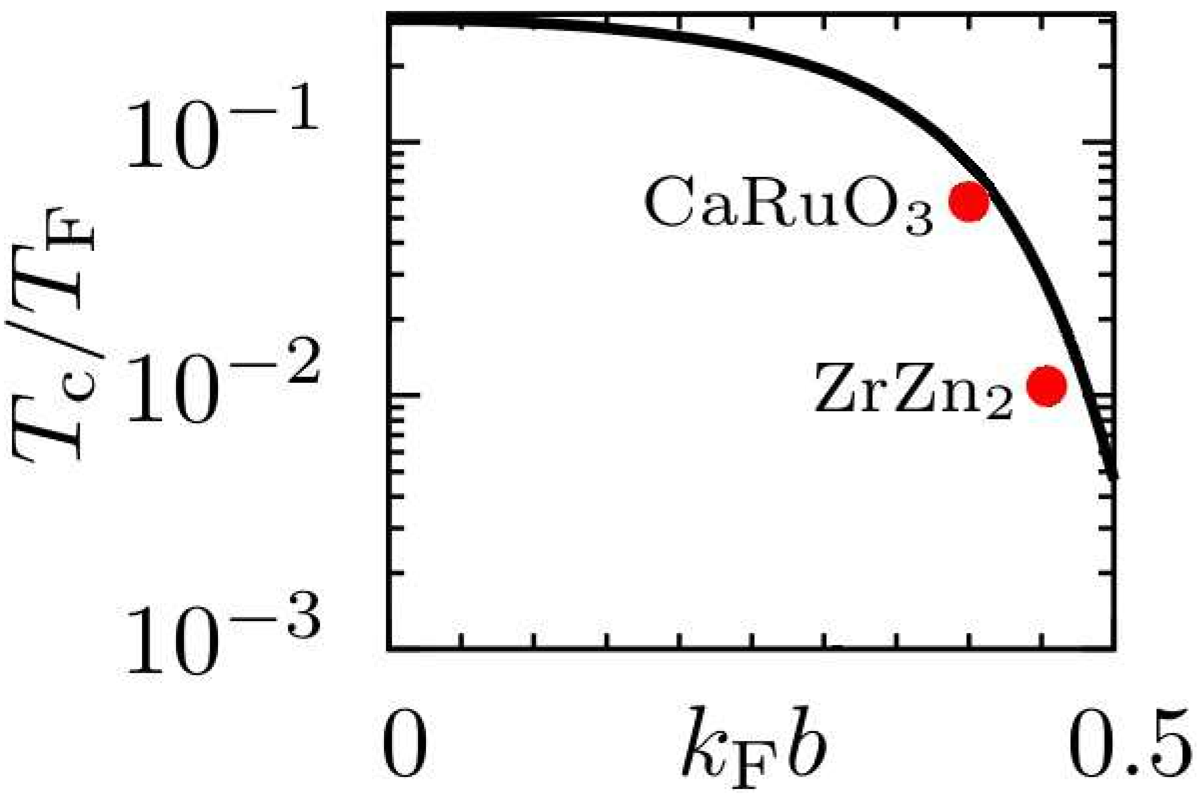
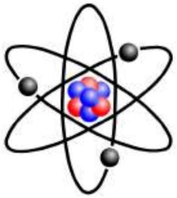
$$+\frac{g^{2}}{(1+2\mathrm{b}^{2}k_{\mathrm{F}}^{2})^{2}}\sum_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3\boldsymbol{k}_4}\frac{n_{\uparrow}(\epsilon_{\boldsymbol{k}_1})n_{\downarrow}(\epsilon_{\boldsymbol{k}_2})[n_{\uparrow}(\epsilon_{\boldsymbol{k}_3})+n_{\downarrow}(\epsilon_{\boldsymbol{k}_4})]}{\epsilon_{\boldsymbol{k}_1}+\epsilon_{\boldsymbol{k}_2}-\epsilon_{\boldsymbol{k}_3}-\epsilon_{\boldsymbol{k}_4}}$$

