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Ferromagnetism in a few-fermion system

G.J. Conduit, P.O. Bugnion

Theory of Condensed Matter Group, Department of Physics, Cambridge

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G. Zürn, F. Serwane, T. Lompe, A.N. Wenz,
M.G. Ries, J.E. Bohn, S. Jochim

Physikalisches Institut, Ruprecht-Karls-Universität Heidelberg

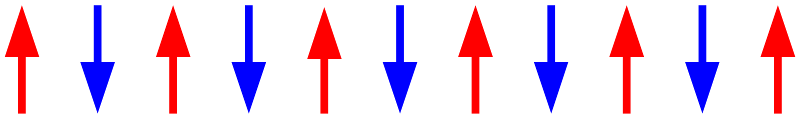
Models for ferromagnetism

Heisenberg model

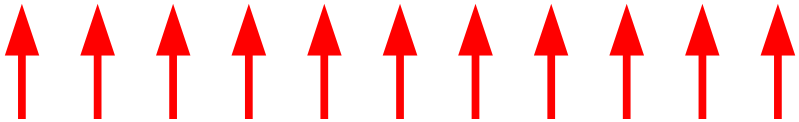
Localized in real space

$$E = -J \sum_{\langle i, j \rangle} S_i \cdot S_j$$

Antiferromagnet



Ferromagnet



Models for ferromagnetism

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Localized in real space

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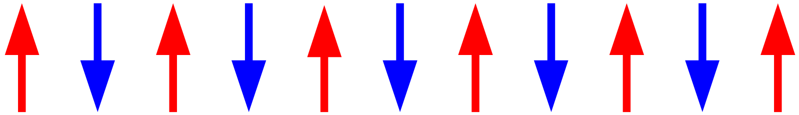
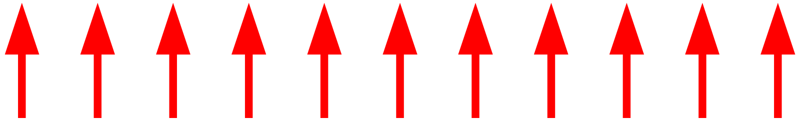
Antiferromagnet



Ferromagnet

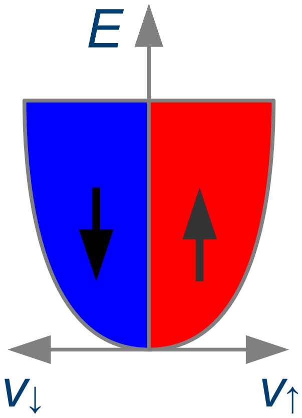


Stoner model

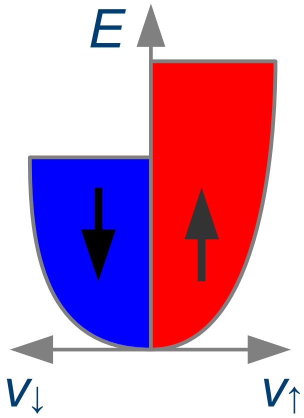
Localized in momentum space

$$E = \sum_{\boldsymbol{k}\sigma} \epsilon_{\boldsymbol{k}} n_{\sigma}(\epsilon_{\boldsymbol{k}}) + g N_{\uparrow} N_{\downarrow}$$

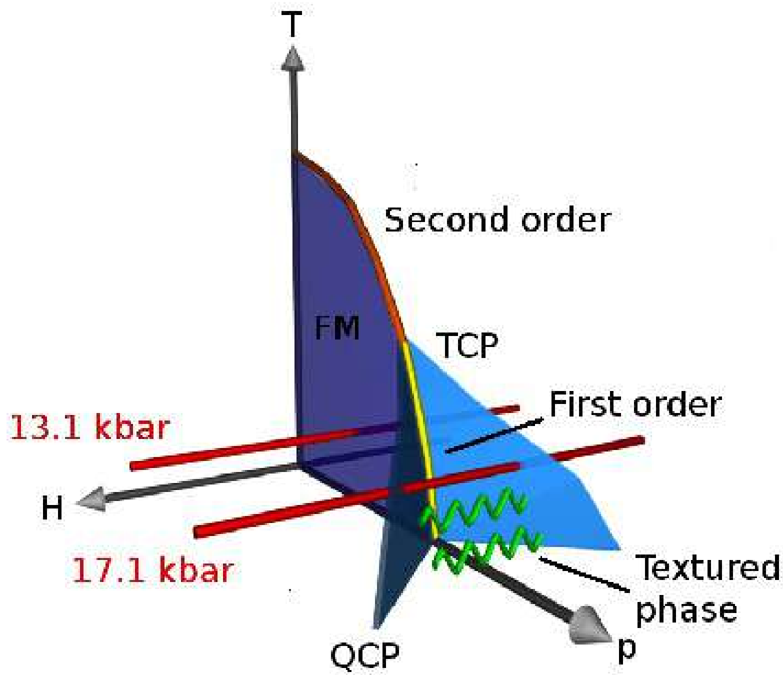
Paramagnet

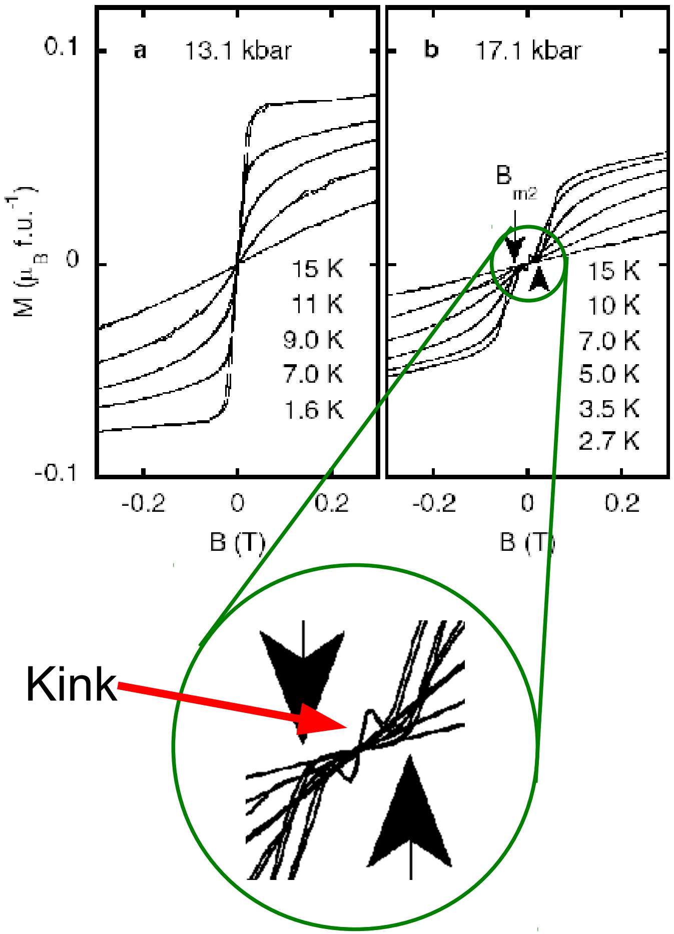


Ferromagnet



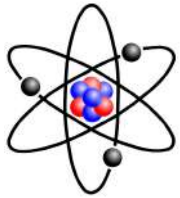
Mysteries in magnetism





Uhlarz *et al.* PRL (2004), Conduit *et al.* PRL (2009)

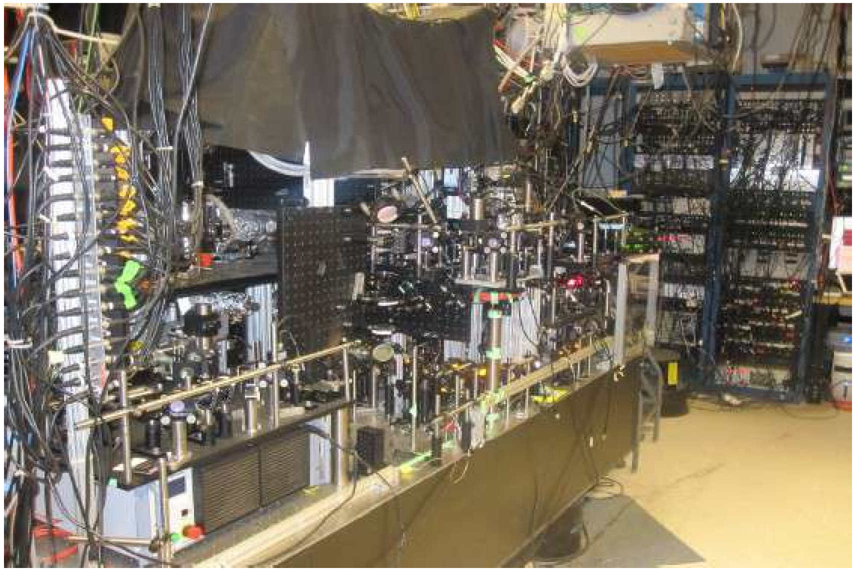
Experimental setup



^{6}Li atom

$|F = 1/2, m_F = 1/2\rangle$ → Up spin electron

$|F = 1/2, m_F = -1/2\rangle$ → Down spin electron



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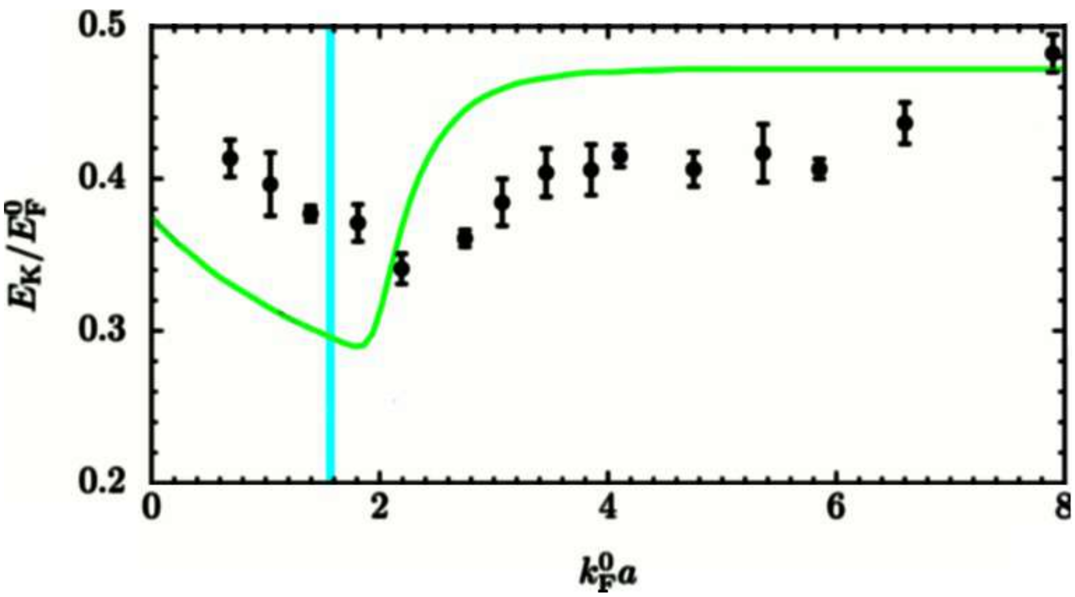
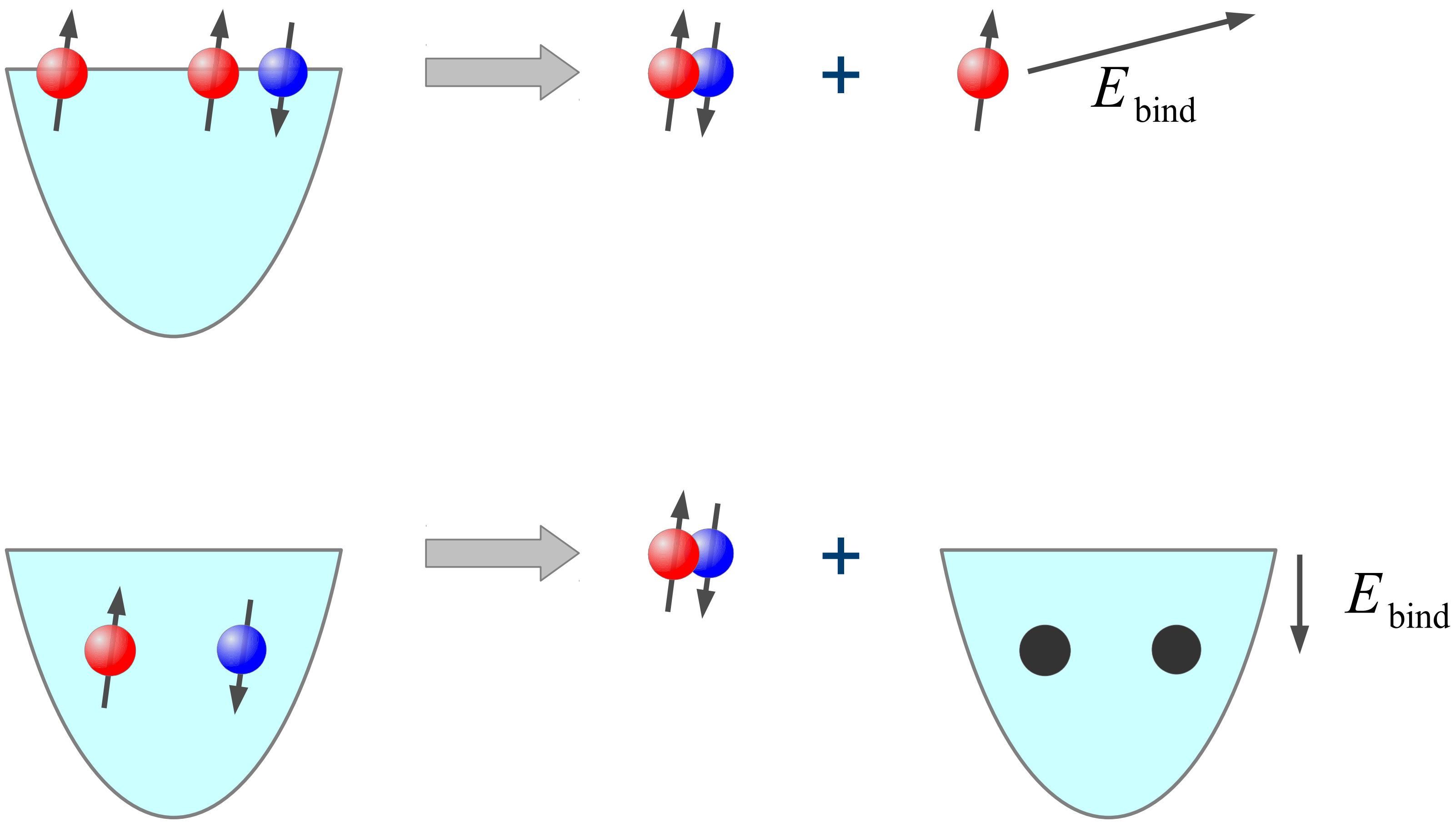
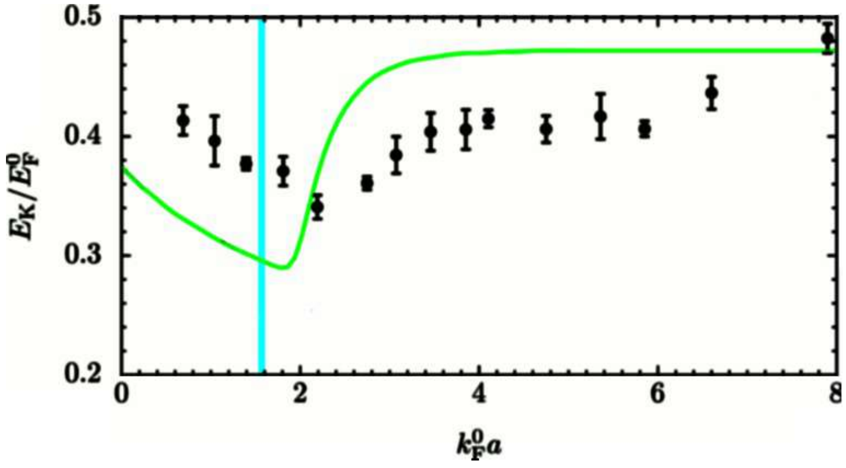


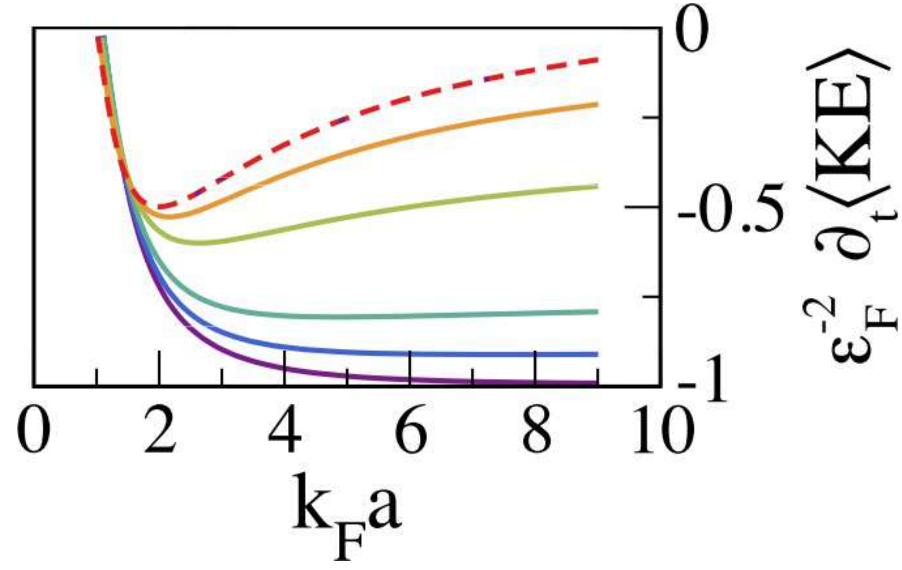
Jo *et al*. Science (2009), Conduit & Simons PRL (2009)

Competing loss processes



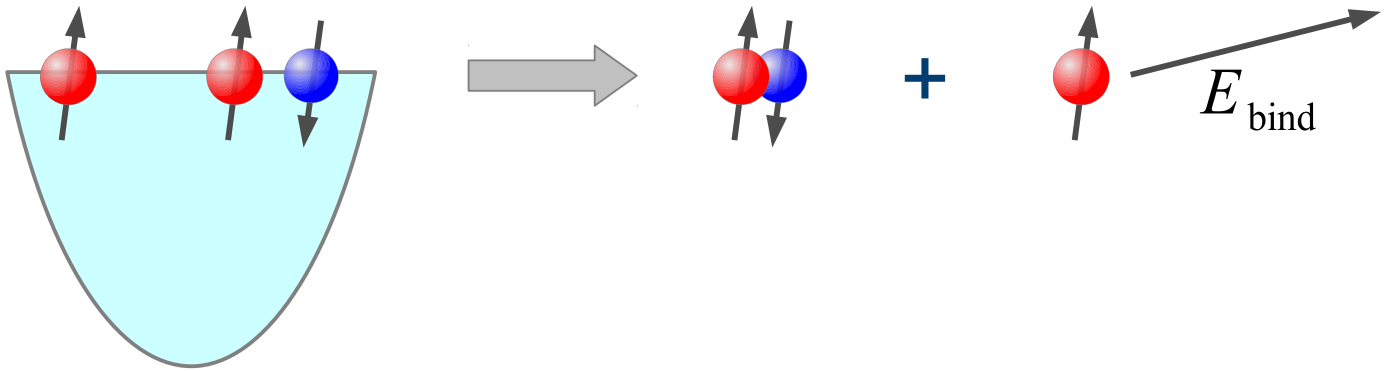
Experimental setup

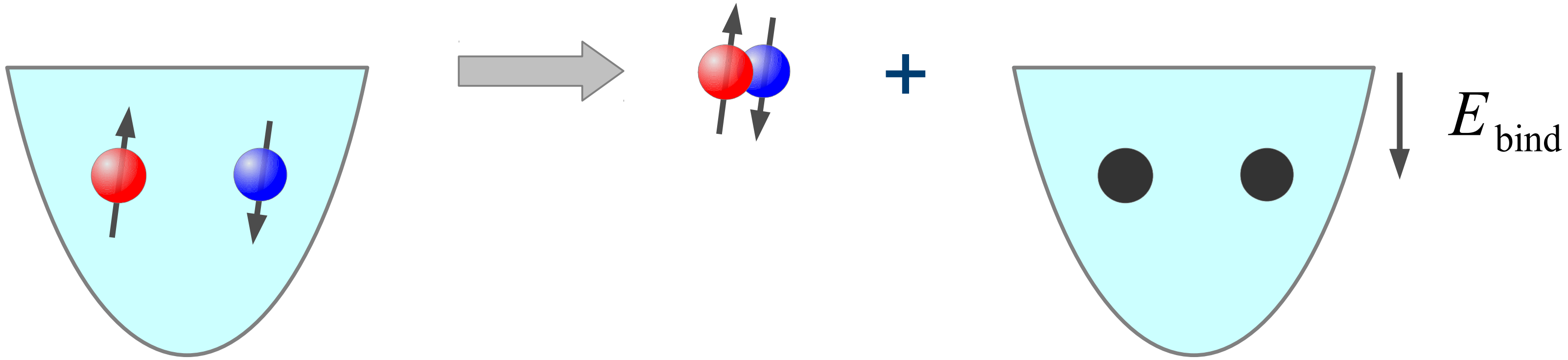




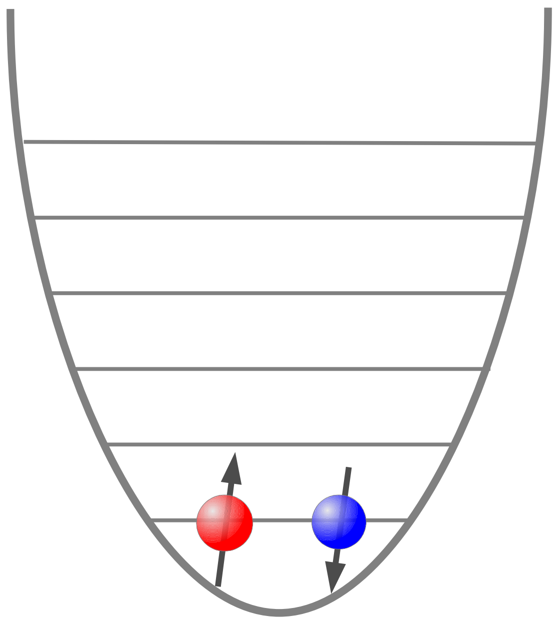
Pekker *et al*. PRL **106**, 050402 (2011)

Competing loss processes



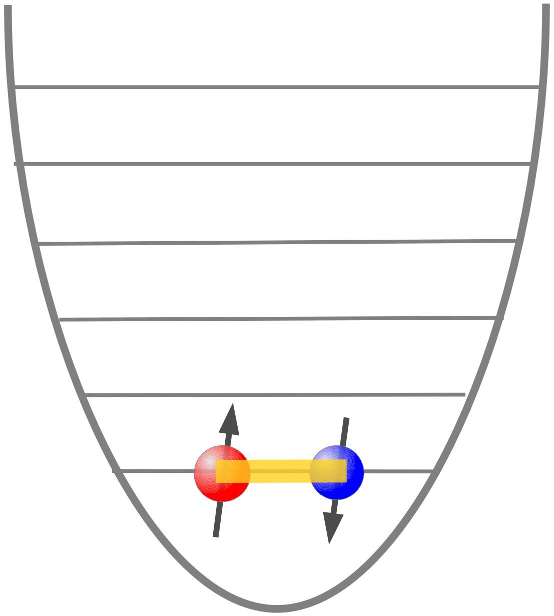


Two distinguishable fermions



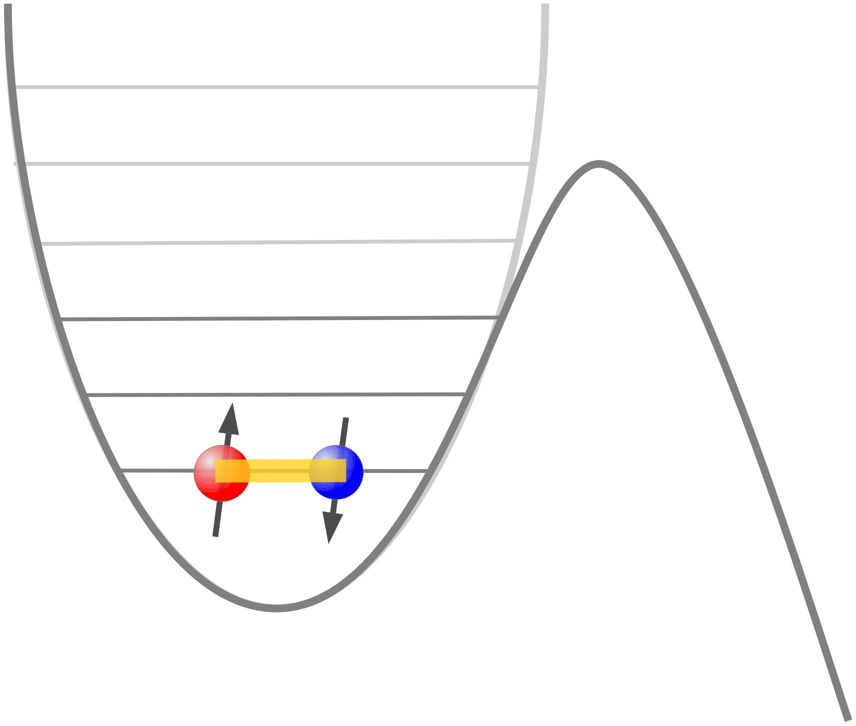
Zürn PRL **108** 075303 (2012)

Two distinguishable fermions

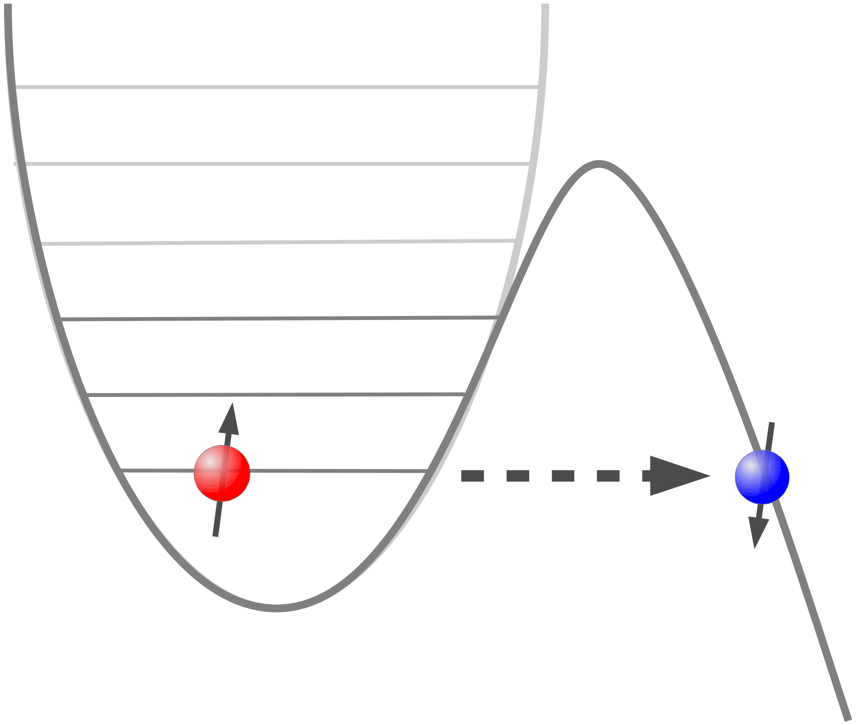


Zürn PRL **108** 075303 (2012)

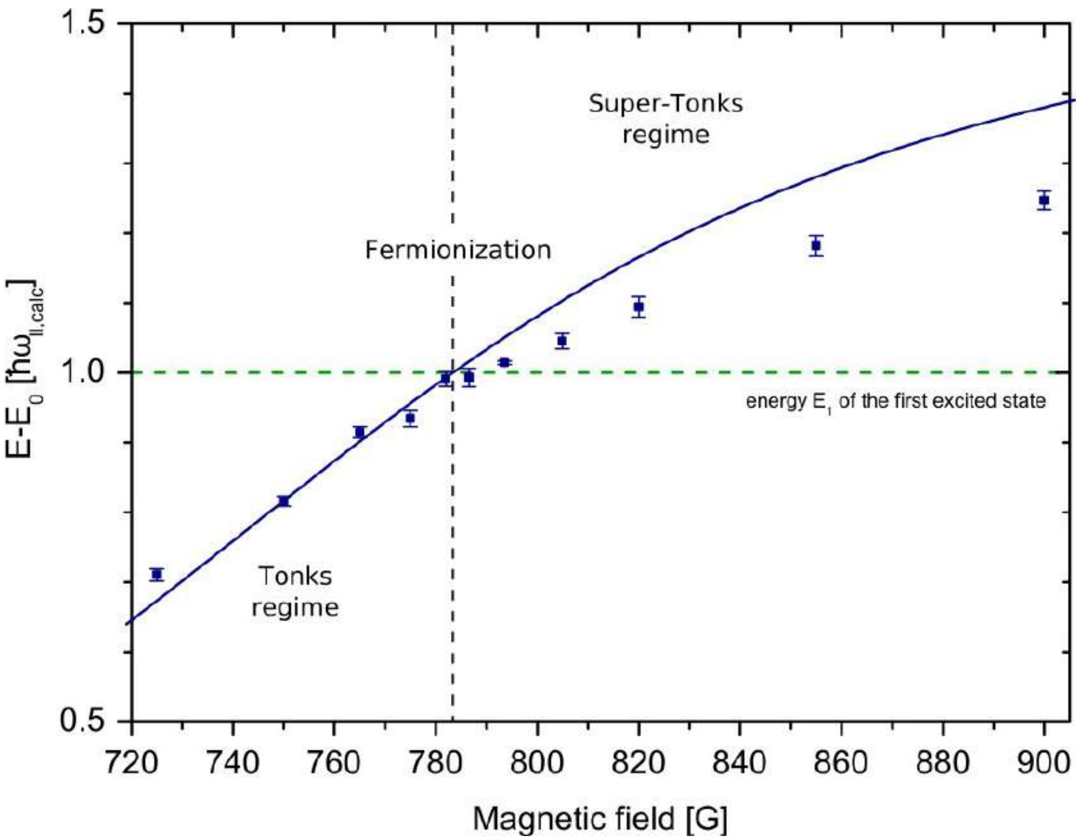
Two distinguishable fermions

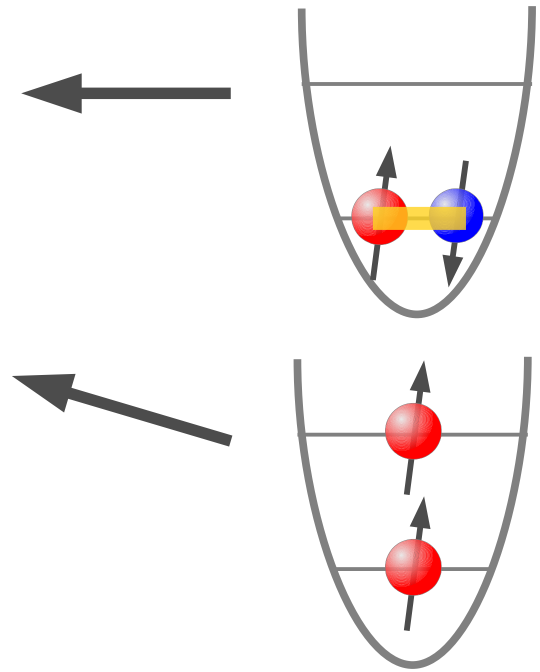


Two distinguishable fermions



Energy of states

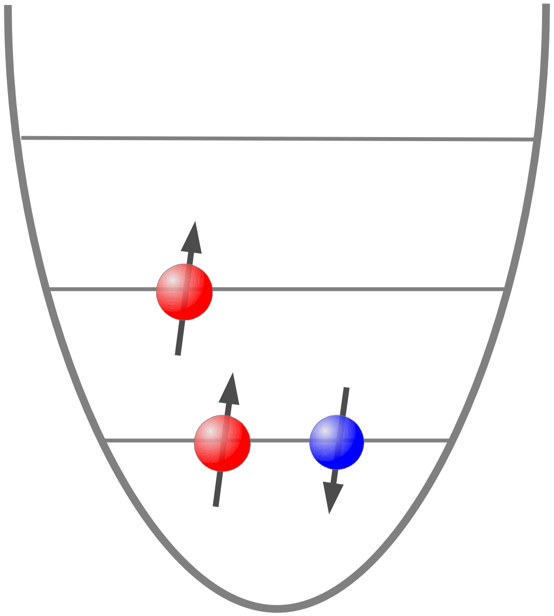




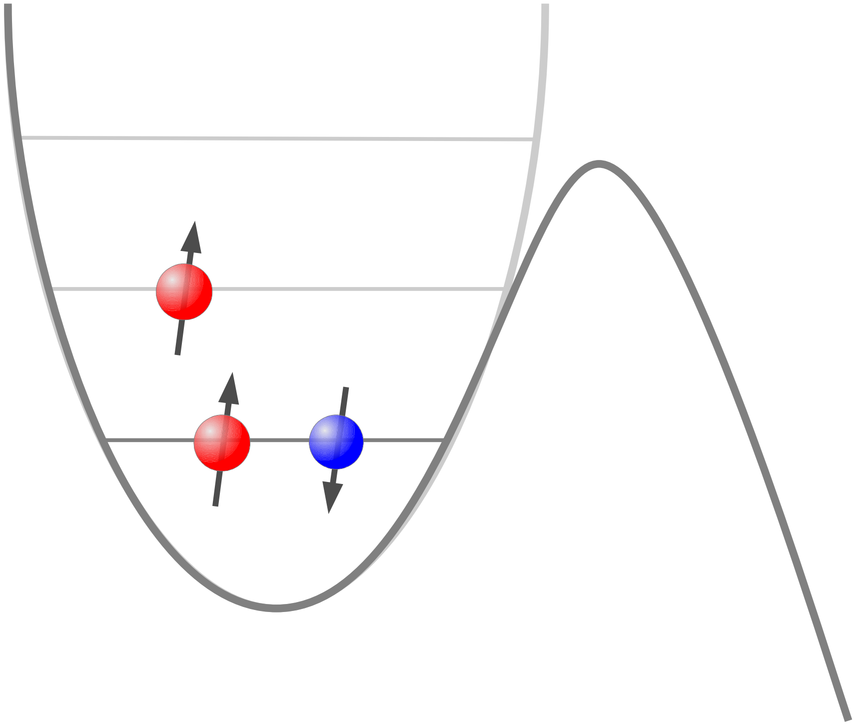
Weak repulsion

Strong repulsion

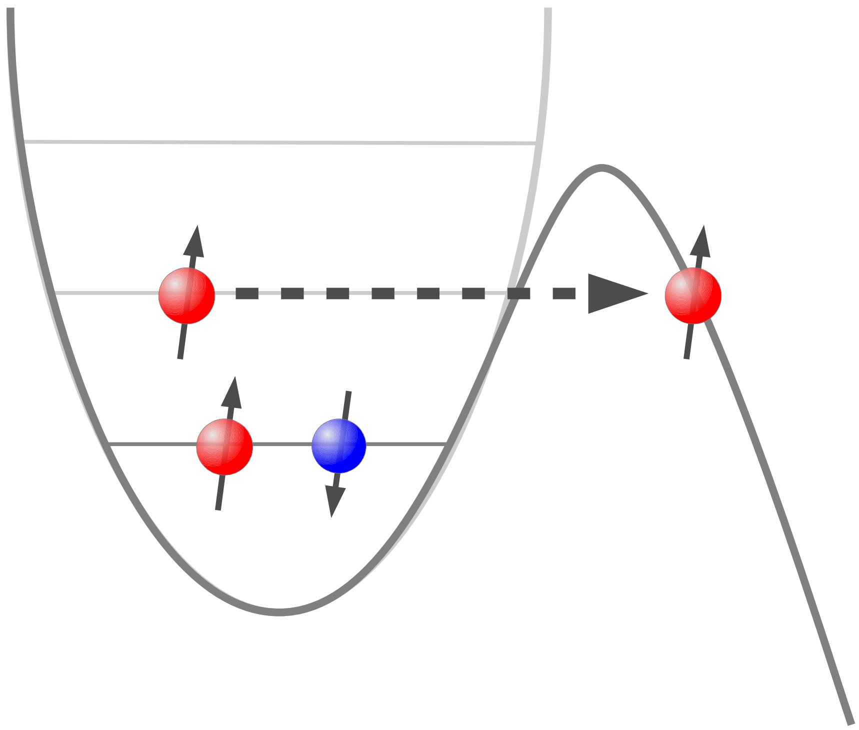
Polaron state



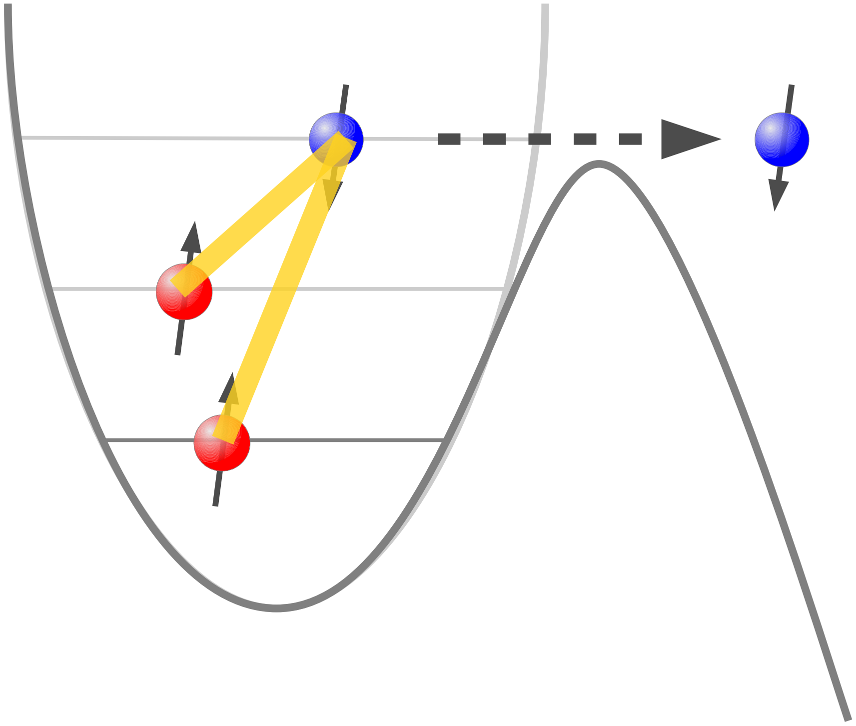
Polaron state



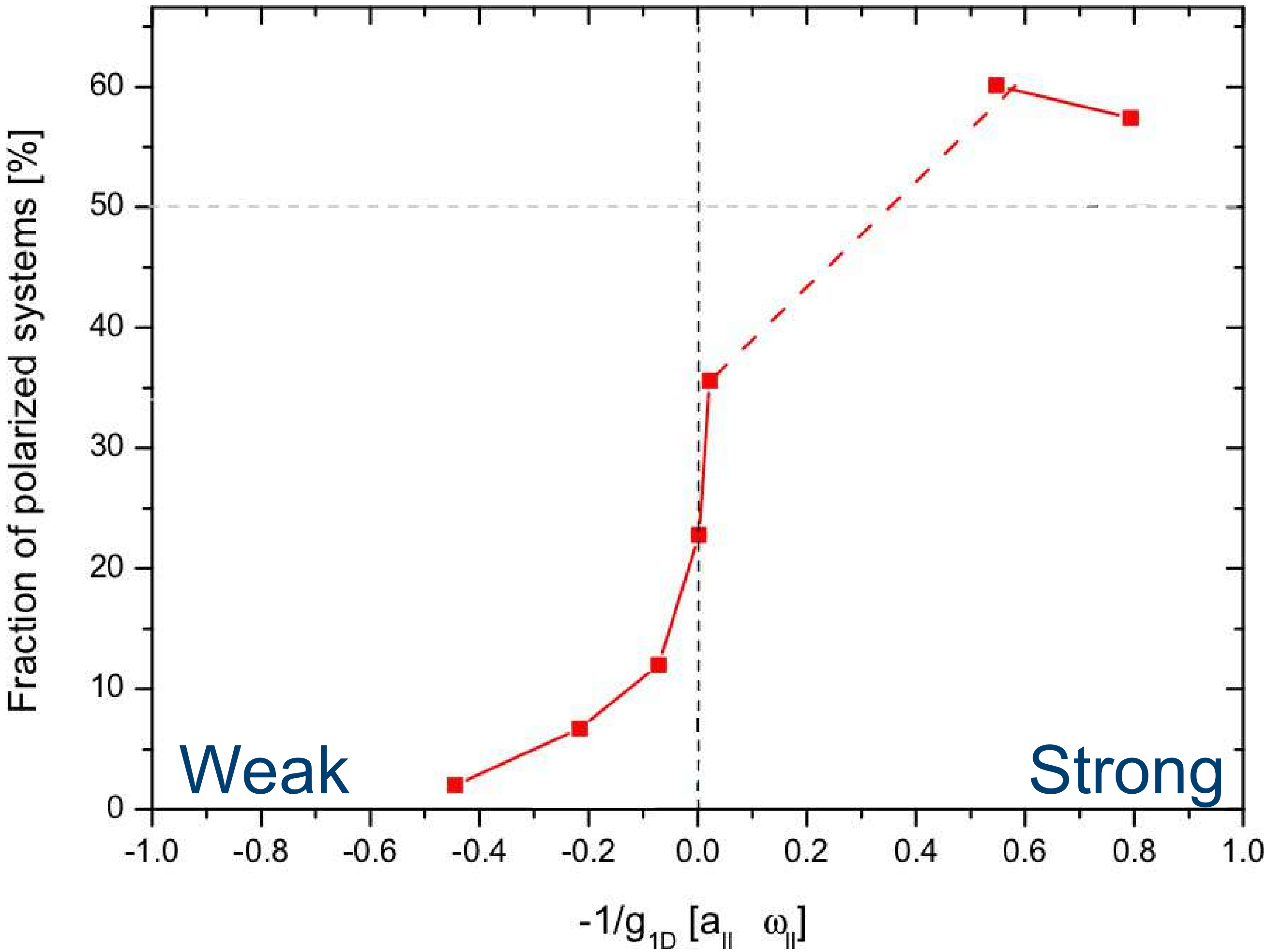
Polaron state

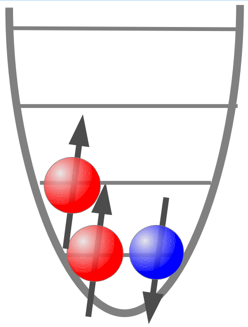


Polaron state



Tunneling probability





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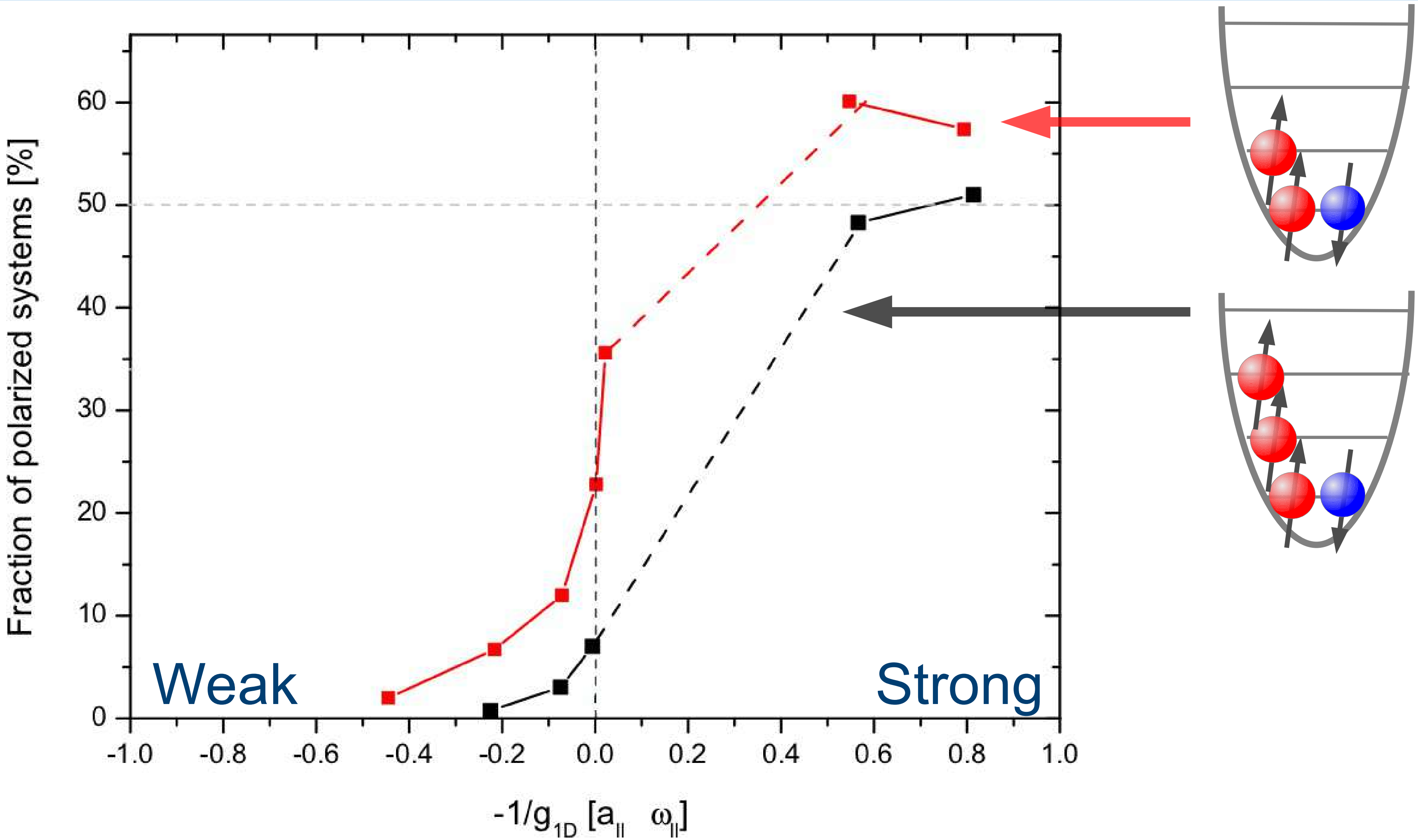

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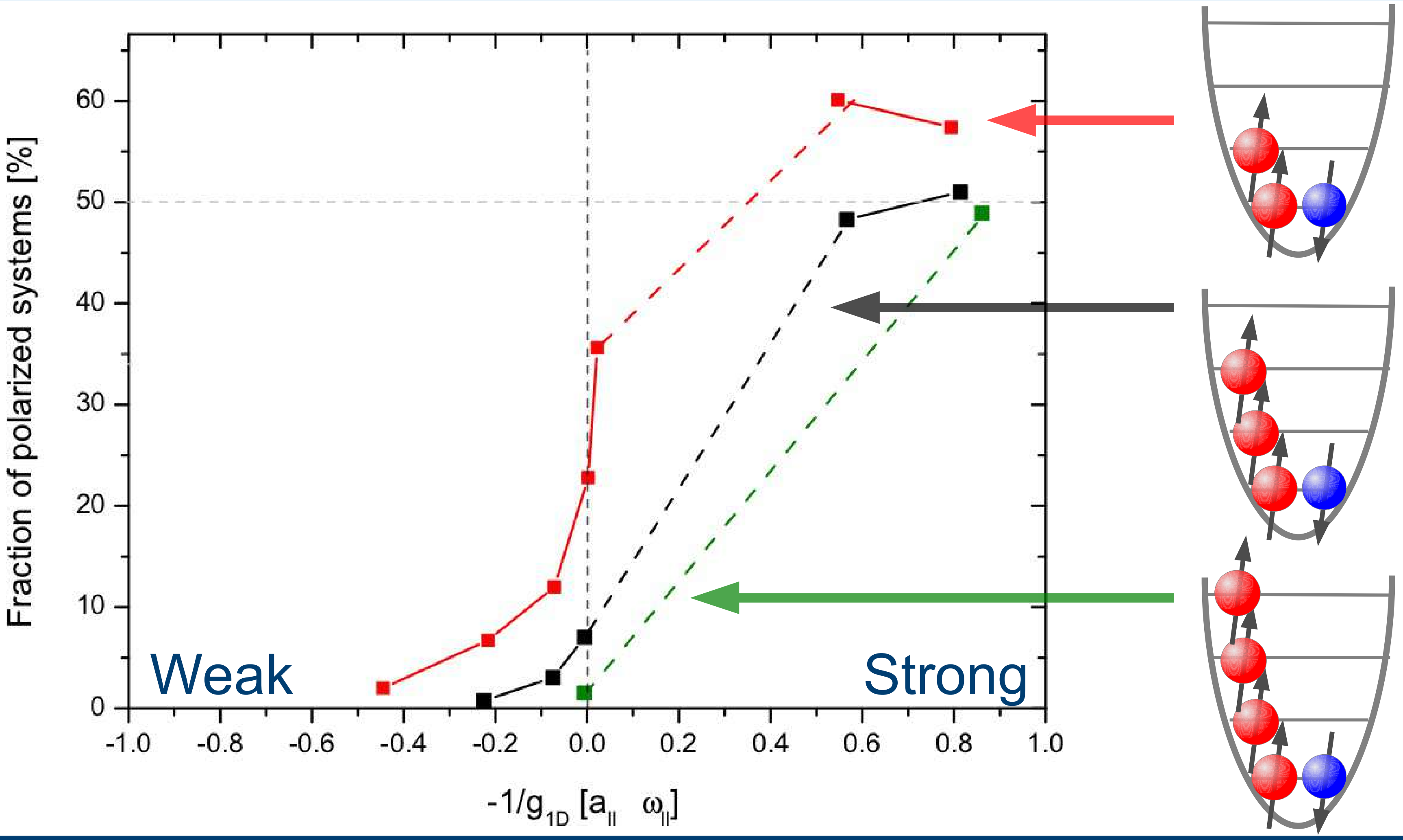