Finite ranged interactions tricritical point



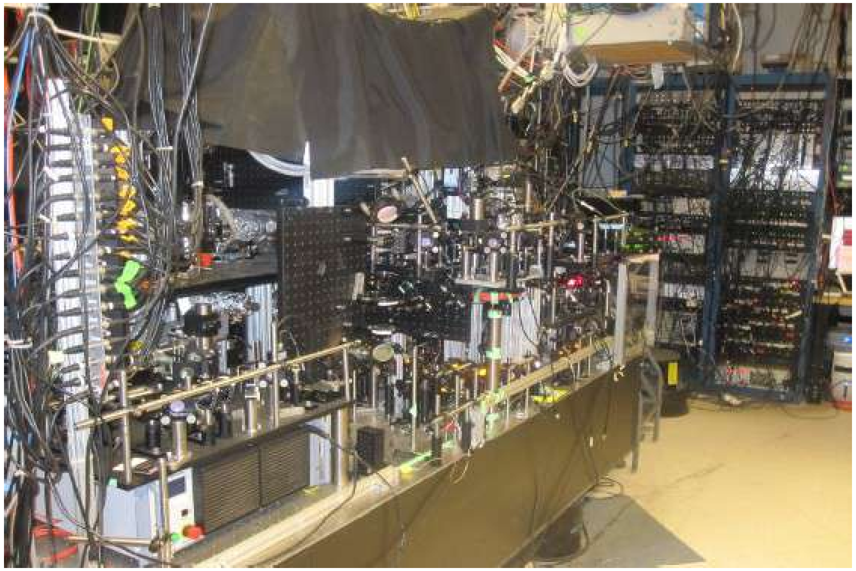
Experimental setup



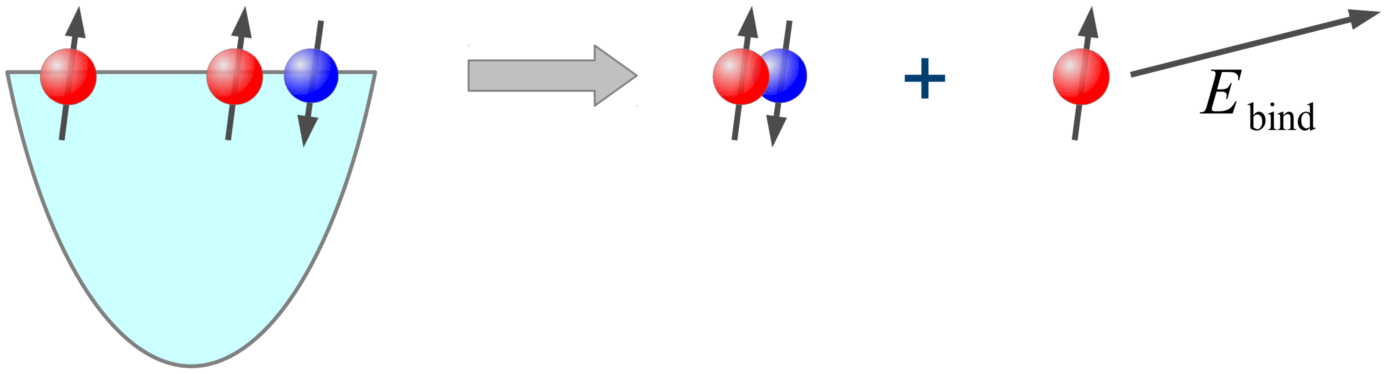
^{6}Li atom

$|F = 1/2, m_F = 1/2\rangle$ → Up spin electron

$|F = 1/2, m_F = -1/2\rangle$ → Down spin electron



Competing loss processes



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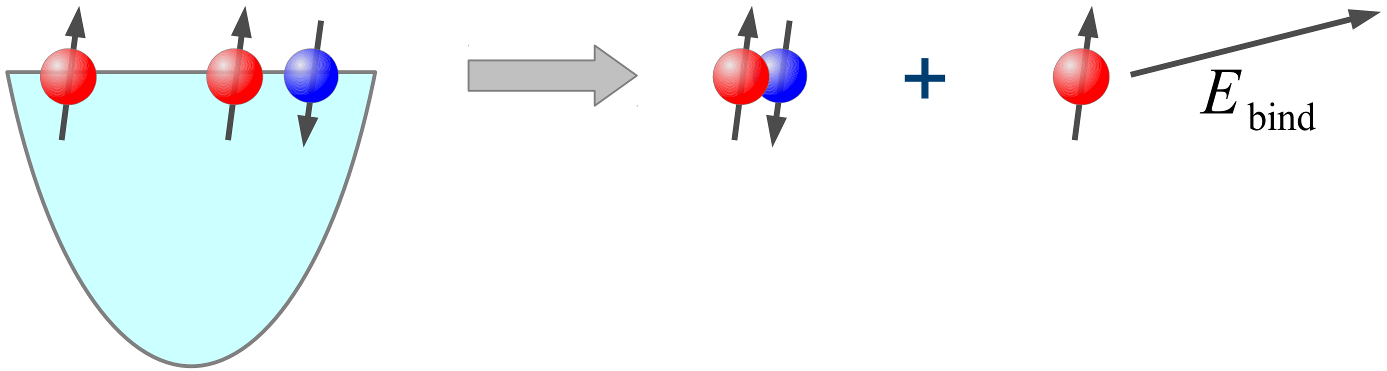
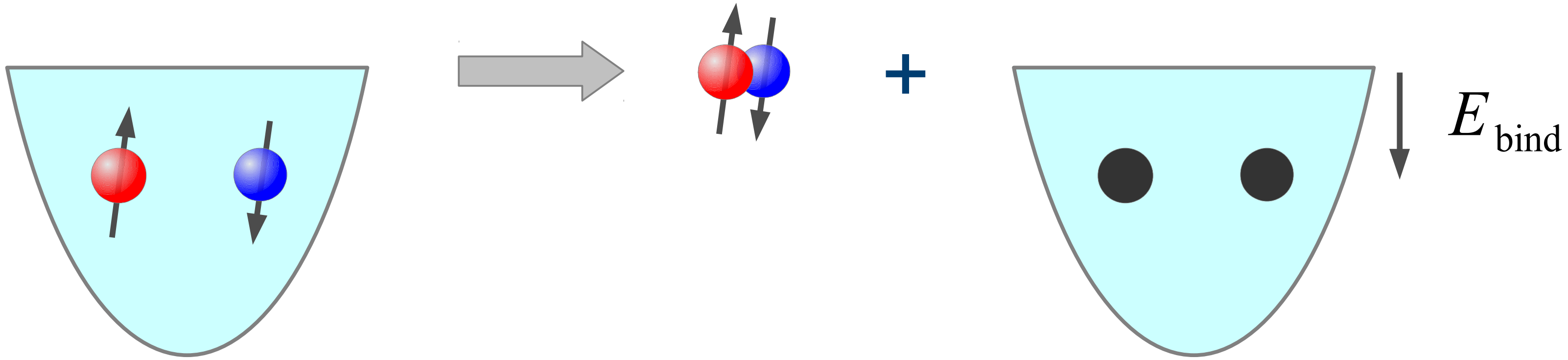
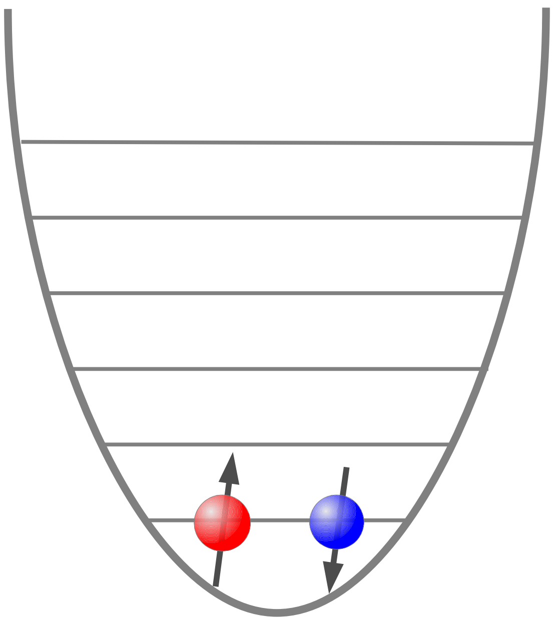