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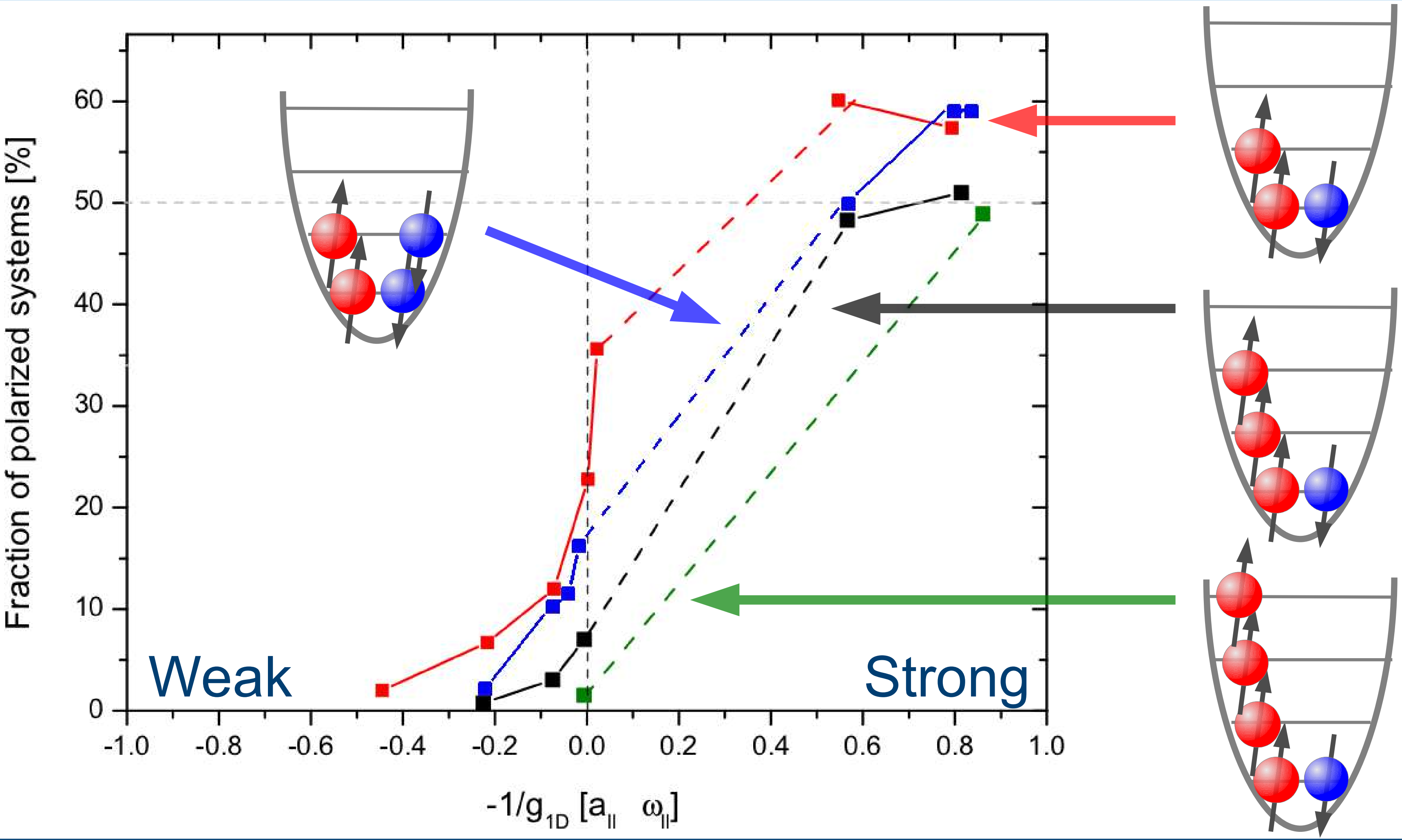
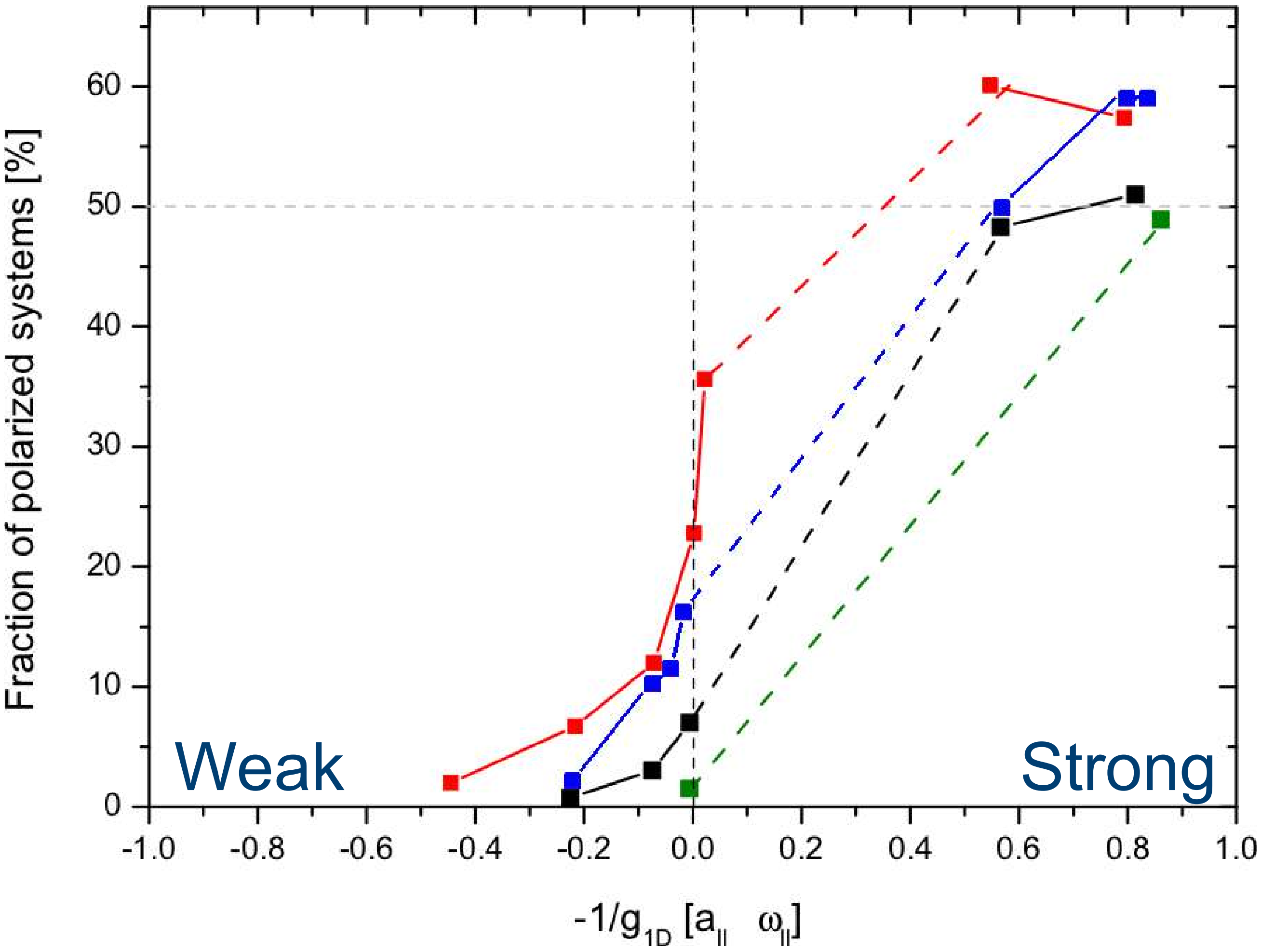
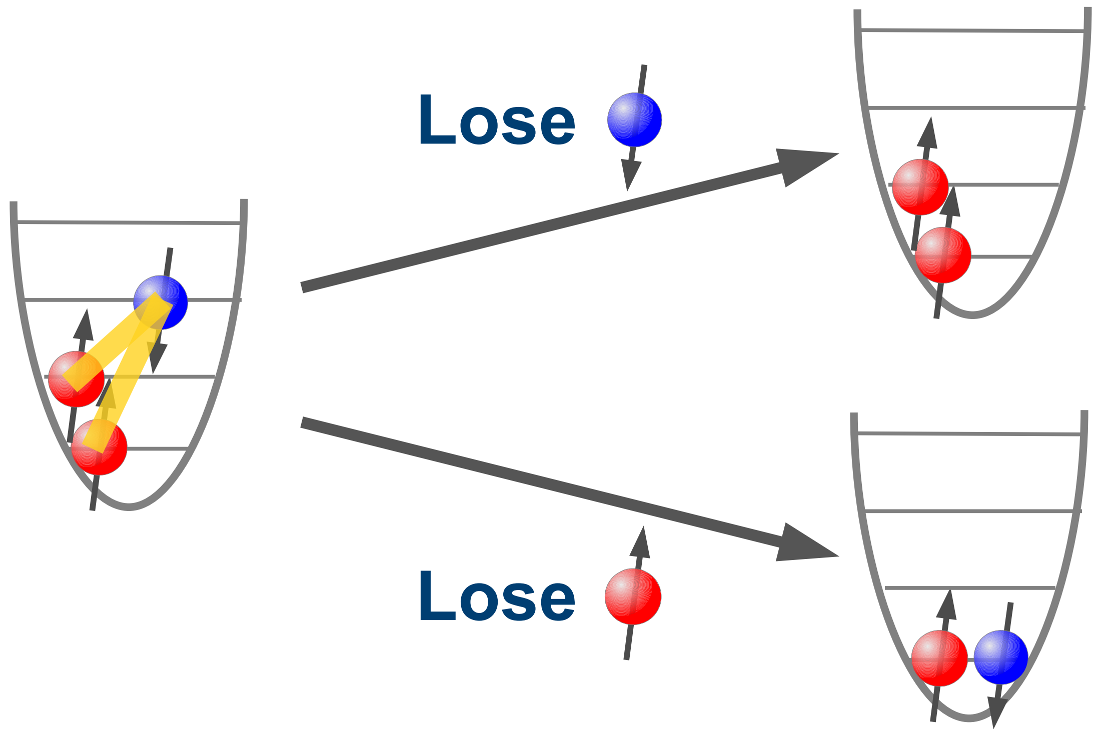


Tunneling probability



Why probability of ½?



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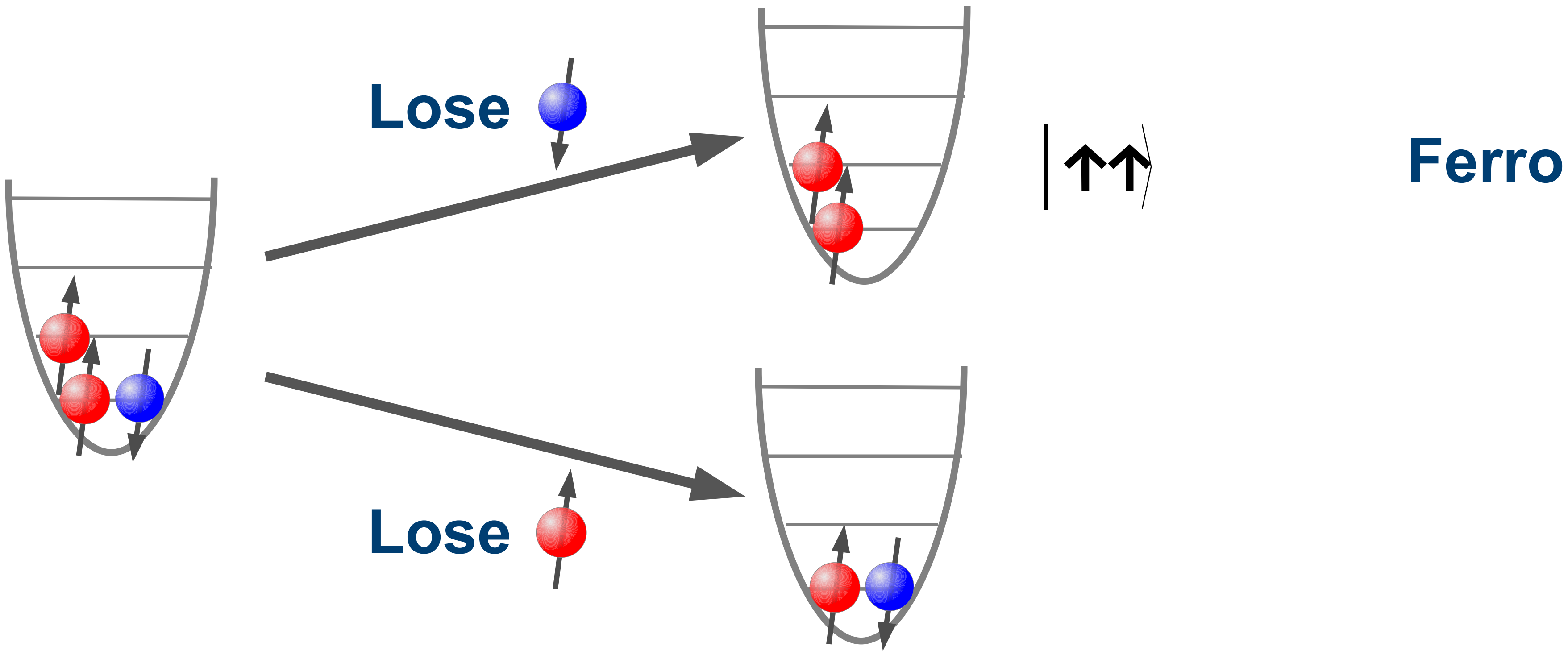

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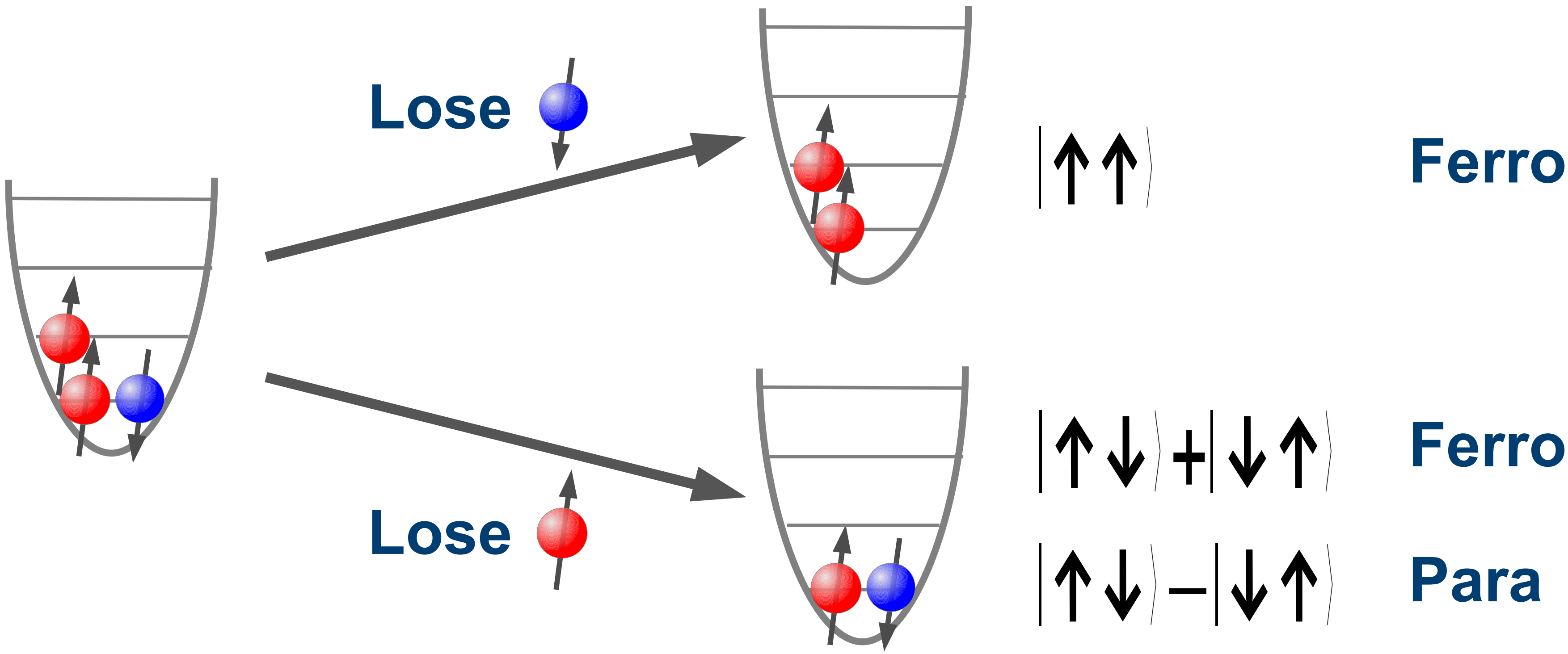
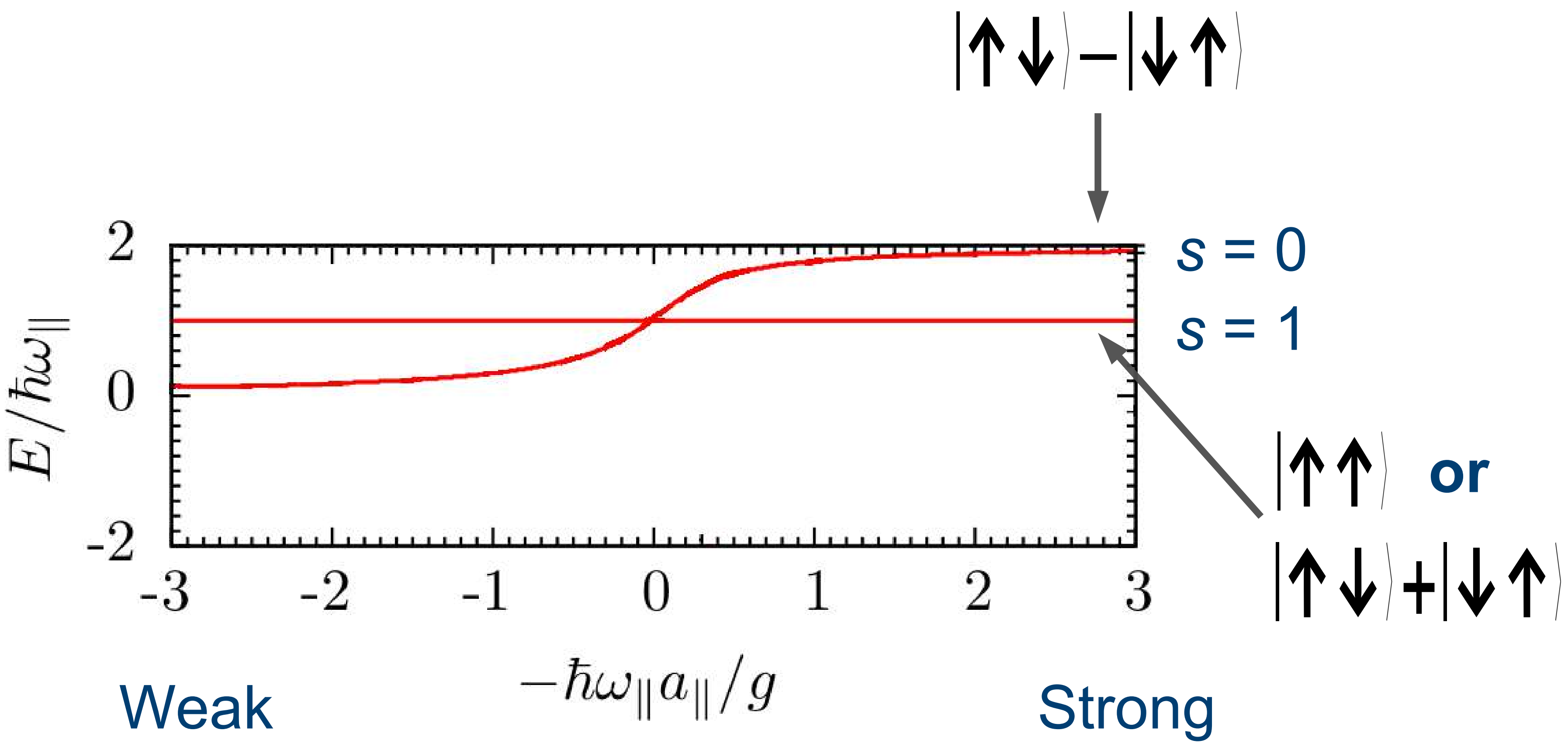


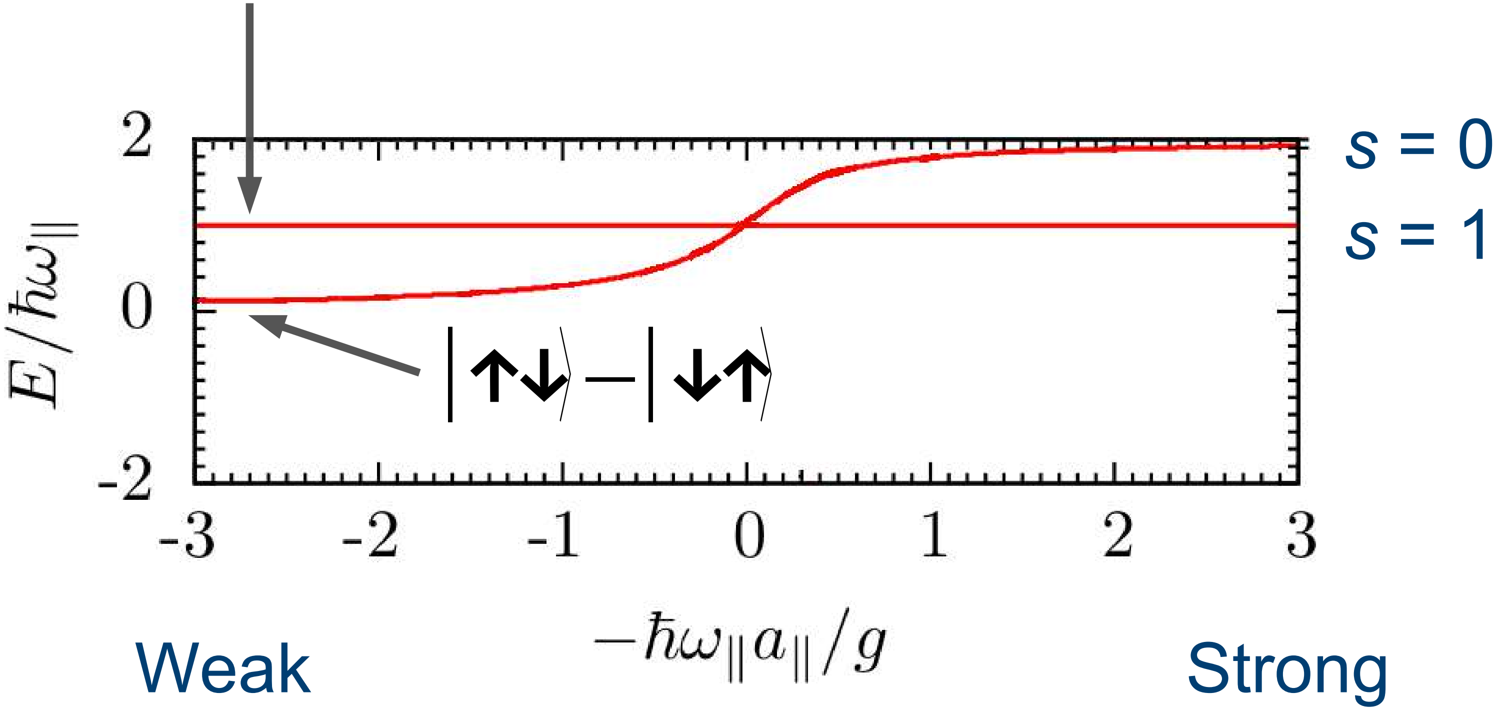
Finding the missing probability



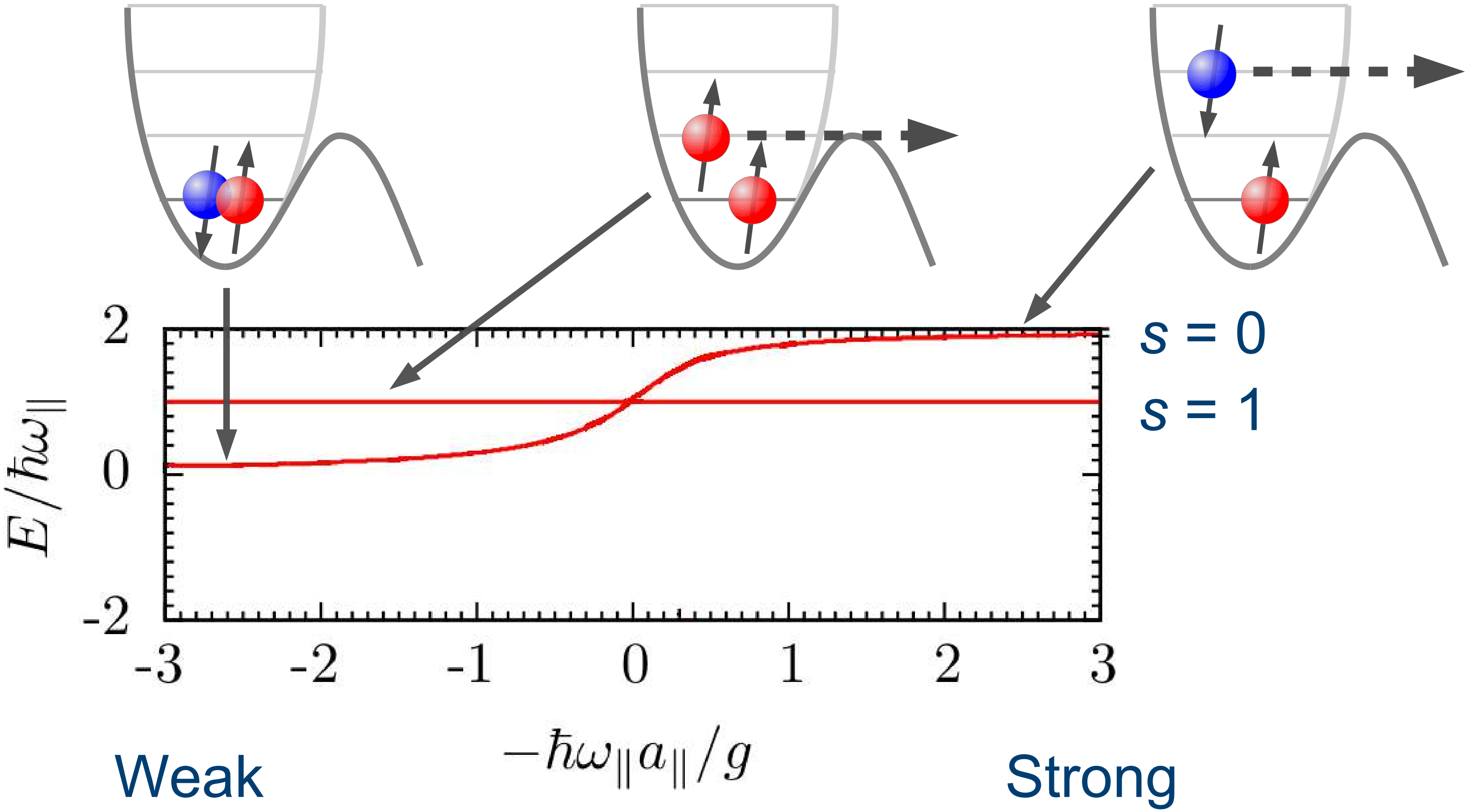
Finding the missing probability

$|\uparrow\uparrow\rangle$ **or**

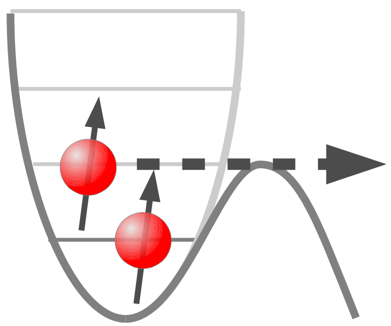
$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$