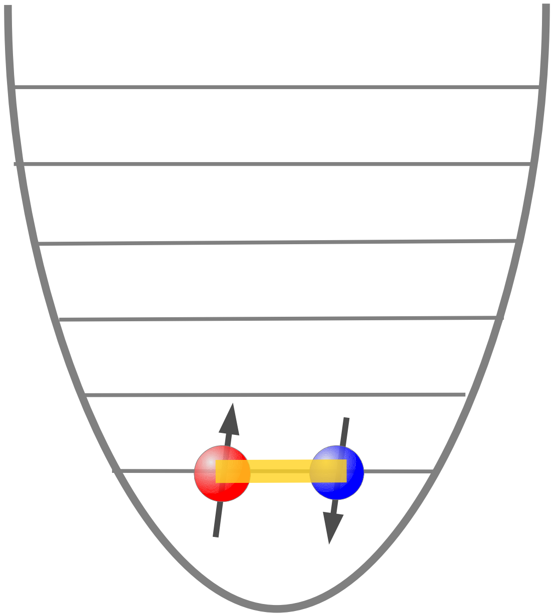


Two distinguishable fermions

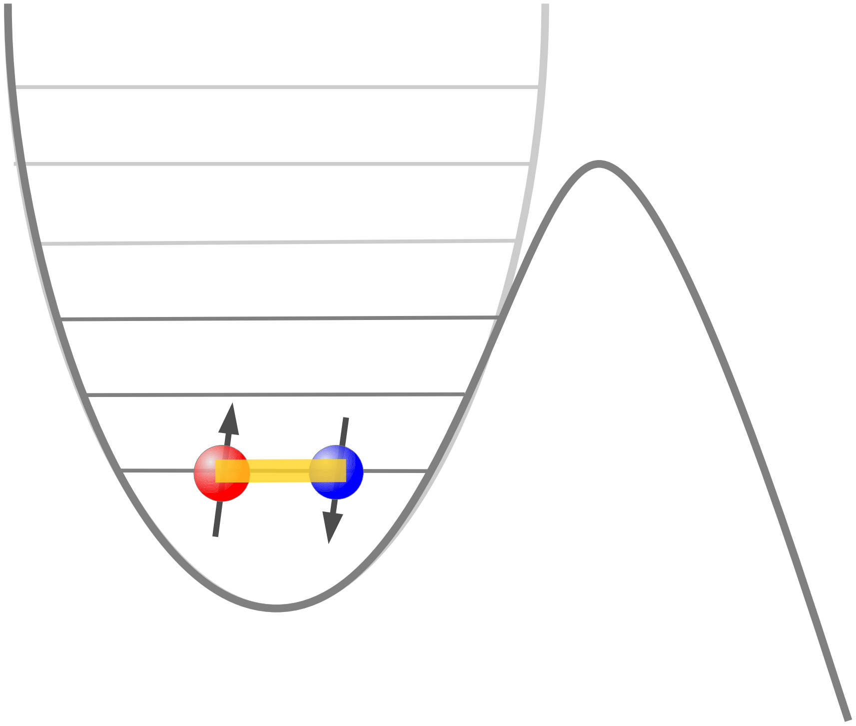


Zürn *et al*. PRL **108** 075303 (2012)