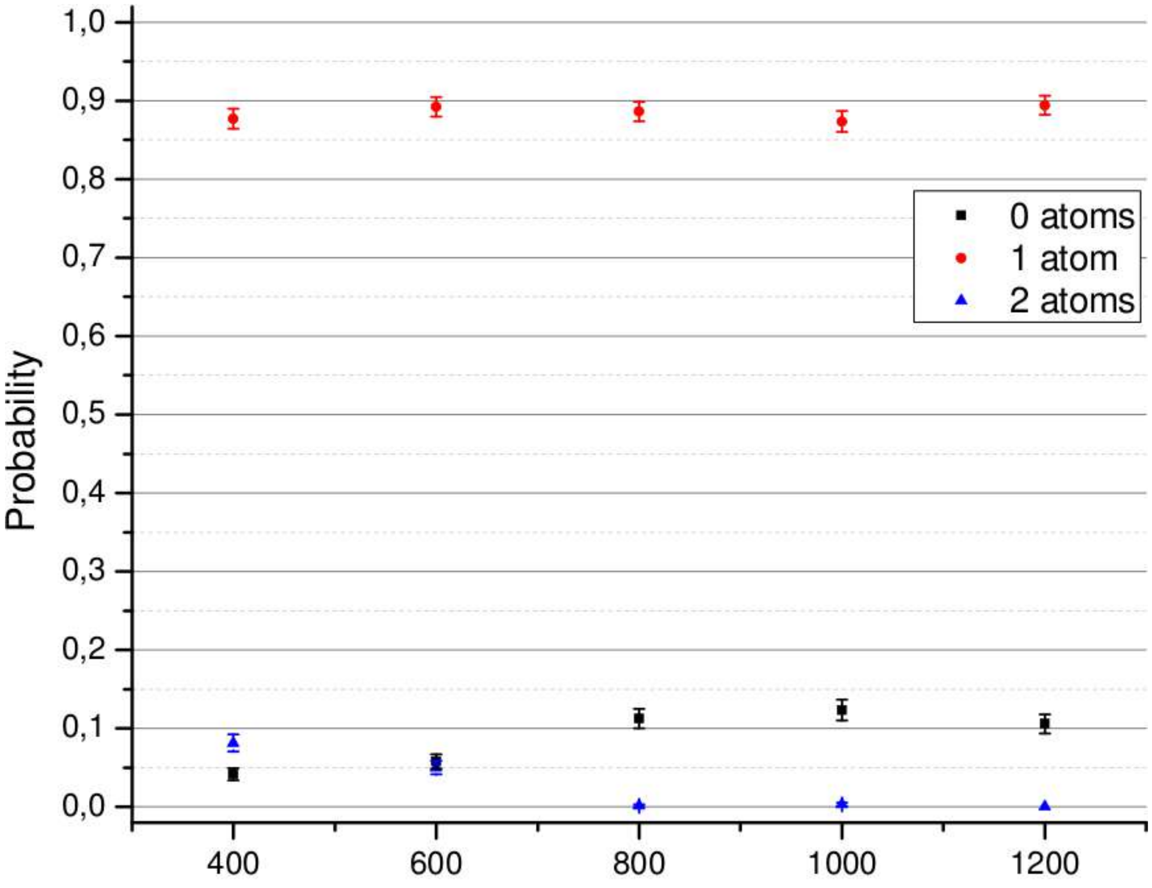


Finding the missing probability



Tunneling probability

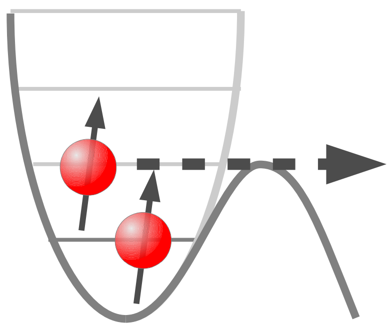




Weak

Strong

Tunneling probability



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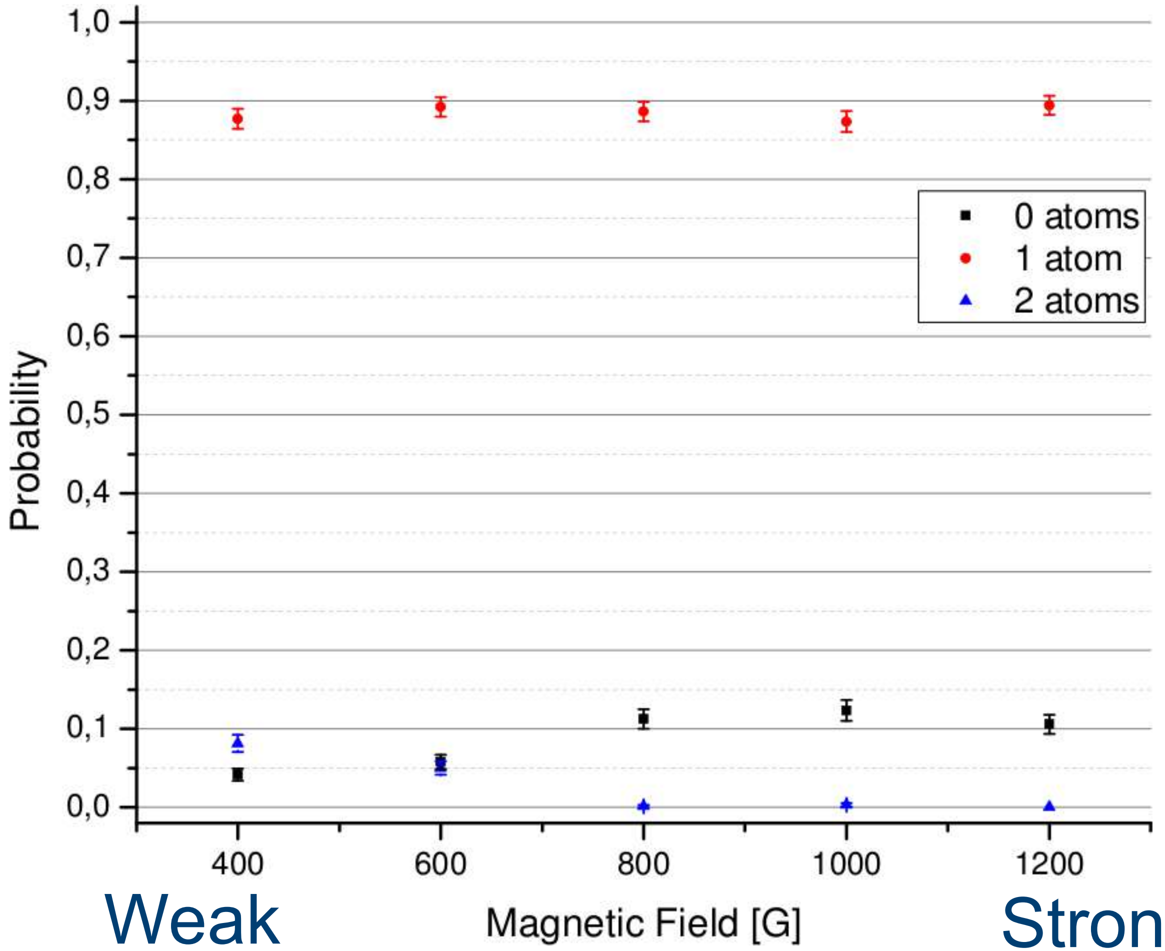


Weak

Strong

Lieb-Mattis theorem

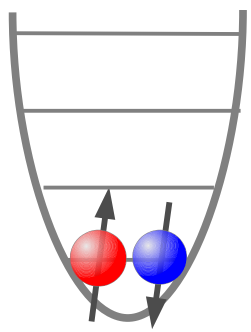
PHYSICAL REVIEW VOLUME 125, NUMBER 1 JANUARY 1, 1962

Theory of Ferromagnetism and the Ordering of Electronic Energy Levels

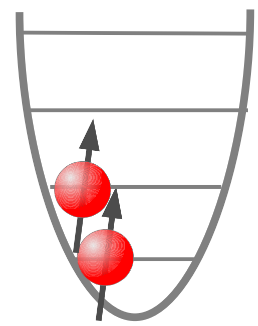
Elliott Lieb and Daniel Mattis
Thomas J. Watson Research Center, International Business Machines Corporation, Yorktown Heights, New York

(Received May 25, 1961; revised manuscript received September 11, 1961)

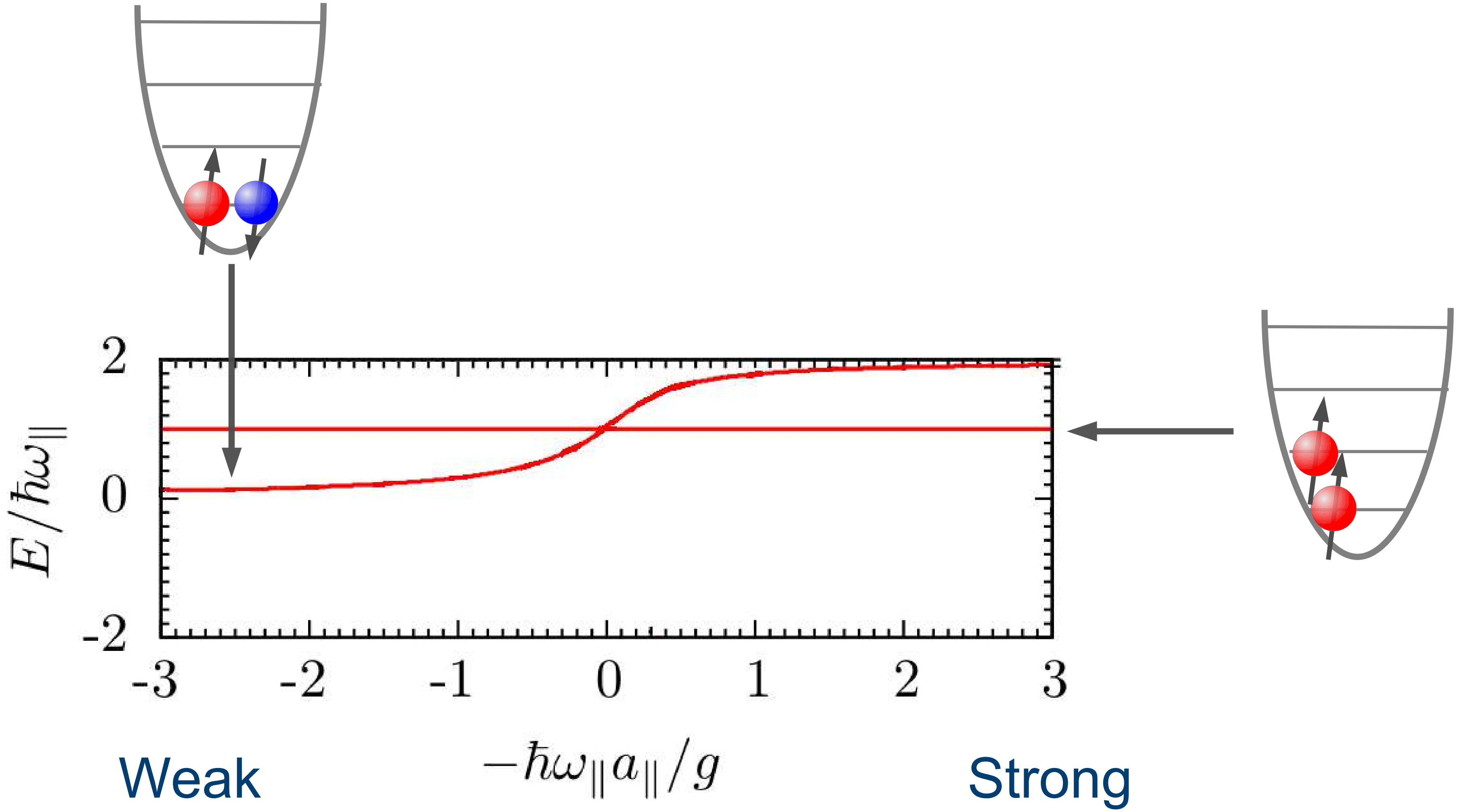
Consider a system of N electrons in one dimension subject to an arbitrary symmetric potential, $V(x_1,\cdots,x_N)$, and let $E(S)$ be the lowest energy belonging to the total spin value S. We have proved the following theorem: $E(S) < E(S')$ if $S < S'$. Hence, the ground state is unmagnetized. The theorem also holds