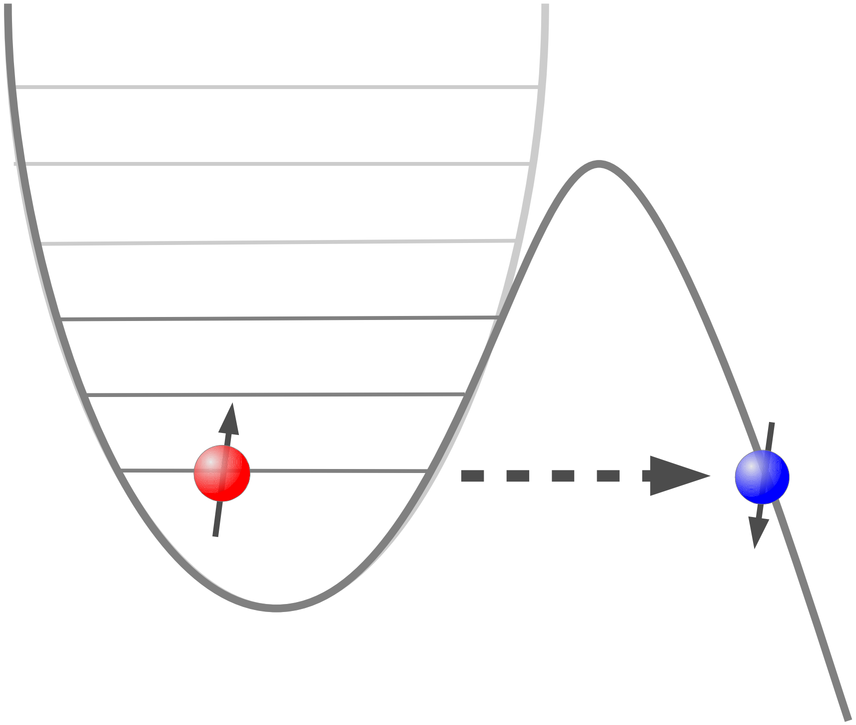
Two distinguishable fermions



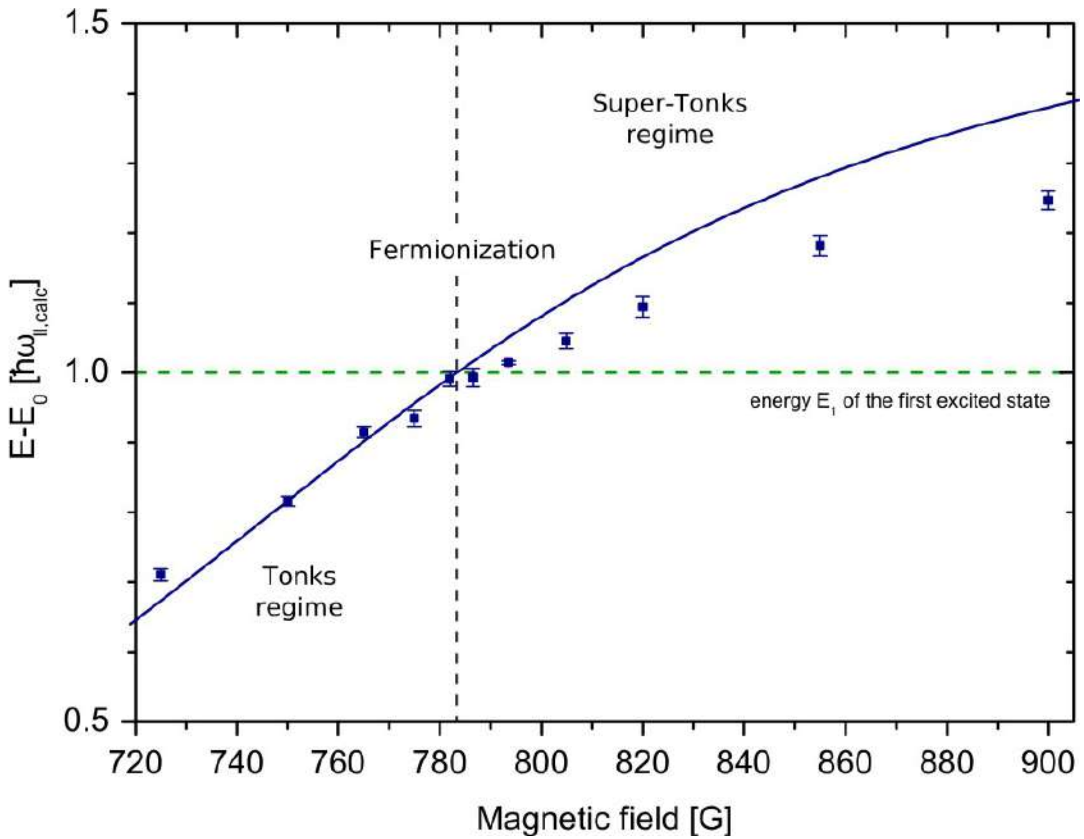
Two distinguishable fermions

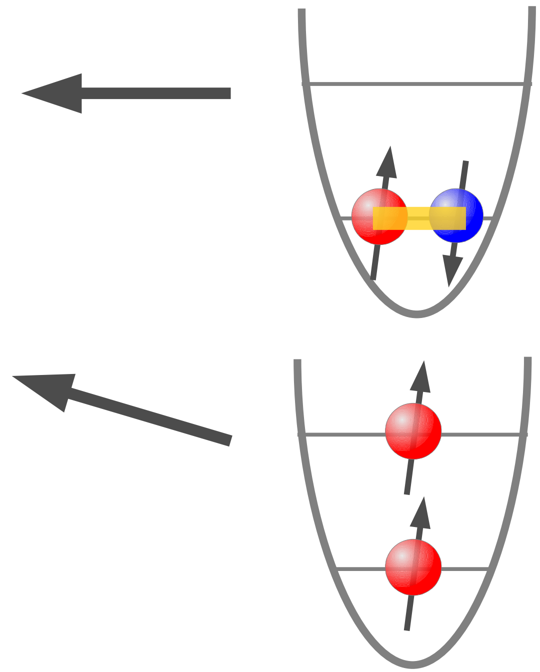


Two distinguishable fermions