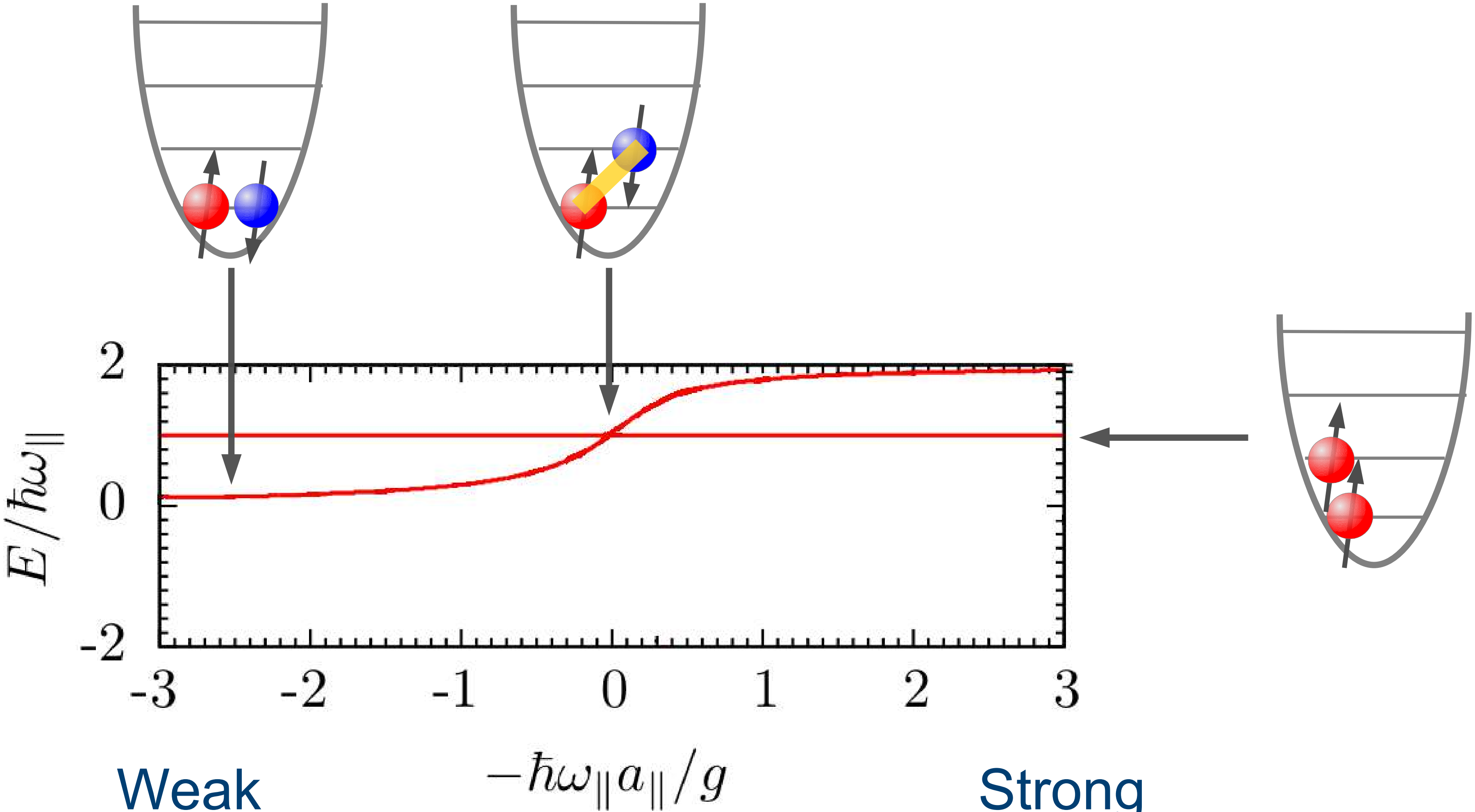
has lower energy than



Lieb-Mattis theorem



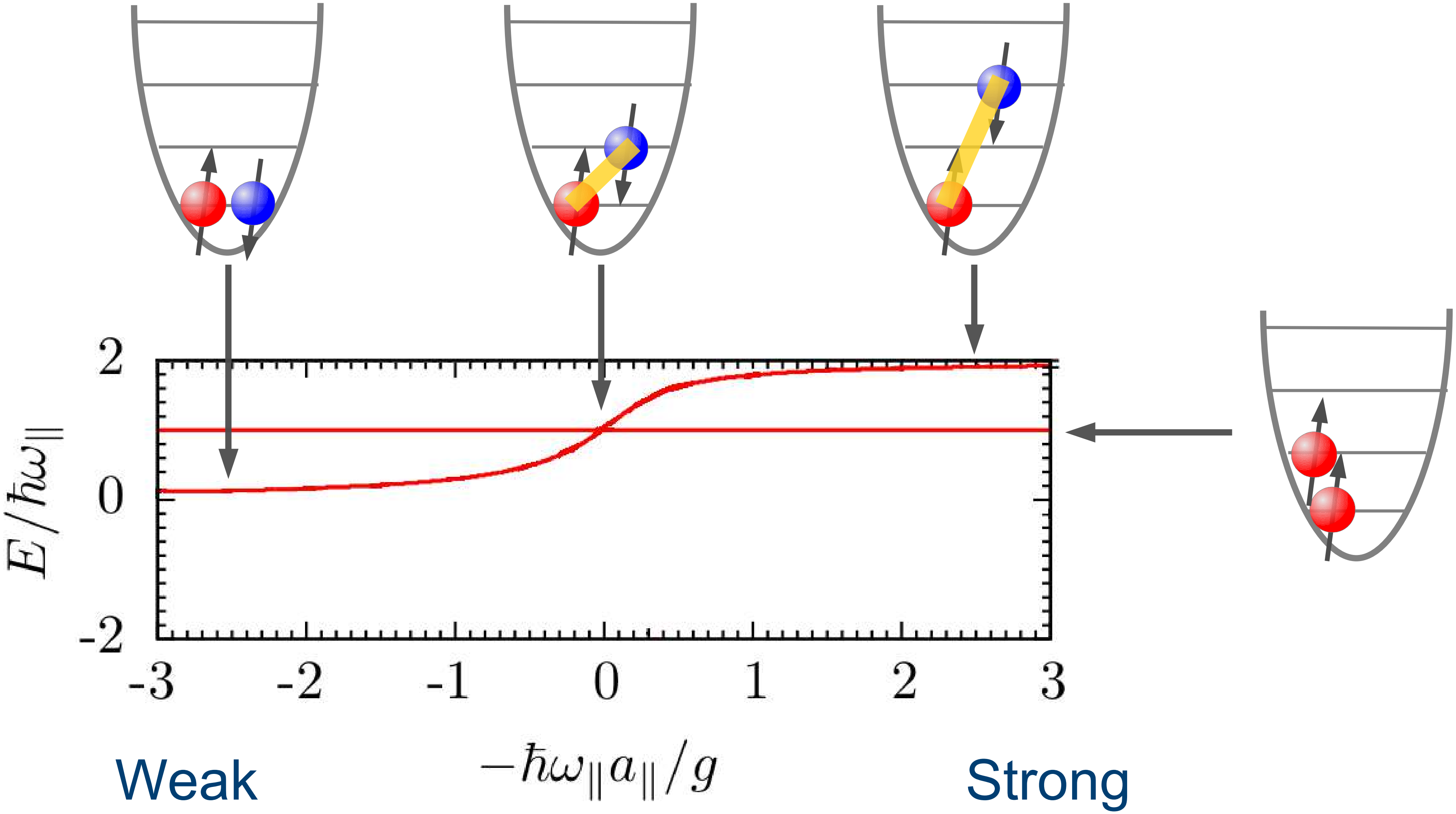
Lieb-Mattis theorem



Weak

Strong

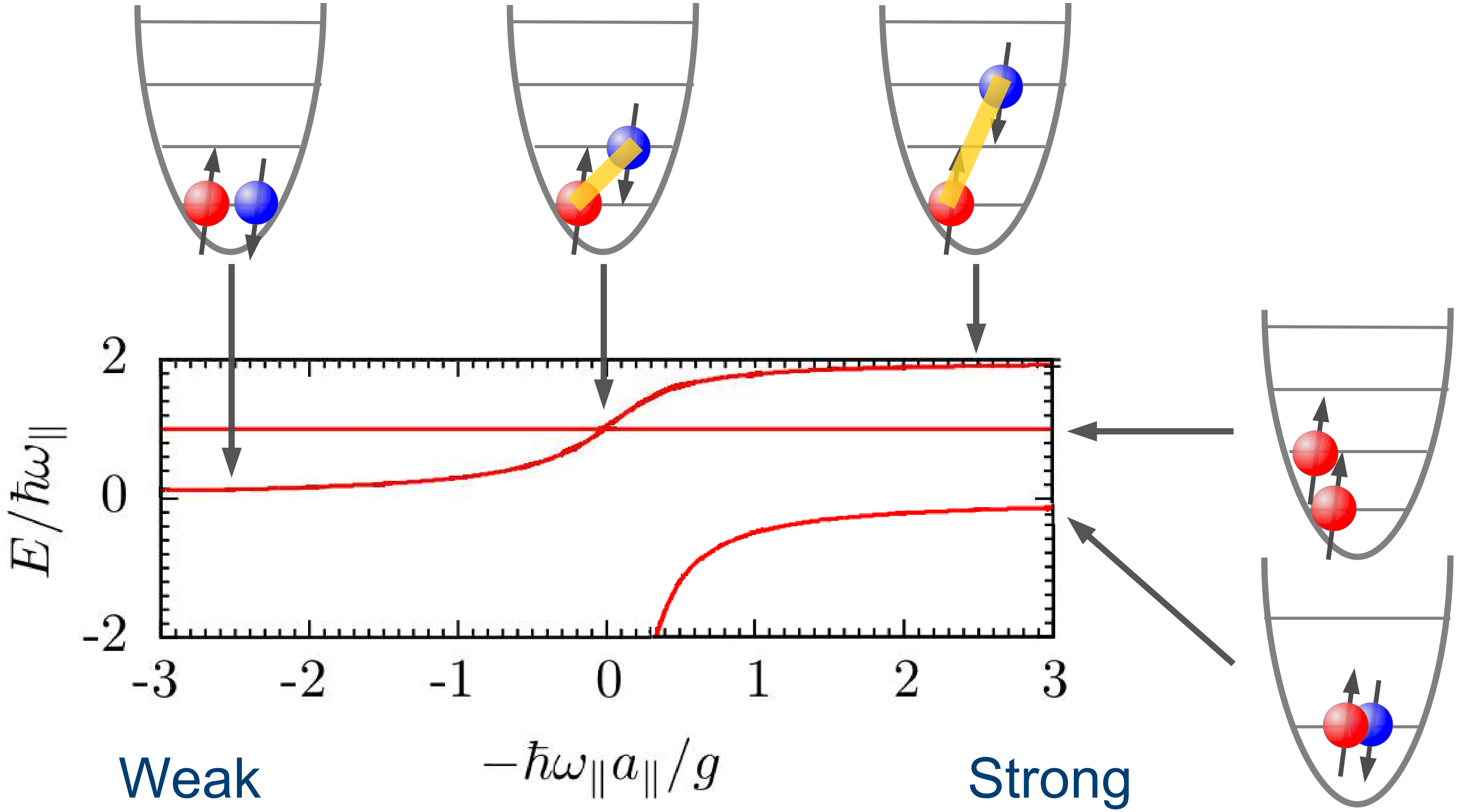
Lieb-Mattis theorem



Weak

Strong

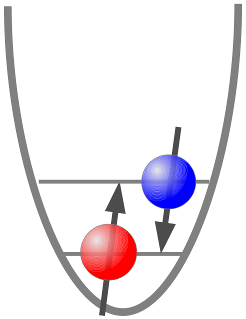
Lieb-Mattis theorem



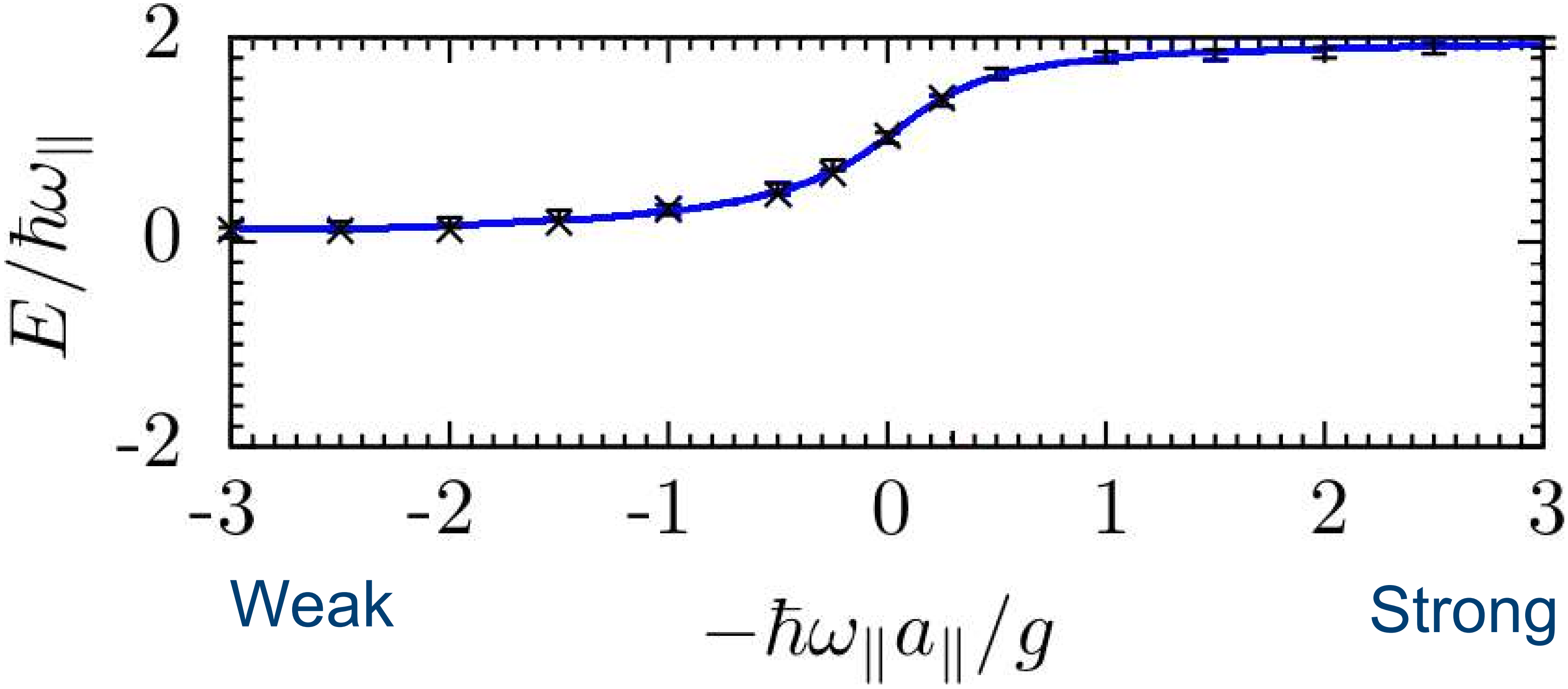
Computational analysis: exact diagonalization

$$\begin{pmatrix} \langle 0\uparrow, 0\downarrow| \\ \langle 1\uparrow, 0\downarrow| \\ \langle 0\uparrow, 1\downarrow| \\ \langle 1\uparrow, 1\downarrow| \end{pmatrix} \hat{H} \left(|0\uparrow, 0\downarrow\rangle \quad |1\uparrow, 0\downarrow\rangle \quad |0\uparrow, 1\downarrow\rangle \quad |1\uparrow, 1\downarrow\rangle \right)$$

$$H = \begin{pmatrix} H_{00,00} & H_{00,10} & H_{00,01} & H_{00,11} \\ H_{10,00} & H_{10,10} & H_{10,01} & H_{10,11} \\ H_{01,00} & H_{01,10} & H_{01,01} & H_{01,11} \\ H_{11,00} & H_{11,10} & H_{11,01} & H_{11,11} \end{pmatrix}$$

$|0\uparrow, 1\downarrow\rangle =$ 

Tunneling probability



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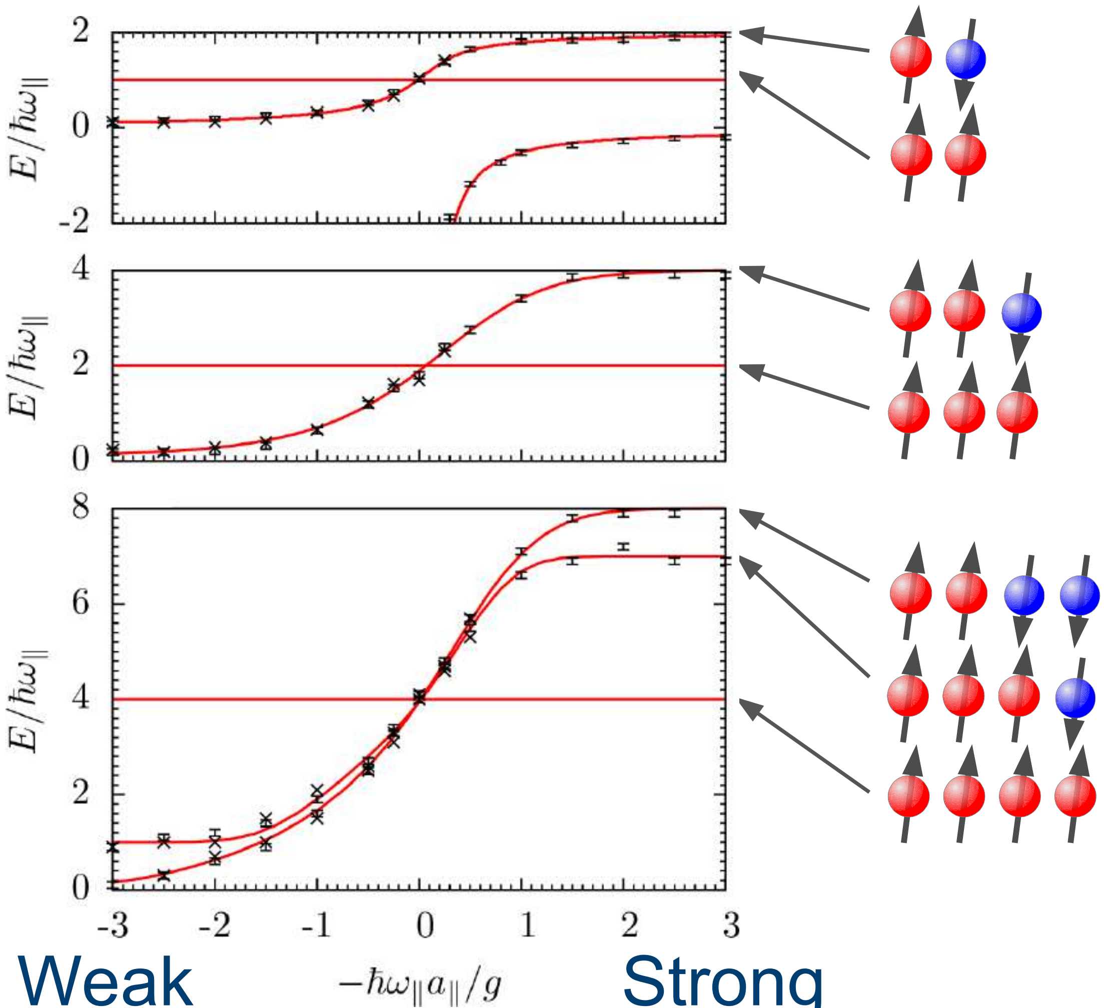

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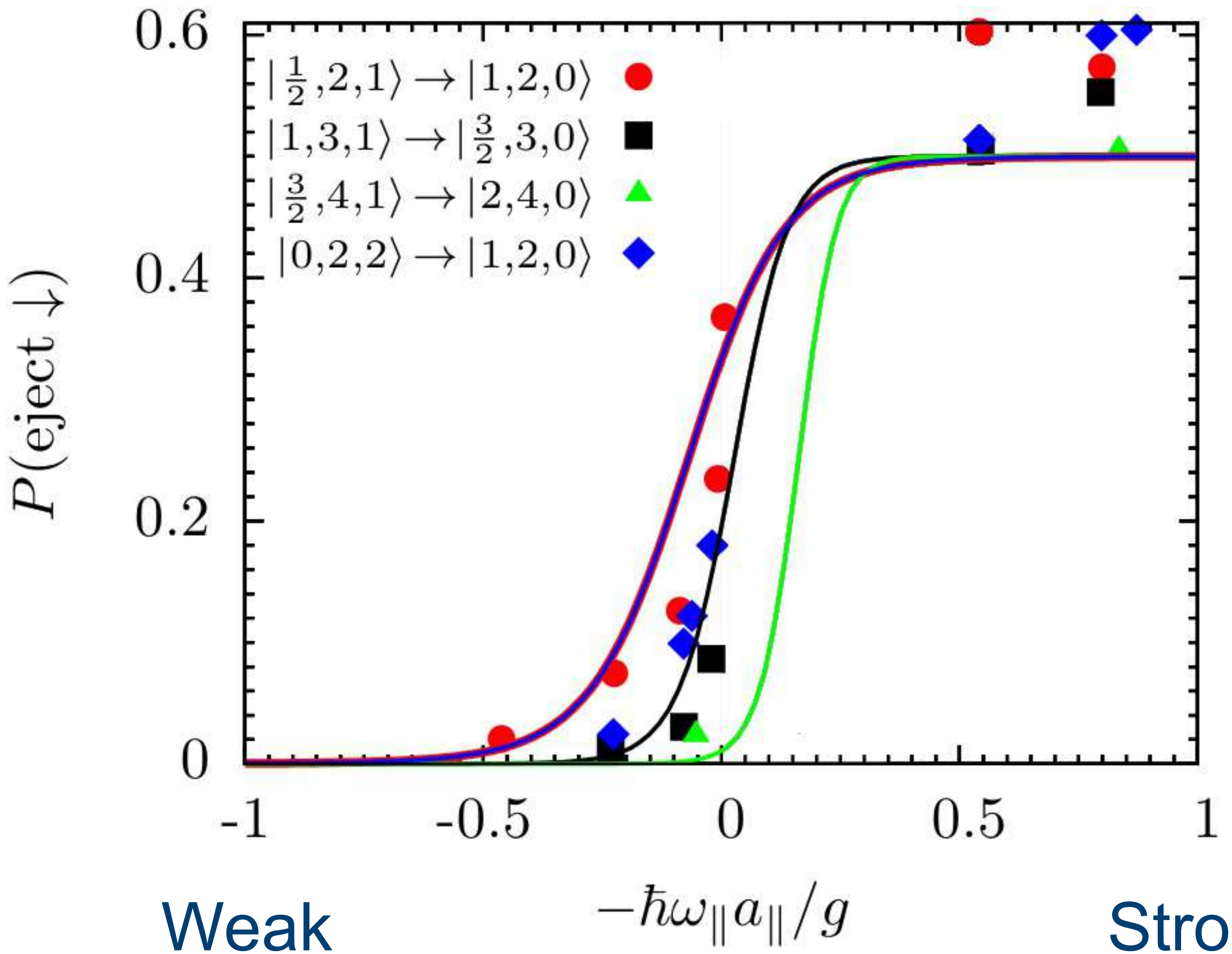
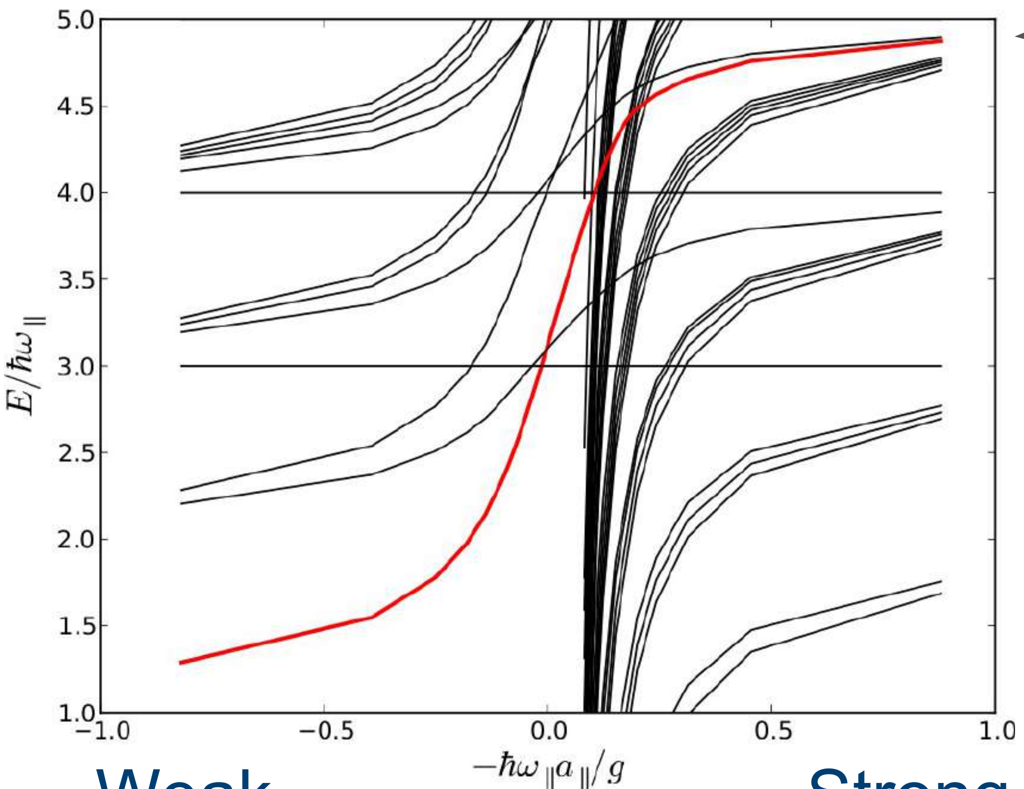


Weak

Strong

Density profiles



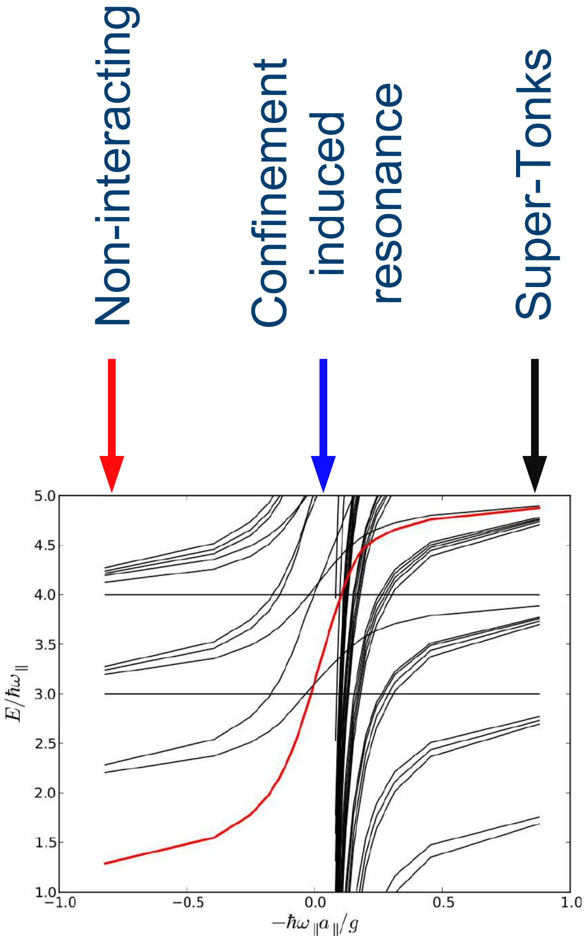
Weak

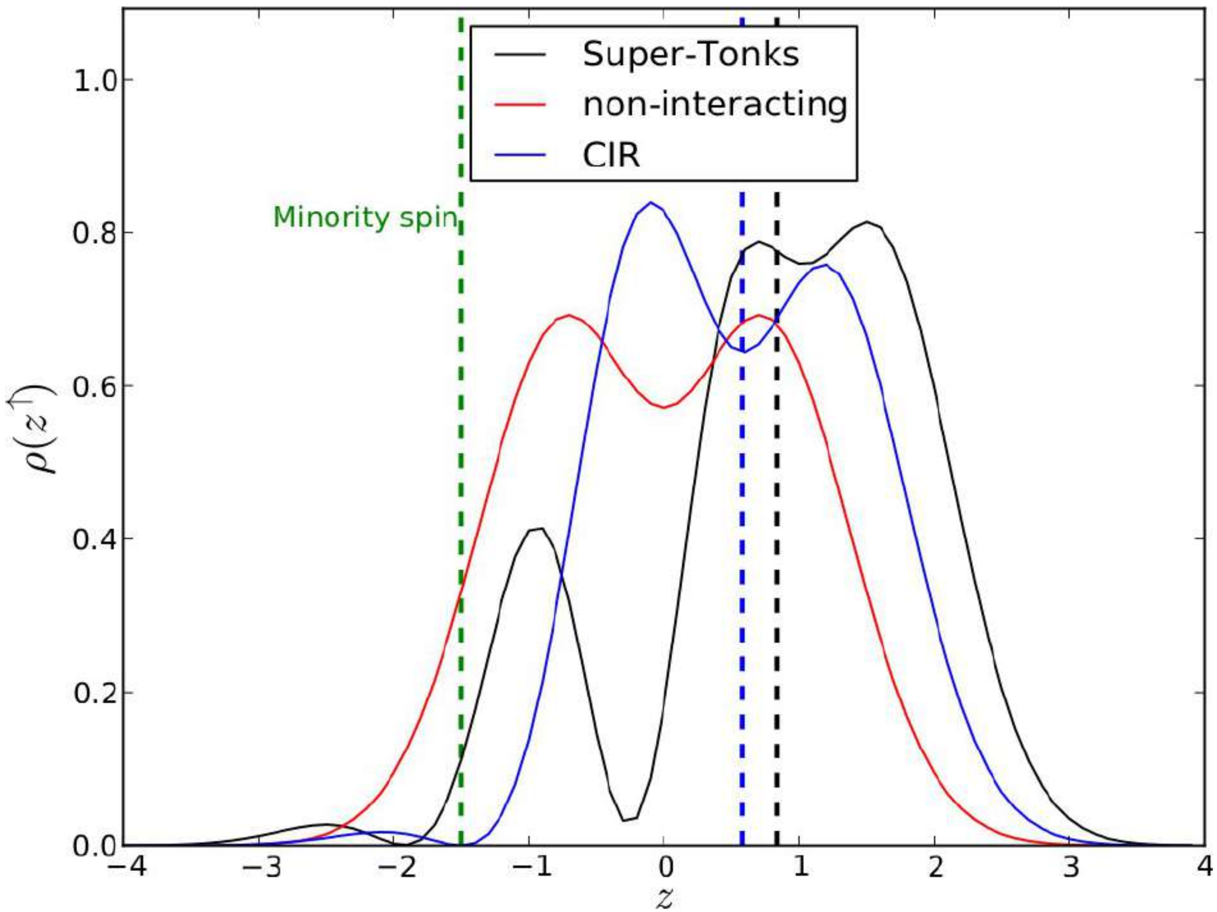
Strong

Three-atom state

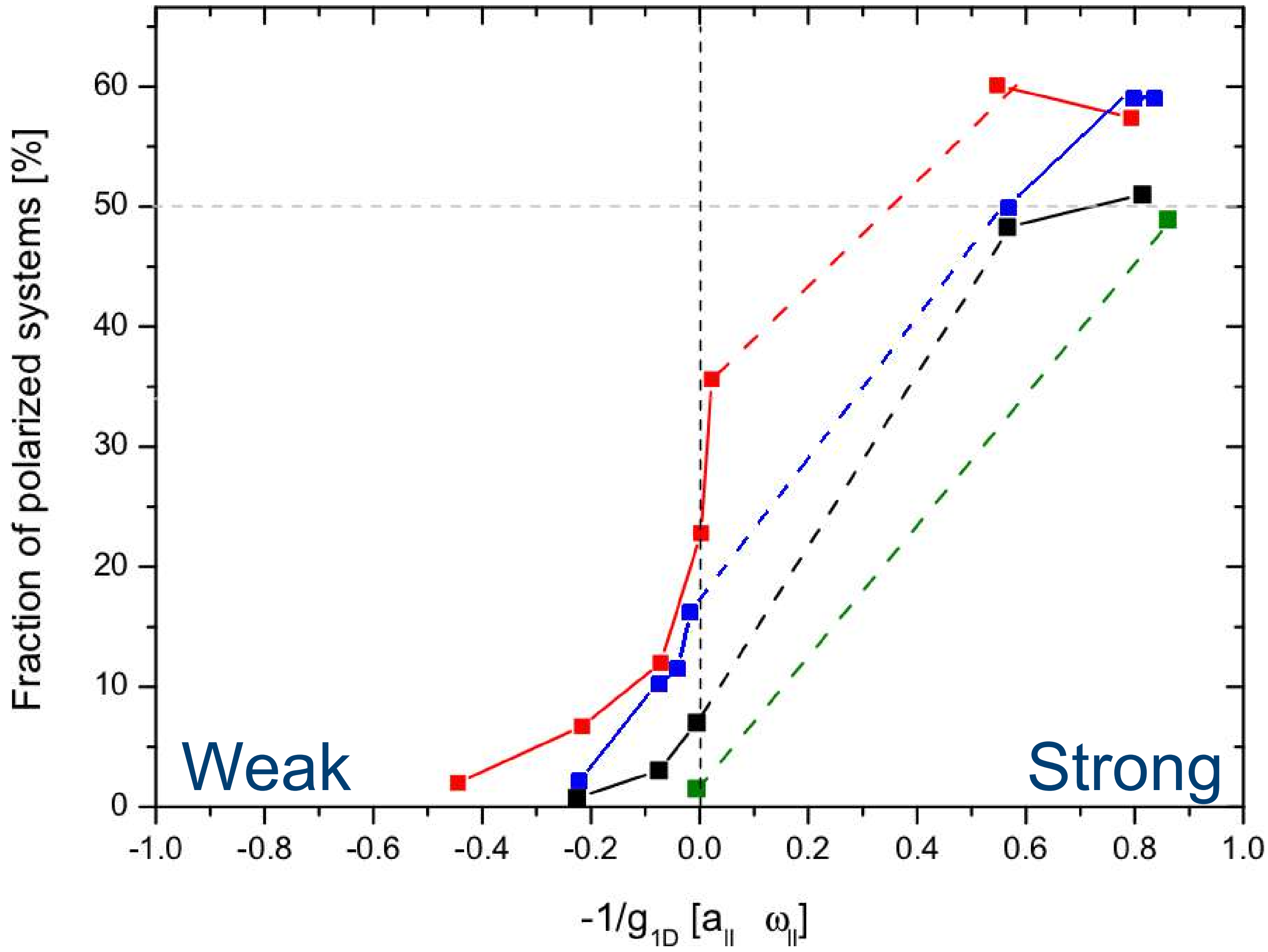
$$
\begin{aligned}
|1/2,2,1\rangle_U = &-0.172 c^+_{0\uparrow} c^+_{1\uparrow} c^+_{4\downarrow} + \\
&0.341 c^+_{0\uparrow} c^+_{2\uparrow} c^+_{3\downarrow} - 0.174 c^+_{0\uparrow} c^+_{3\uparrow} c^+_{2\downarrow} - \\
&0.080 c^+_{0\uparrow} c^+_{4\uparrow} c^+_{1\downarrow} + 0.010 c^+_{0\uparrow} c^+_{5\uparrow} c^+_{0\downarrow} + \\
&0.442 c^+_{1\uparrow} c^+_{3\uparrow} c^+_{1\downarrow} - 0.436 c^+_{1\uparrow} c^+_{2\uparrow} c^+_{2\downarrow} + \\
&0.092 c^+_{1\uparrow} c^+_{4\uparrow} c^+_{0\downarrow} - 0.515 c^+_{2\uparrow} c^+_{3\uparrow} c^+_{0\downarrow} |\rangle
\end{aligned}
$$