Energy of states

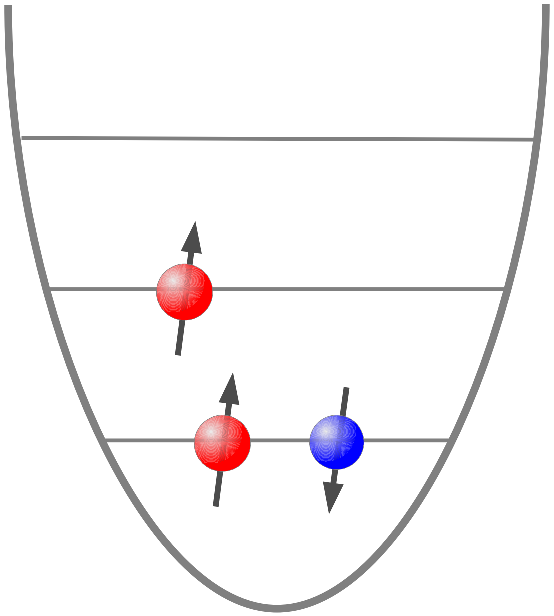




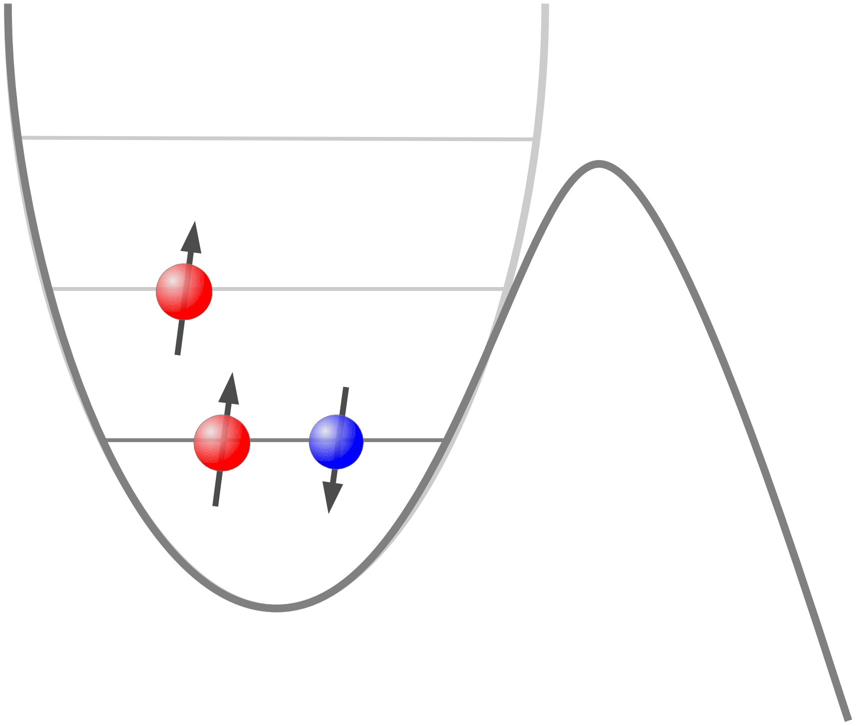
Weak
repulsion

Strong
repulsion

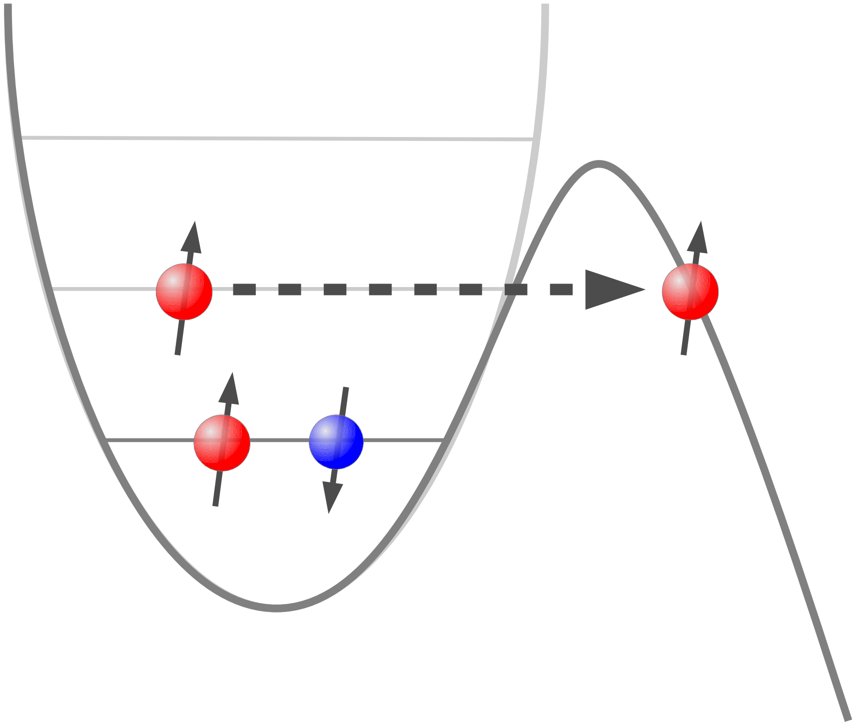
Polaron state



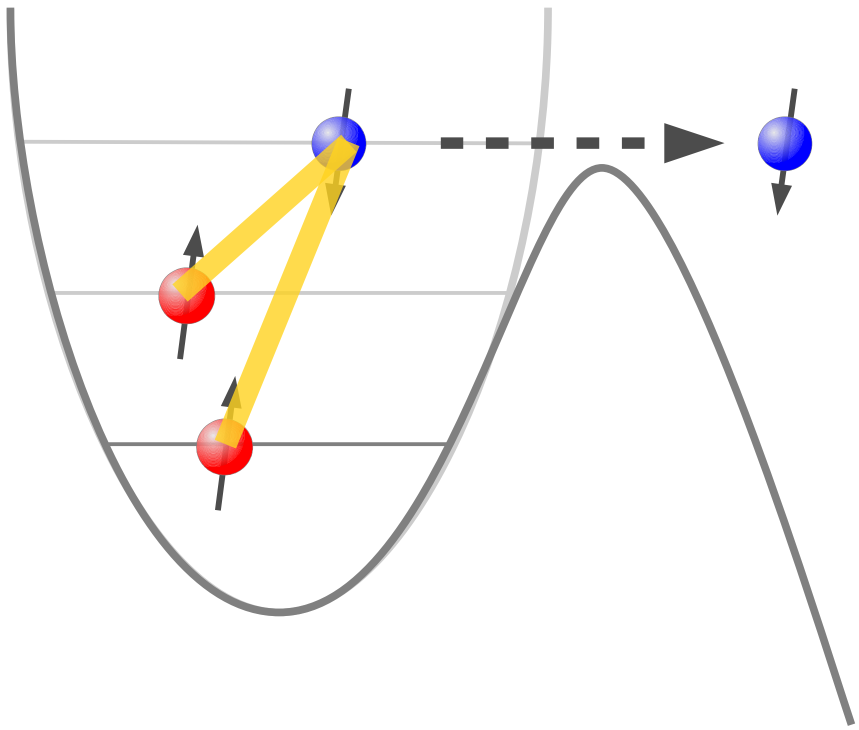
Polaron state



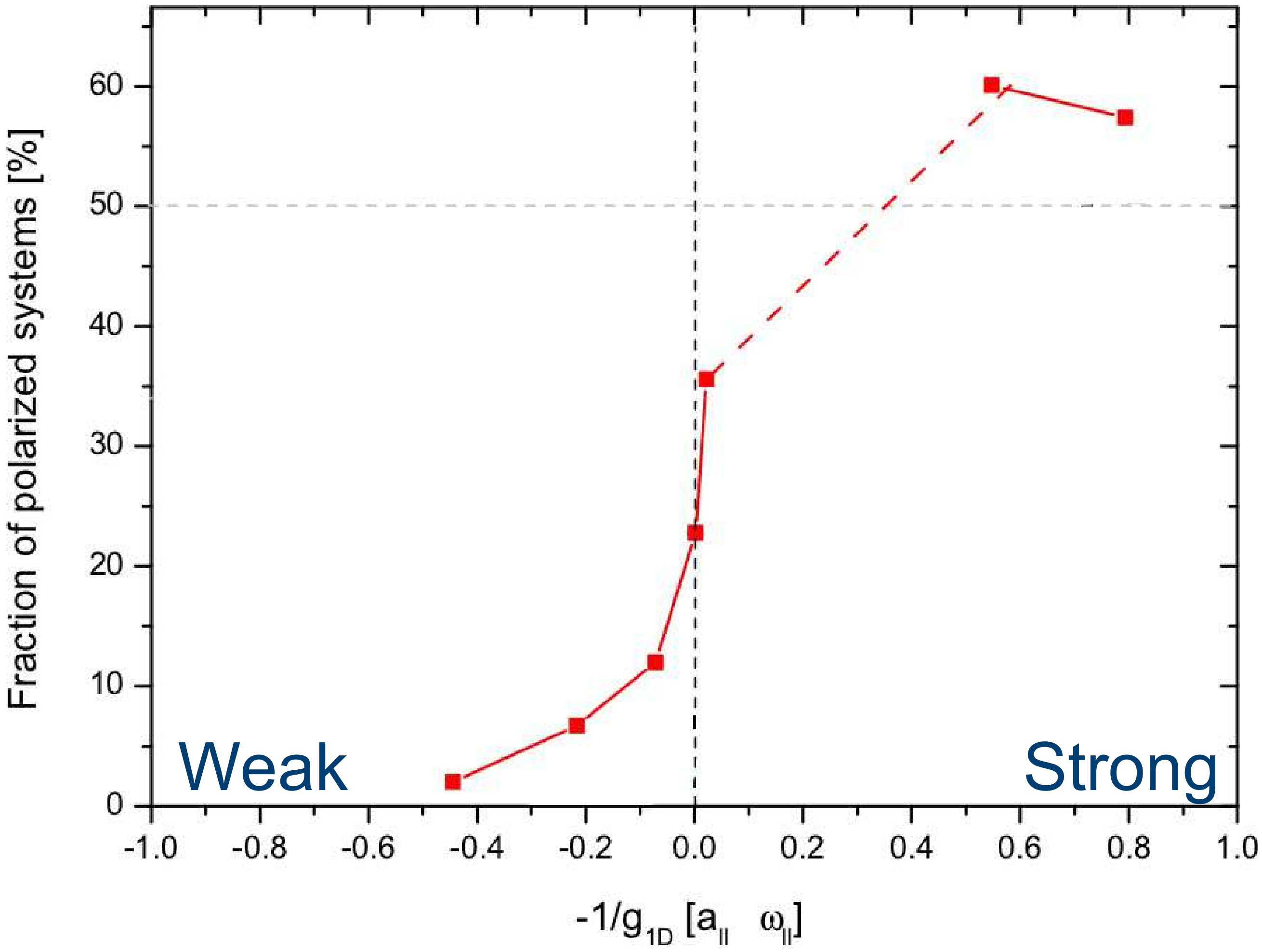
Polaron state

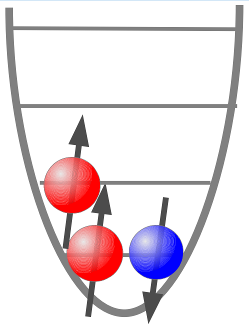


Polaron state

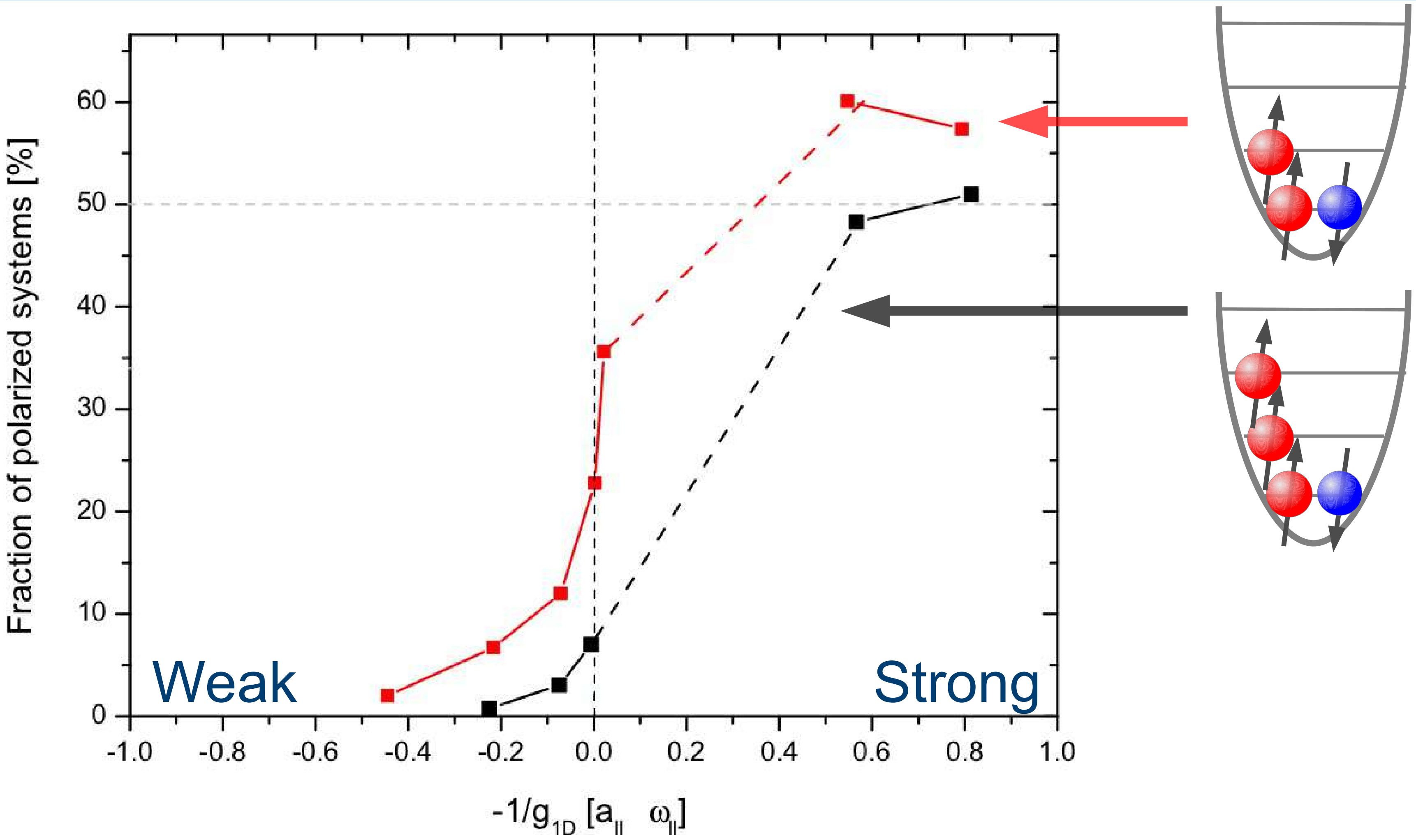


Tunneling probability