Density profiles





Losses

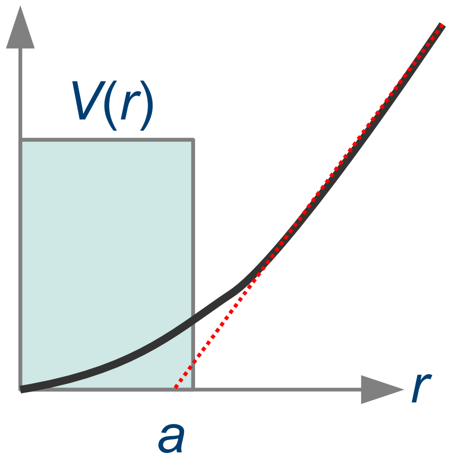


Fraction of polarized systems [%]
60
50
40
30
20
10
0
-1.0
-0.8
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
$-1/g_{1D}$ [$a_{||}$ $\omega_{||}$]
Weak
Strong

Two-atom scattering

Underlying repulsive

Effective repulsive



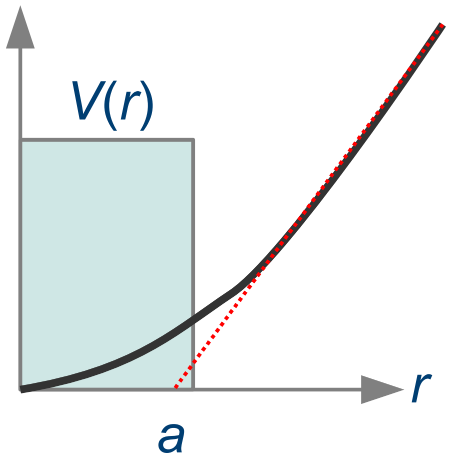
Two-atom scattering

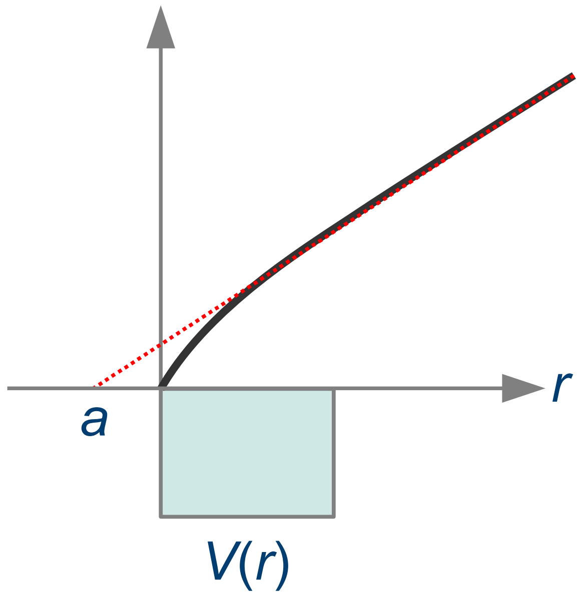
Underlying repulsive

Effective repulsive

Underlying attractive

Effective attractive

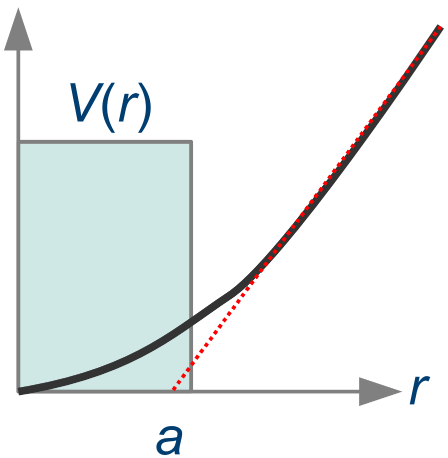




Two-atom scattering

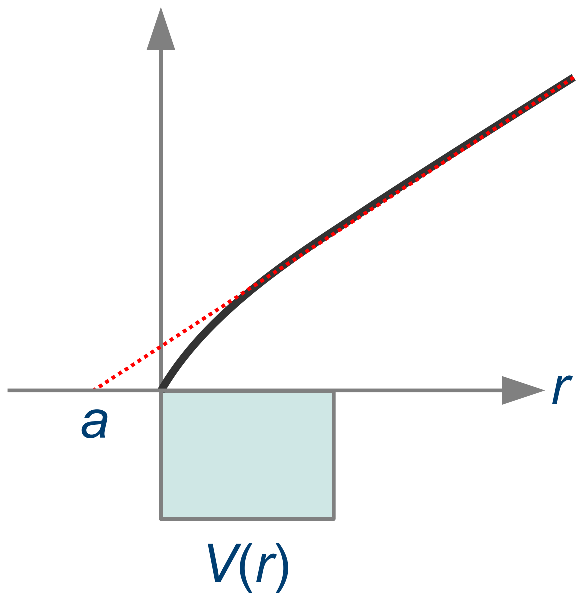
Underlying repulsive

Effective repulsive



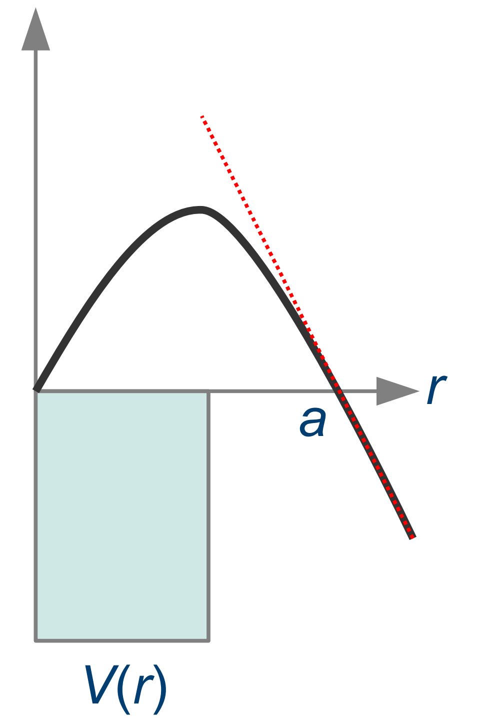
Underlying attractive

Effective attractive



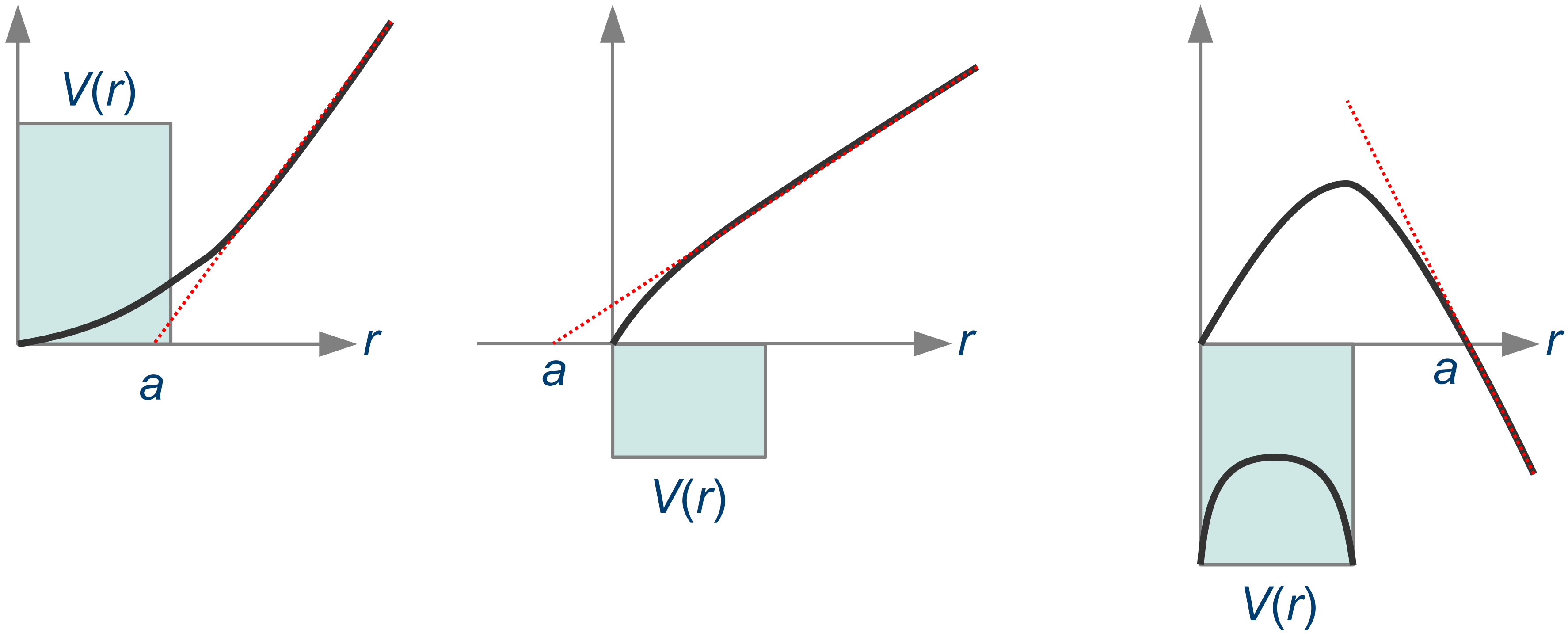
Underlying attractive

Effective repulsive

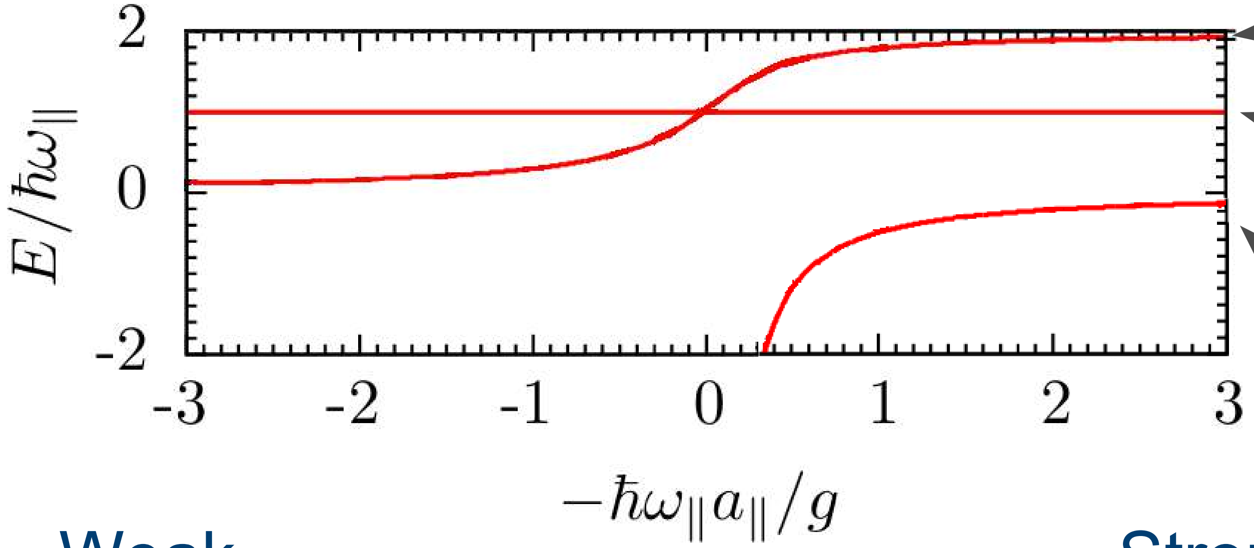


Two-atom scattering

Underlying repulsive	Underlying attractive	Underlying attractive
Effective repulsive	Effective attractive	Effective repulsive

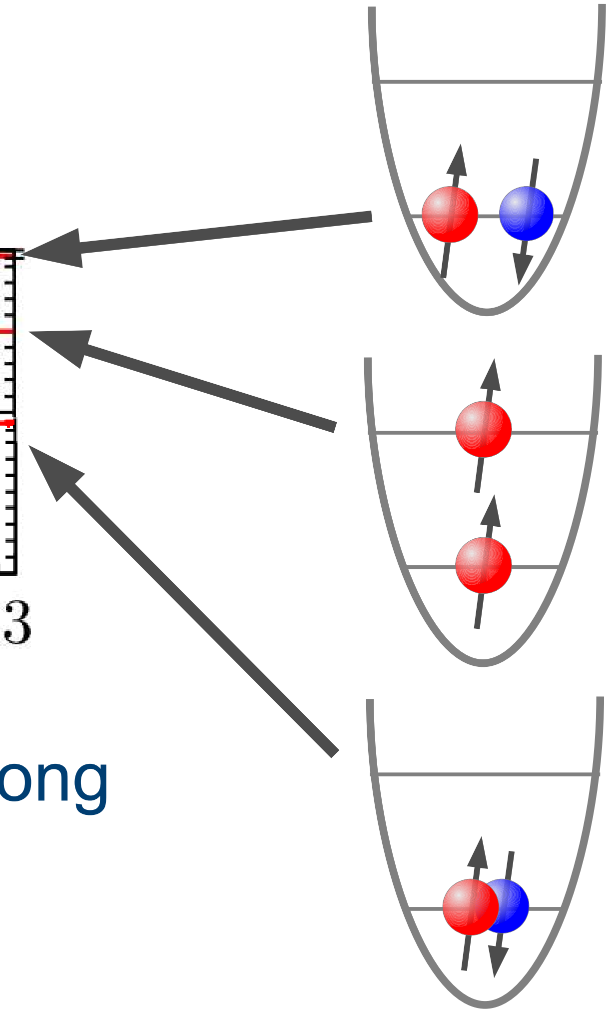


Two-atom bound state

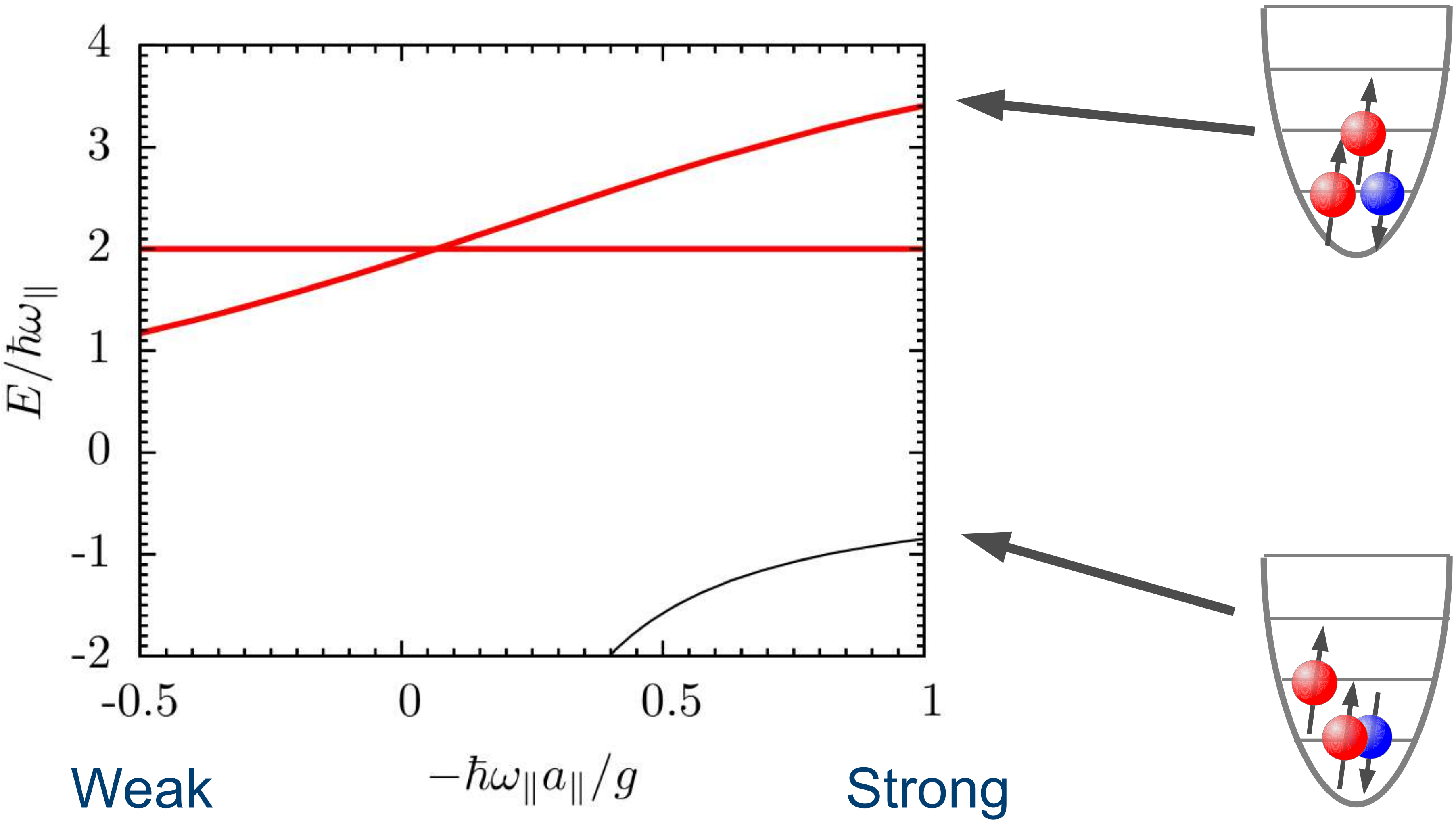


Weak

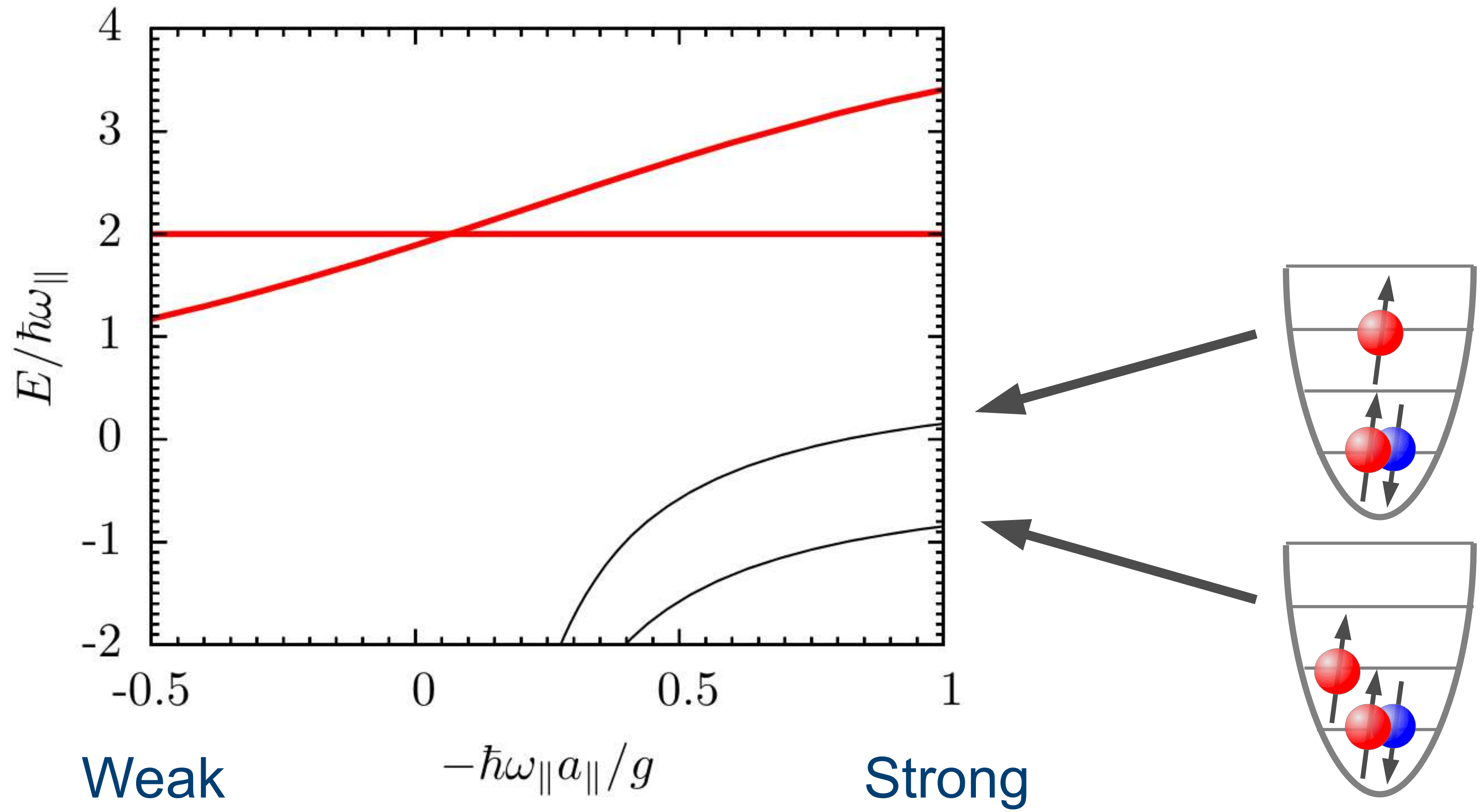
Strong



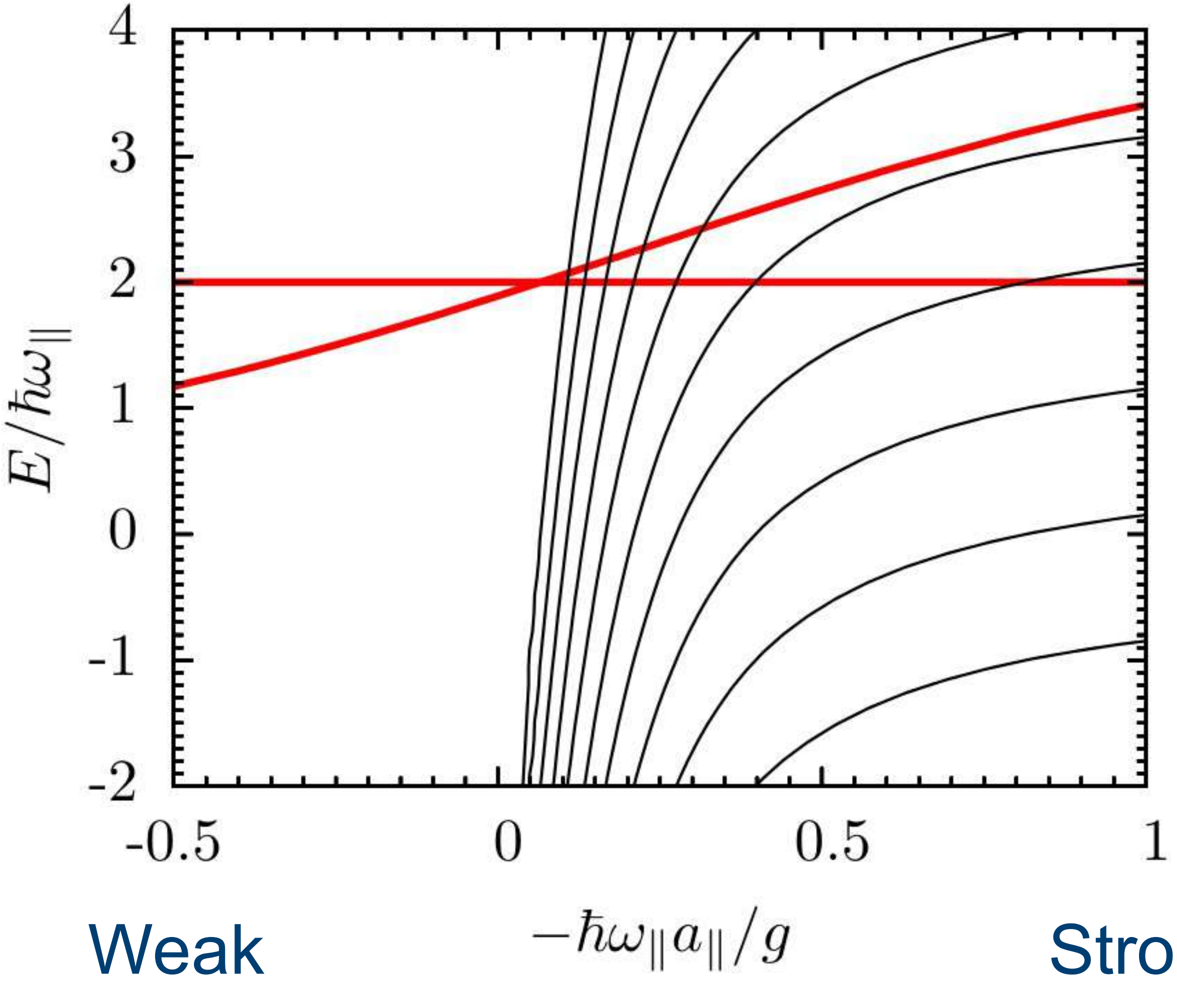
Three-atom bound state

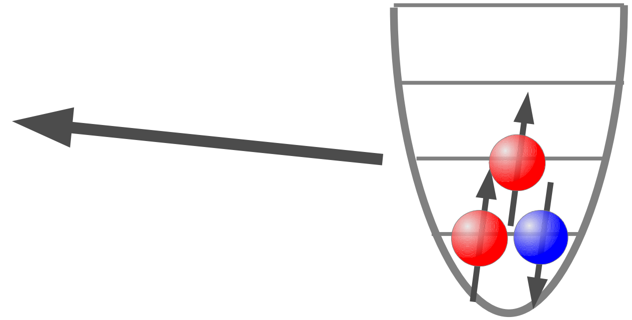


Three-atom bound state



Three-atom bound state

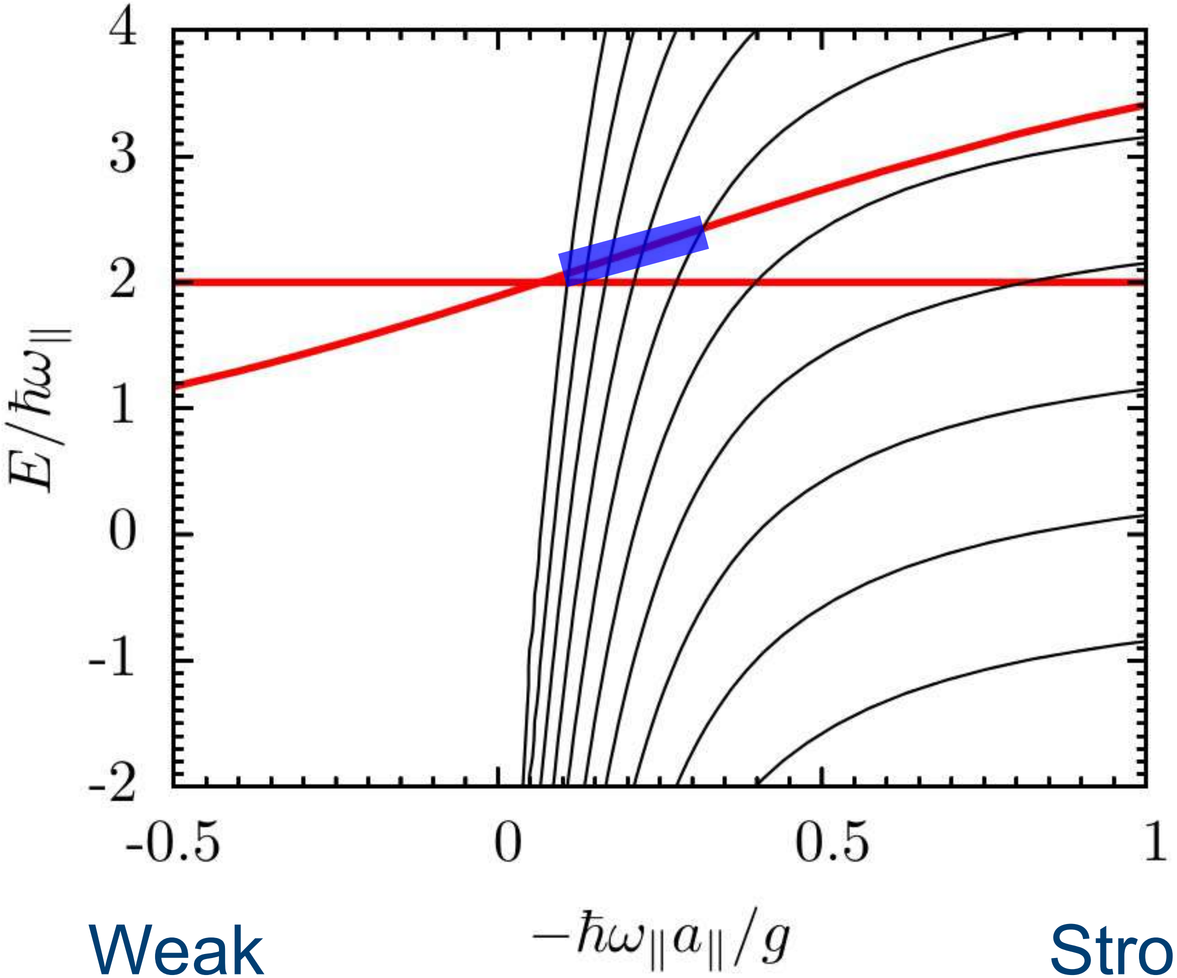


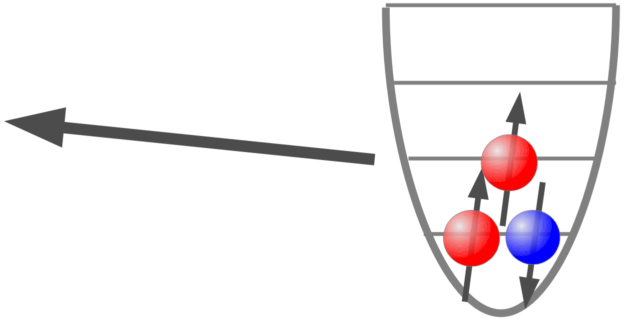


Weak

Strong

Three-atom bound state

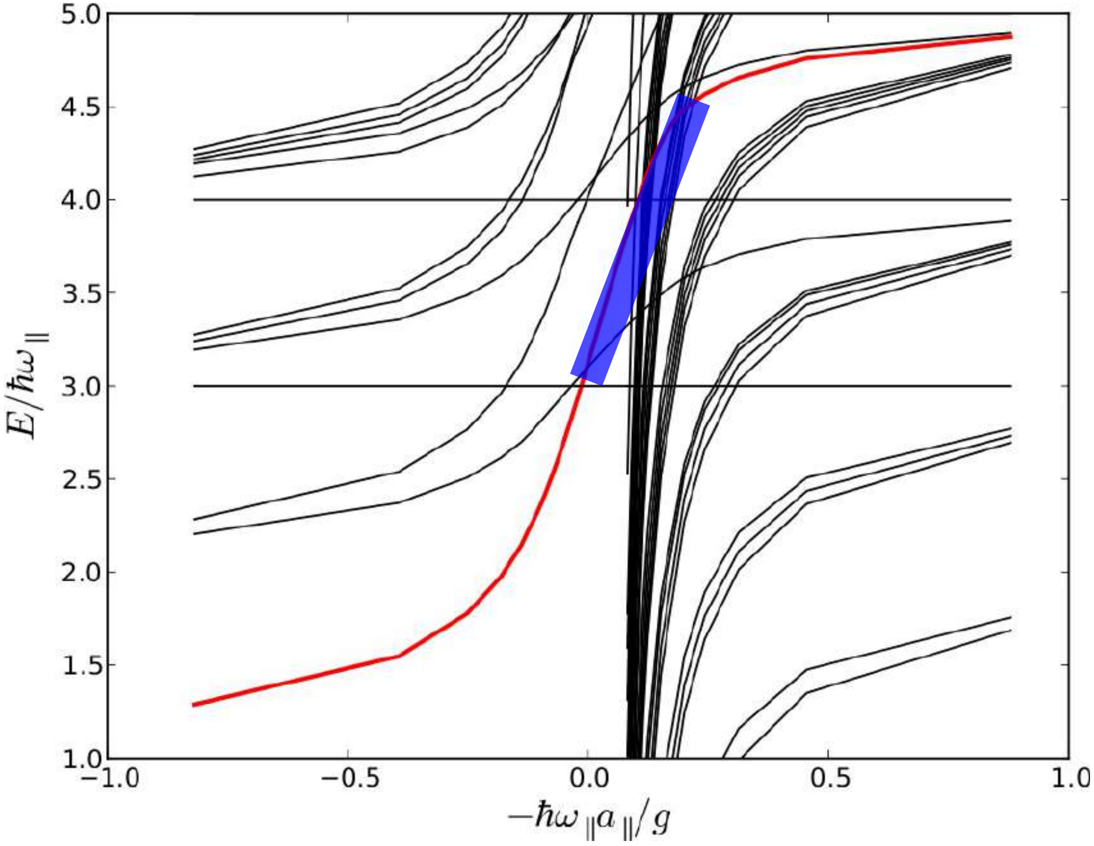




Weak

Strong

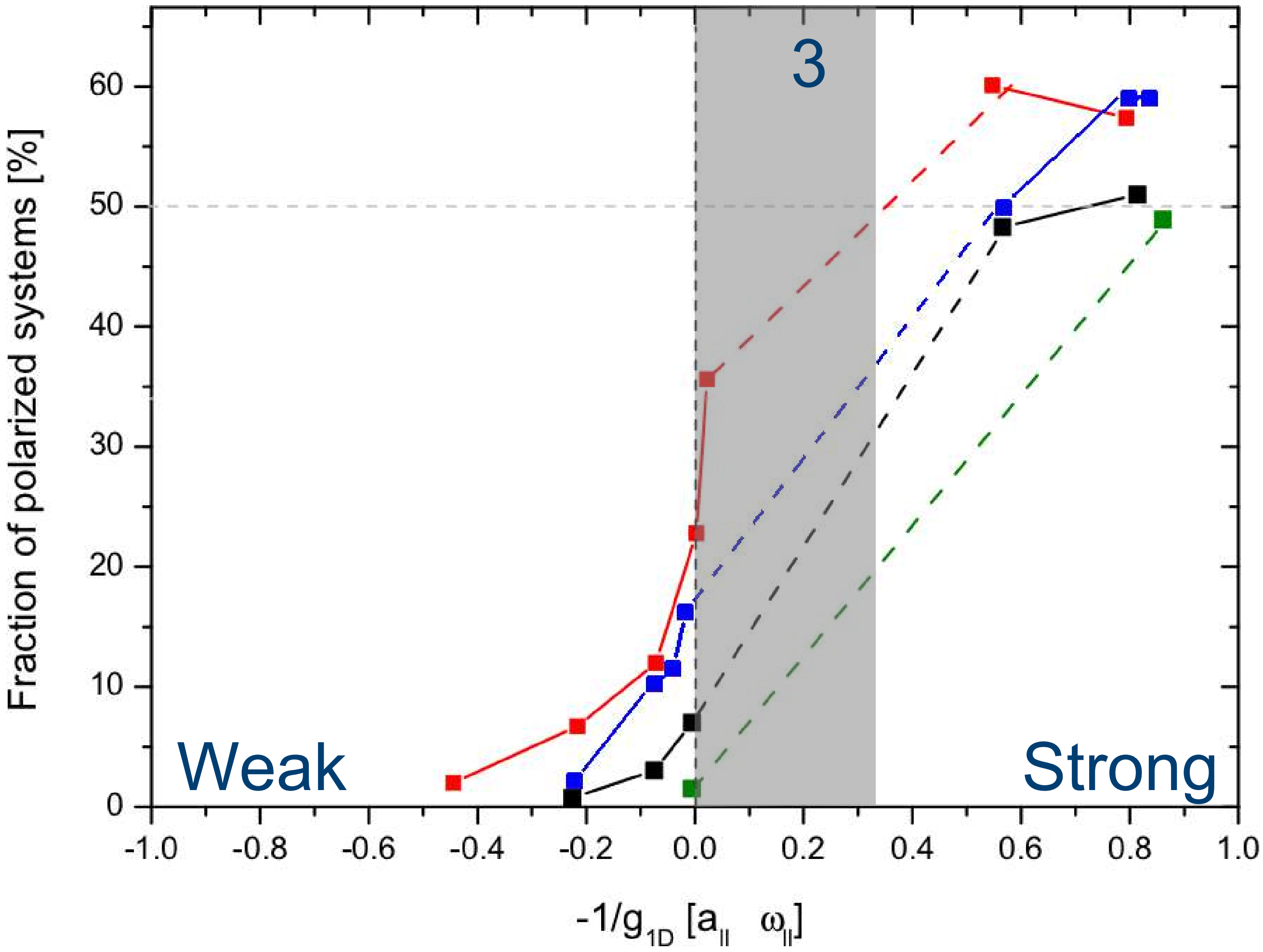
Three-atom bound state



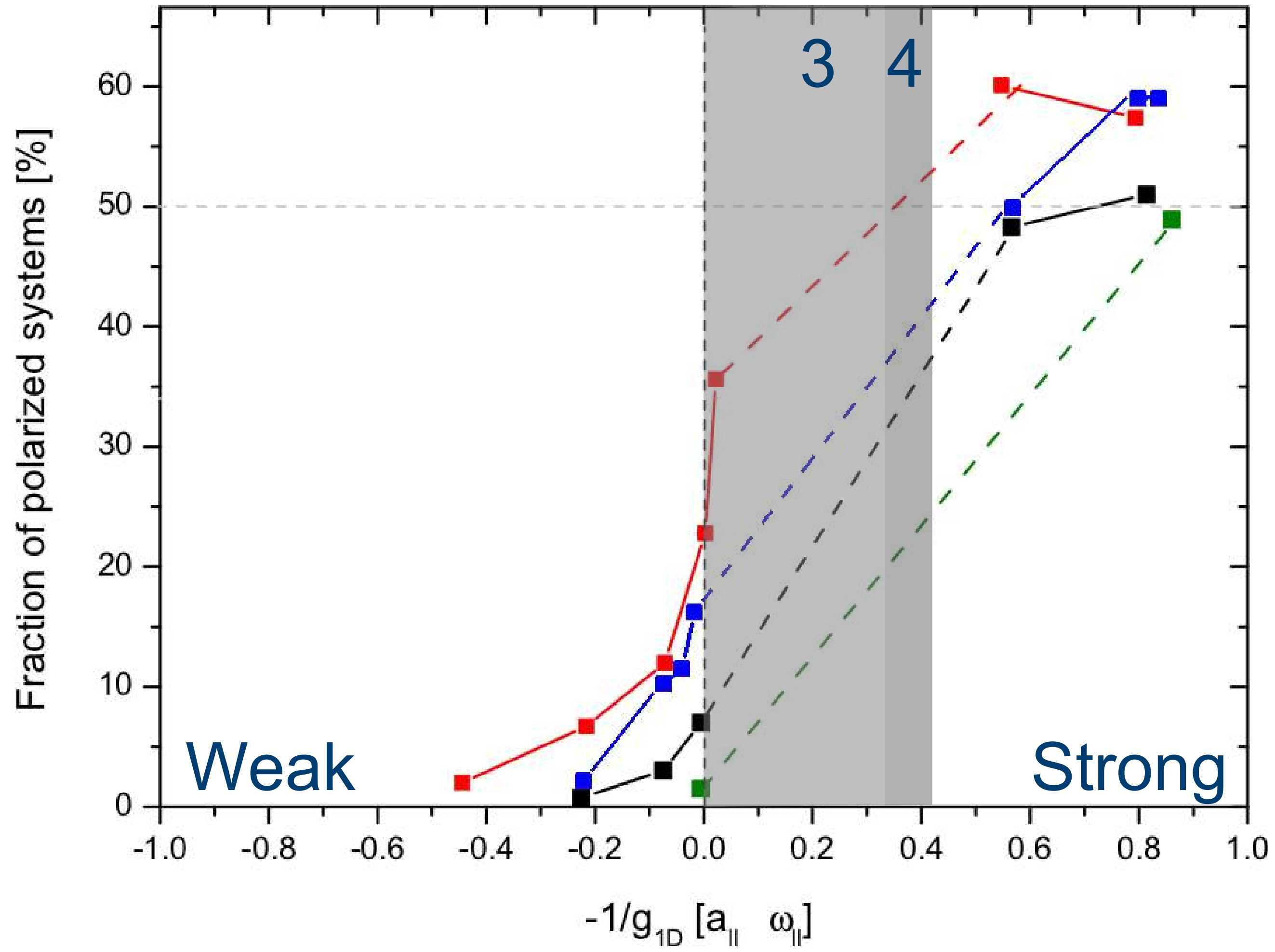
Weak

Strong

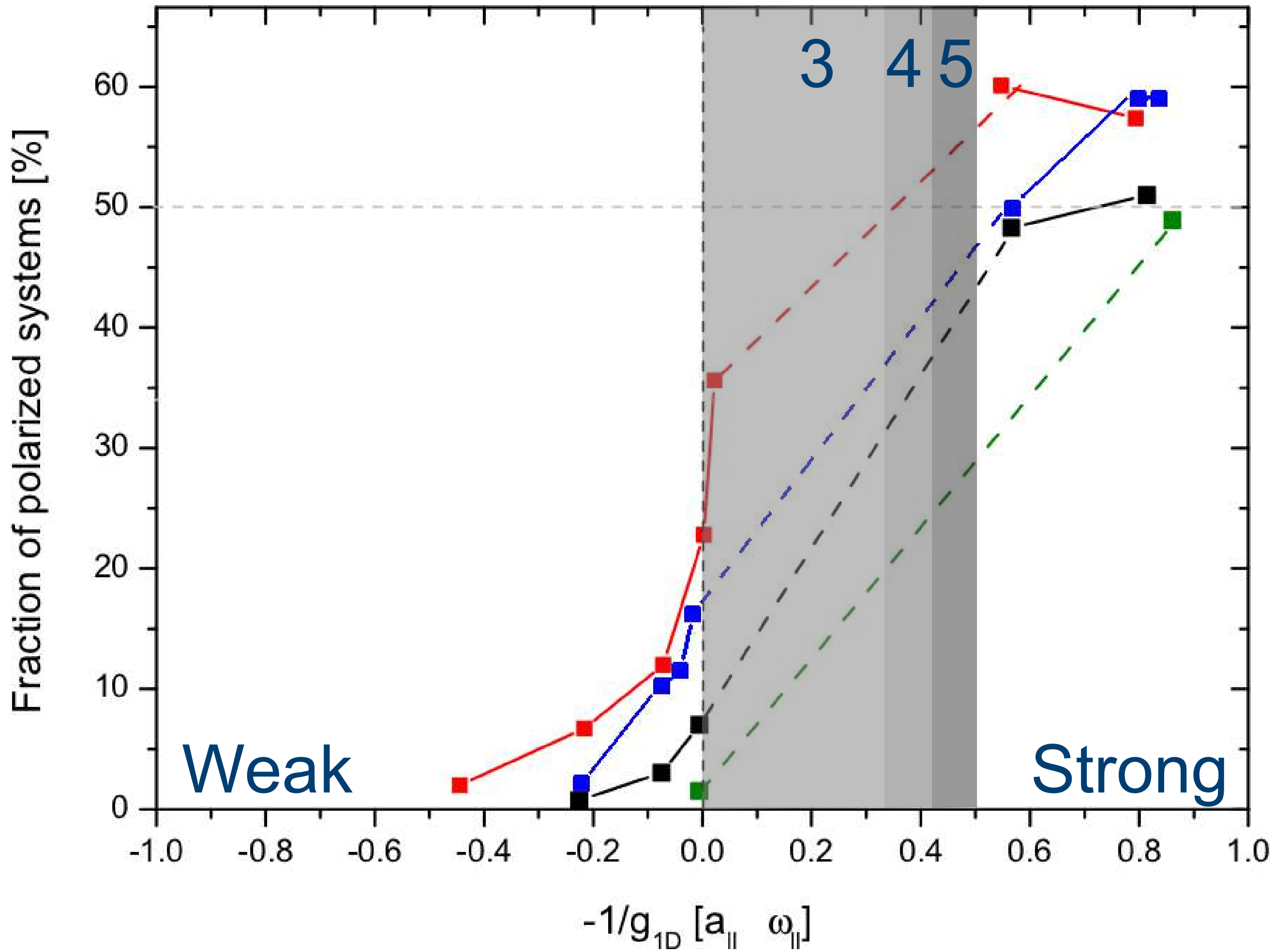
Loss affected region



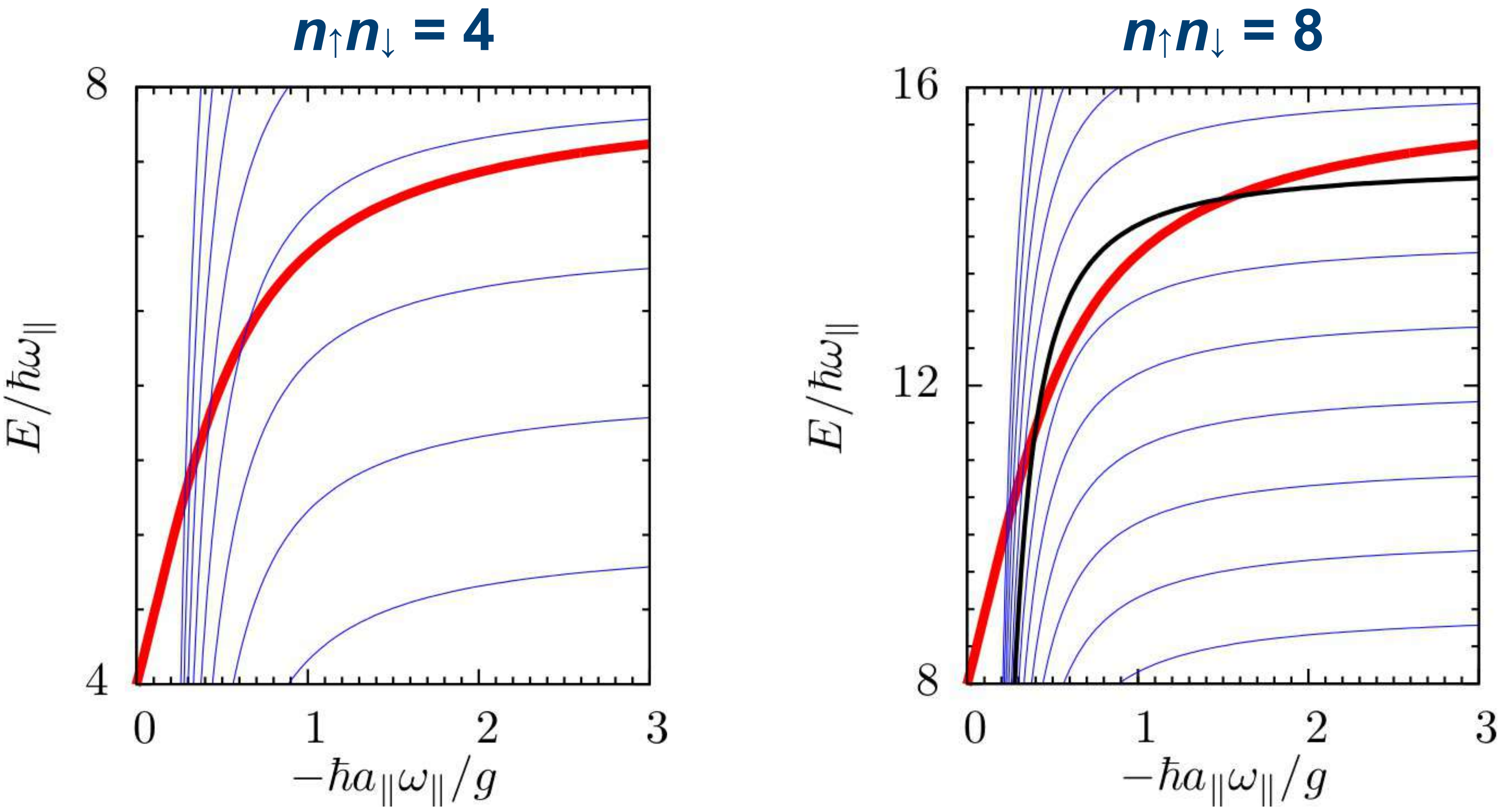
Loss affected region



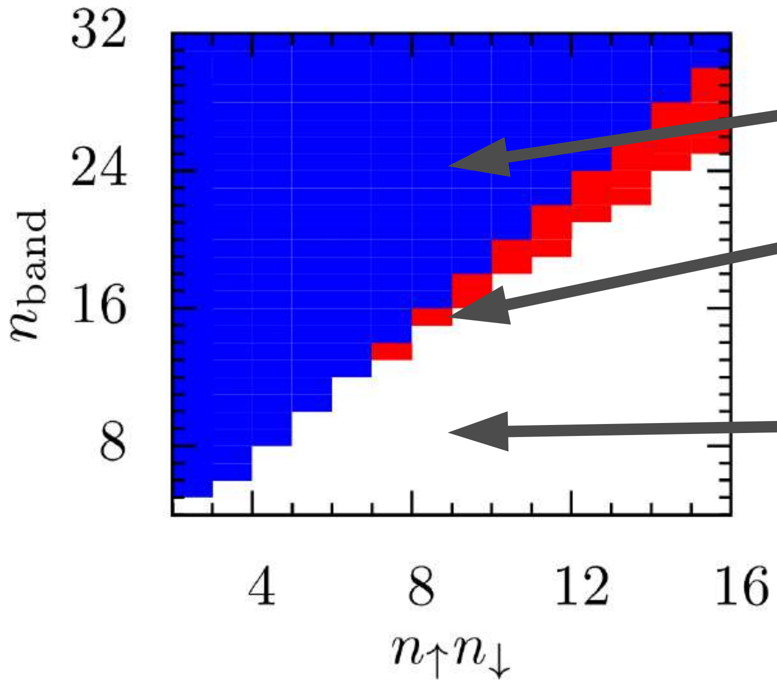