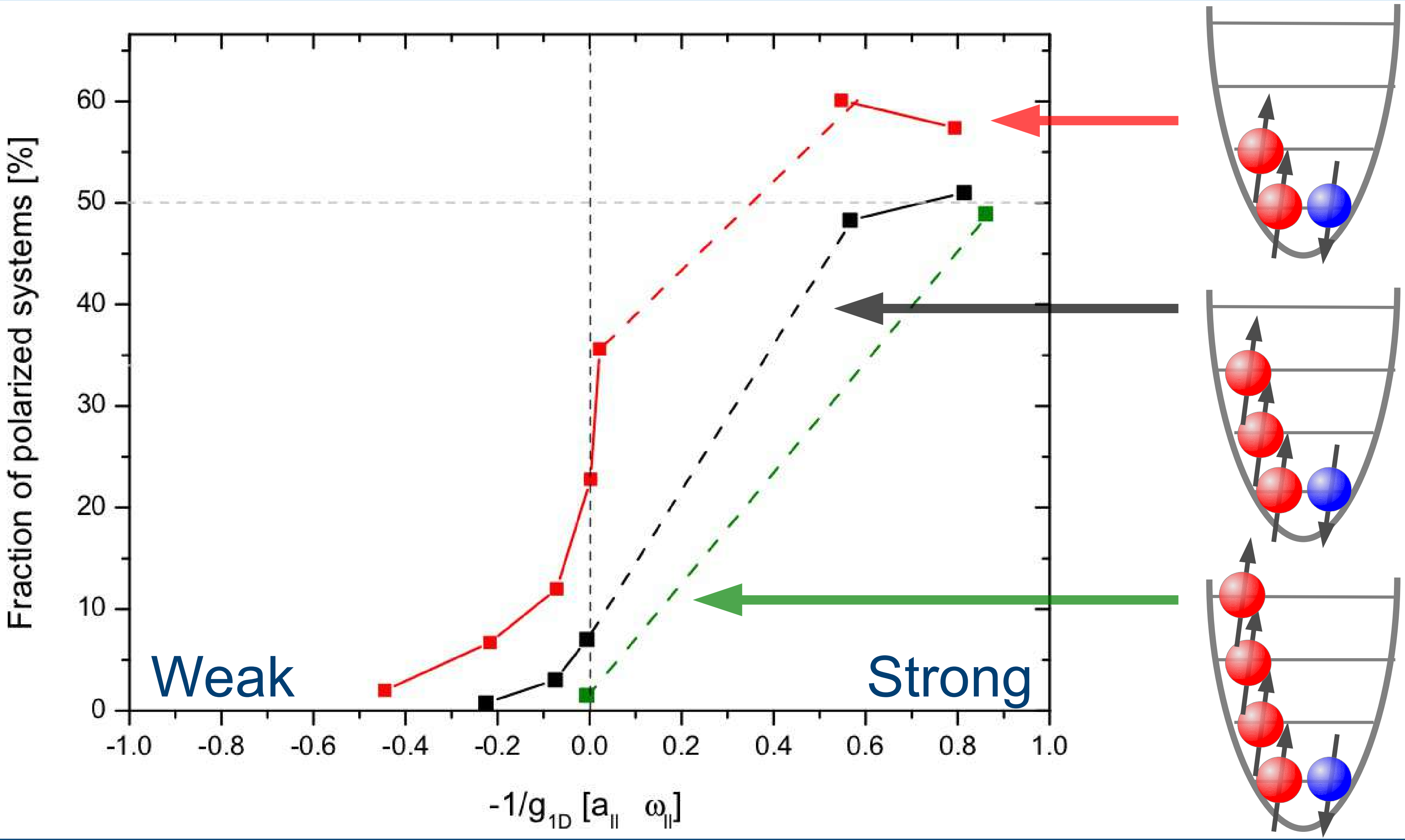




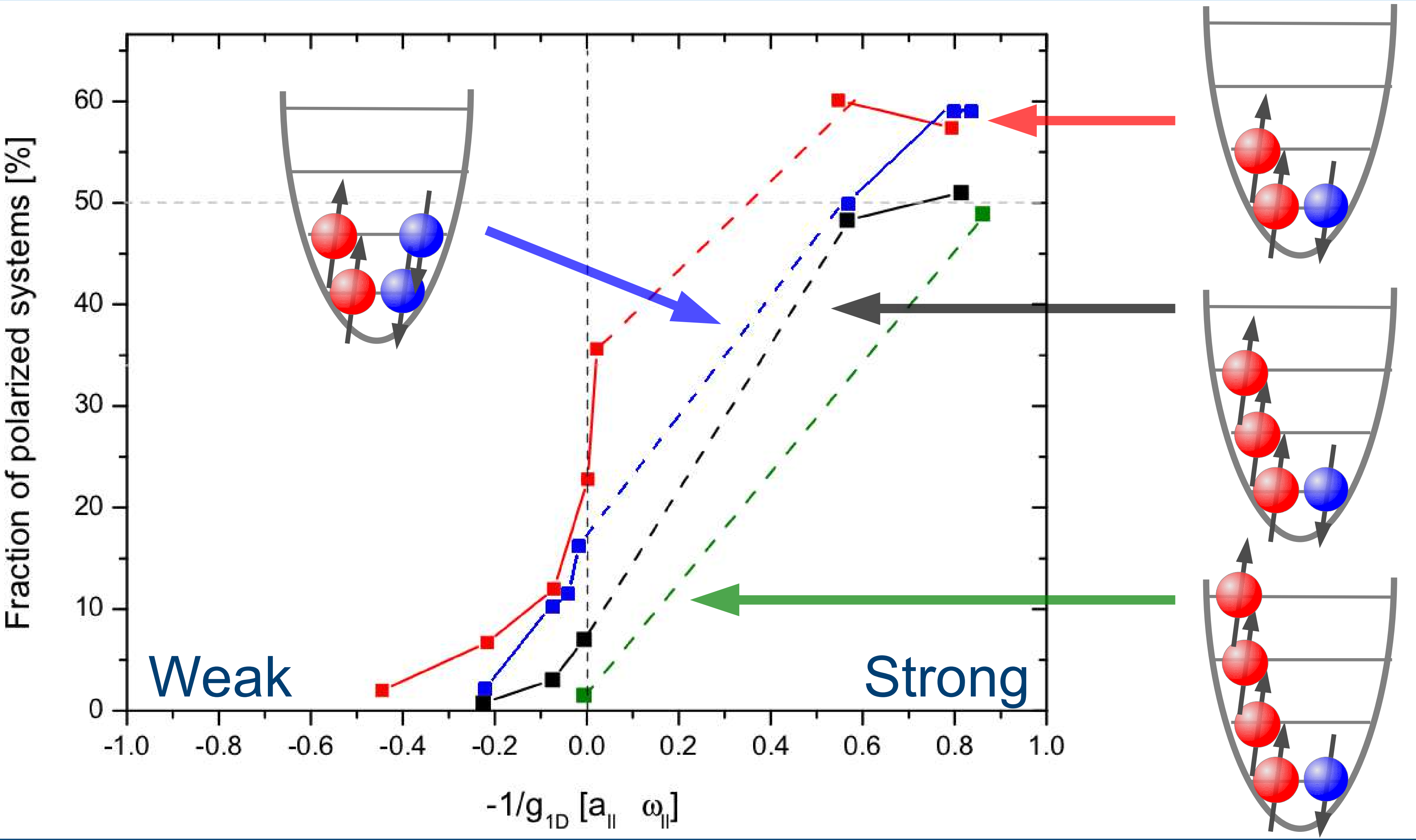
Tunneling probability



Tunneling probability



Tunneling probability



Summary

Fluctuation corrections drive emergence of new ferromagnetic order

Transverse fluctuations drive spin spiral and longitudinal fluctuations a p-wave superconducting phase

Instabilities merge to form a pair density wave

A few-fermion cold atoms system displays ferromagnetic correlations