Loss affected region



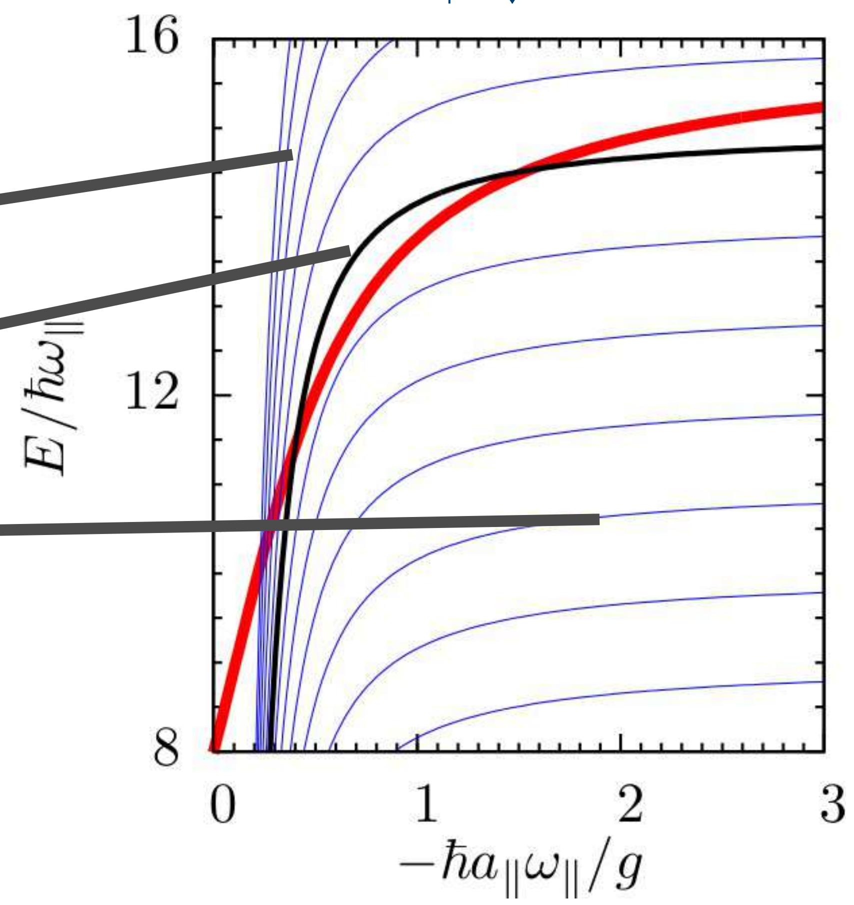
Band crossings



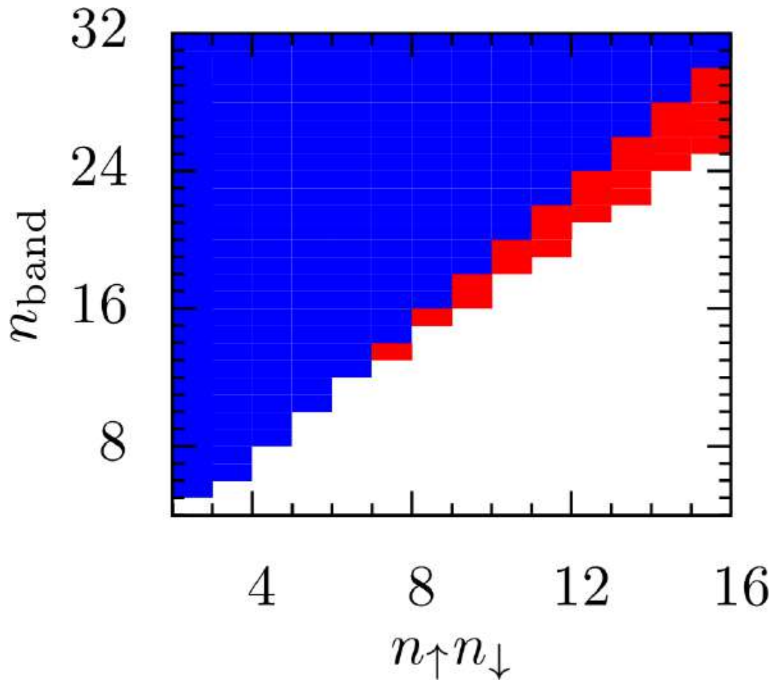
Band crossings

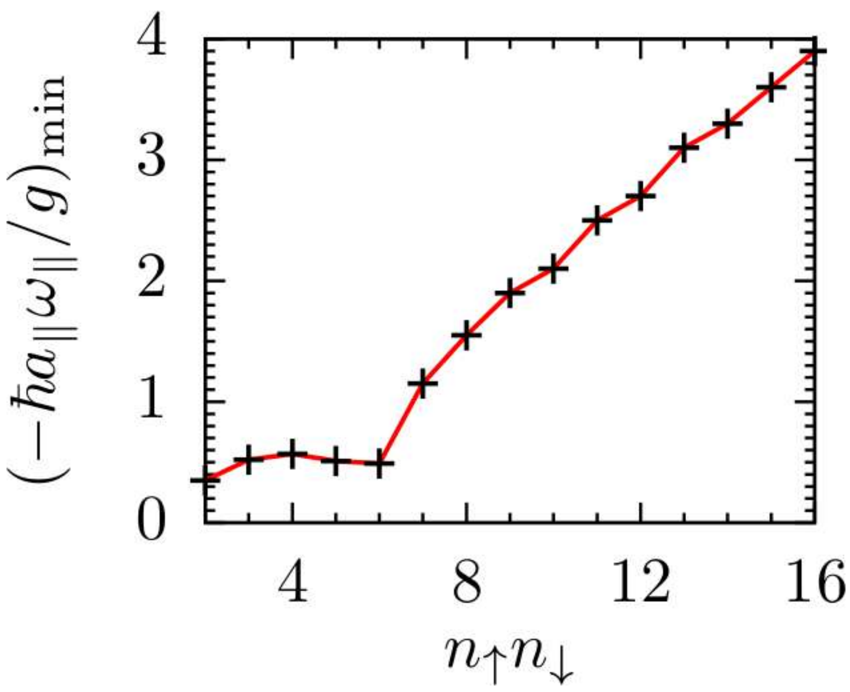


$n_\uparrow n_\downarrow$ = 8

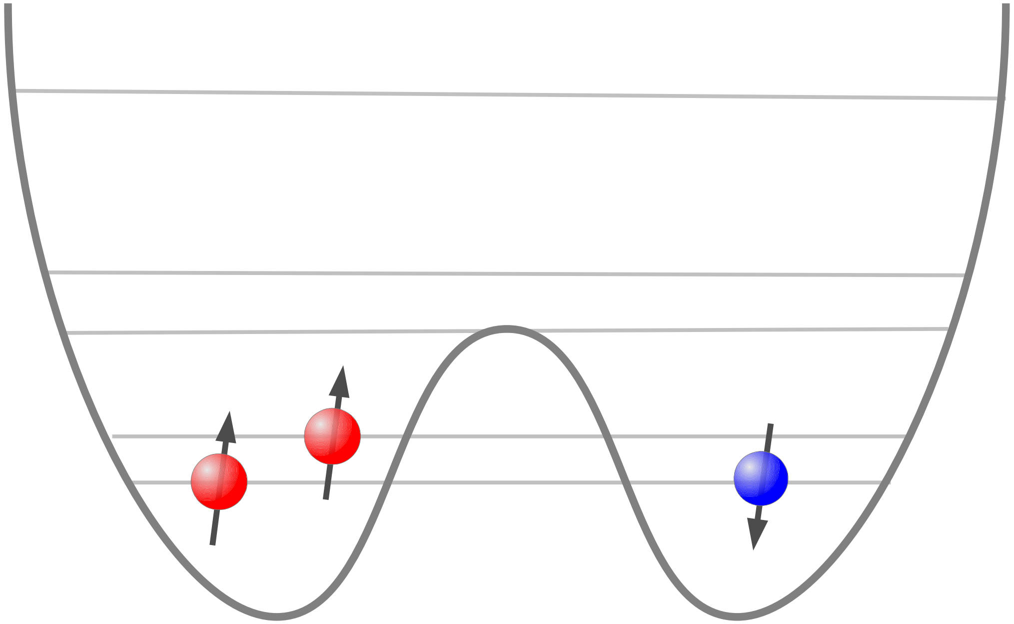


Band crossings

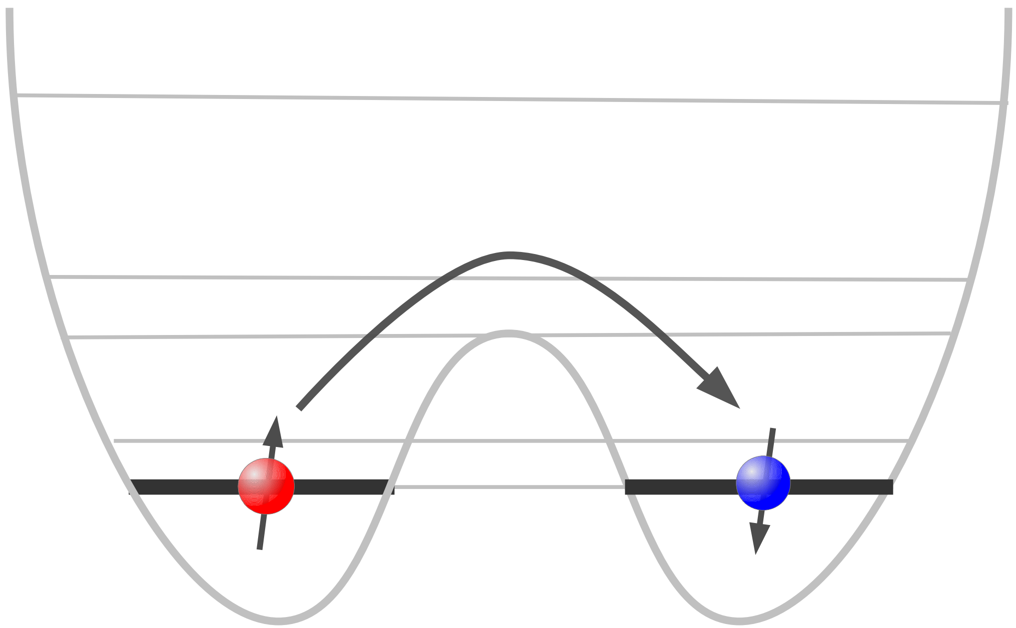




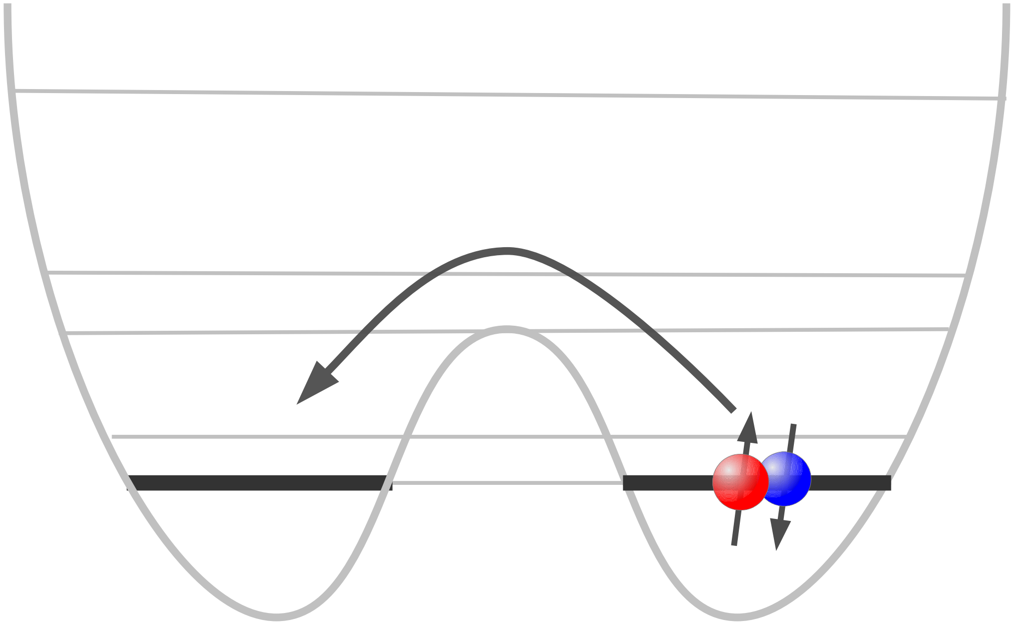
Double well potential



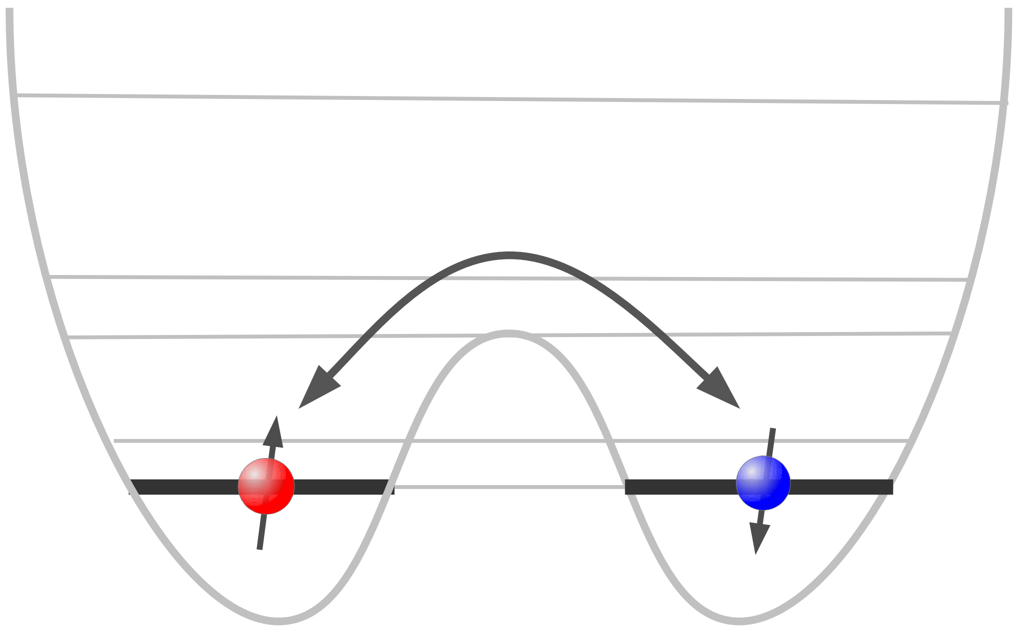
Direct exchange



Direct exchange

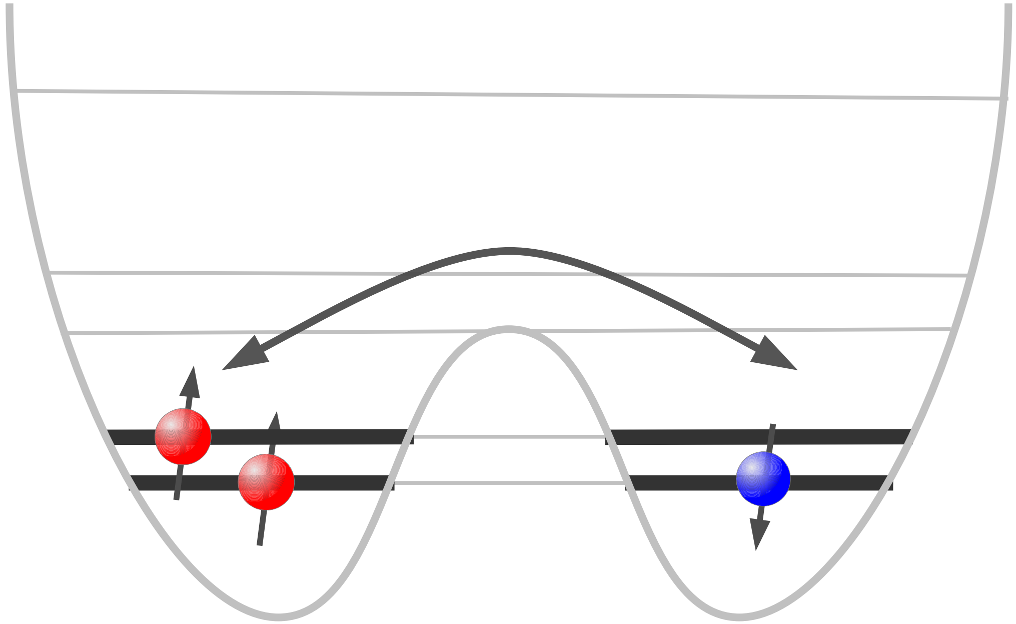


Direct exchange

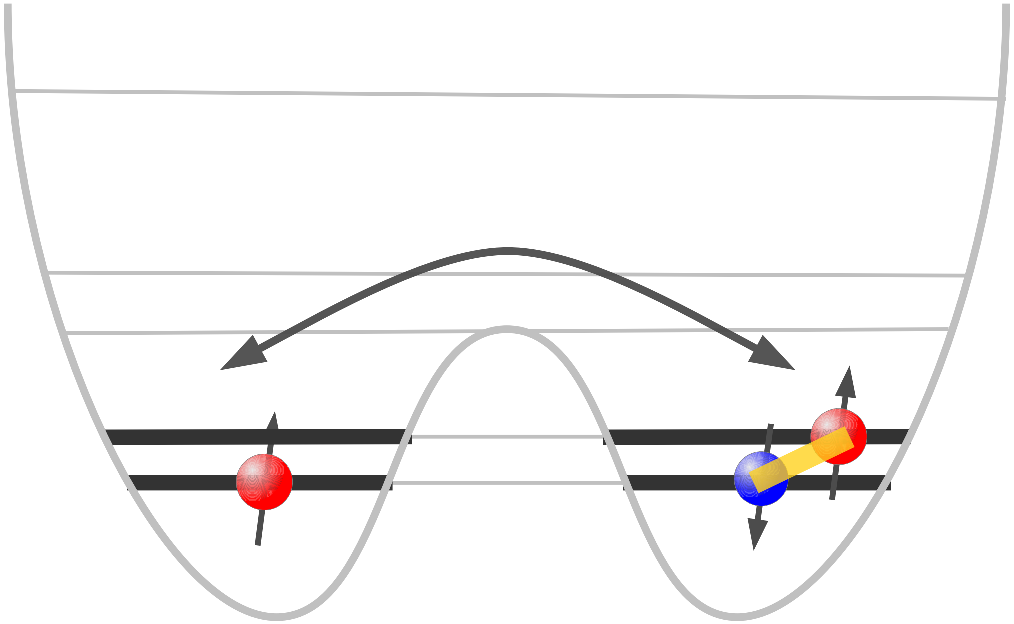


$$\Delta E = -\frac{t^2}{U}$$

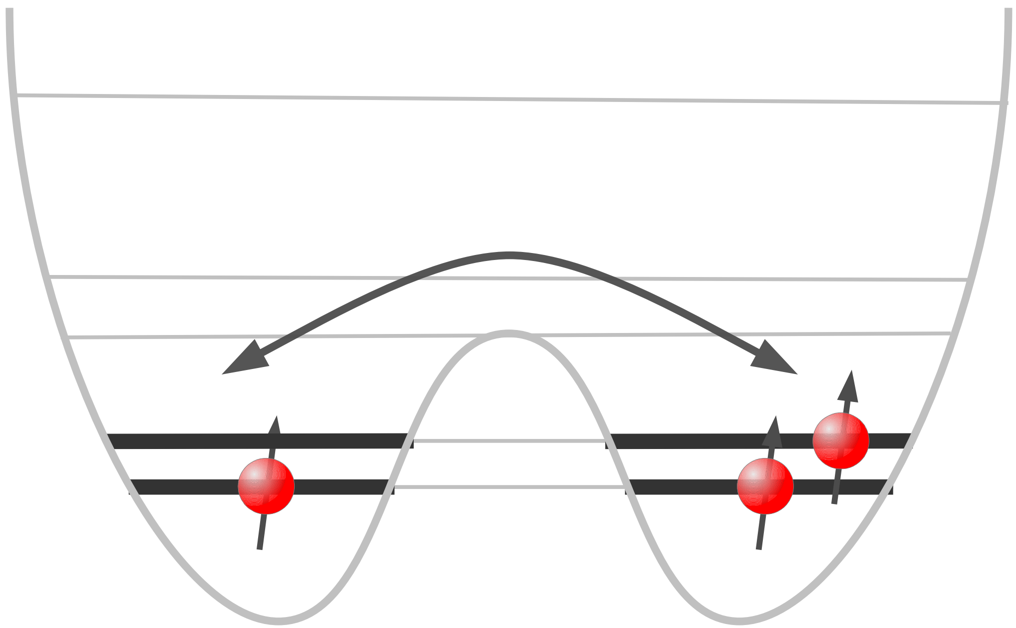
Double exchange



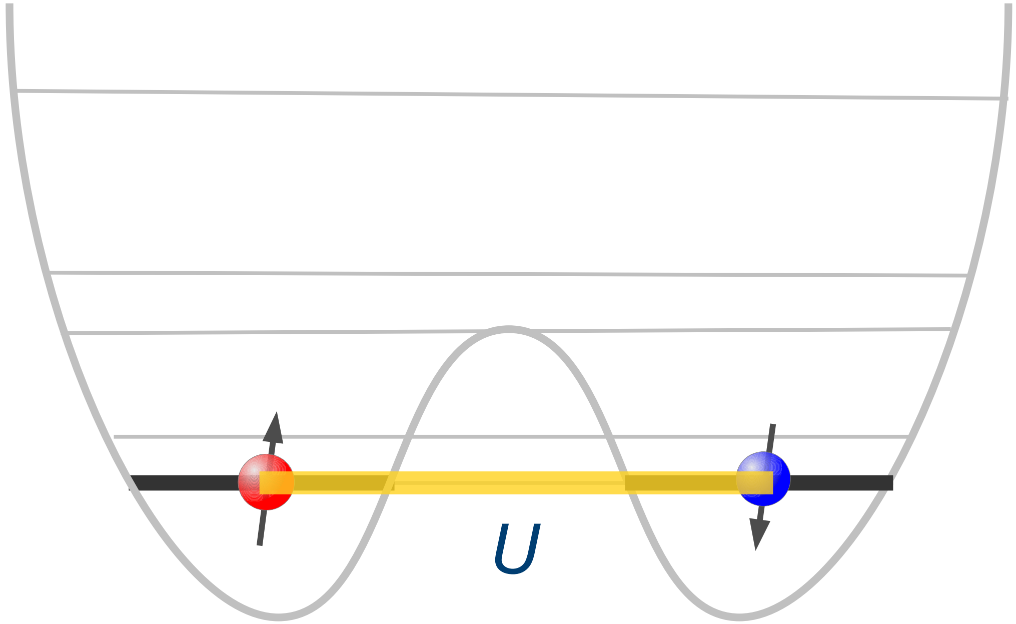
Double exchange



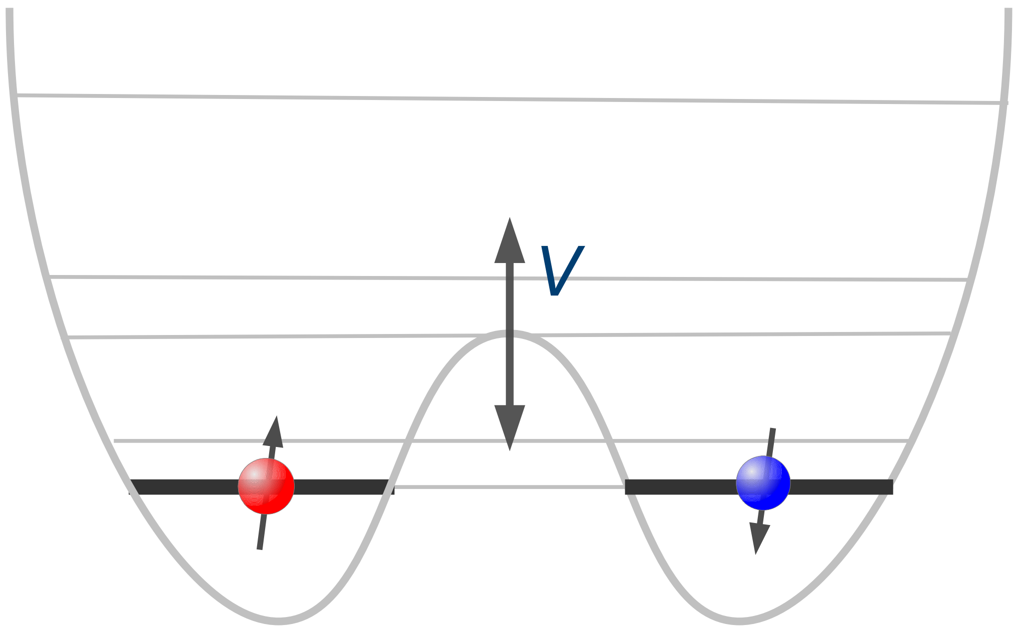
Double exchange



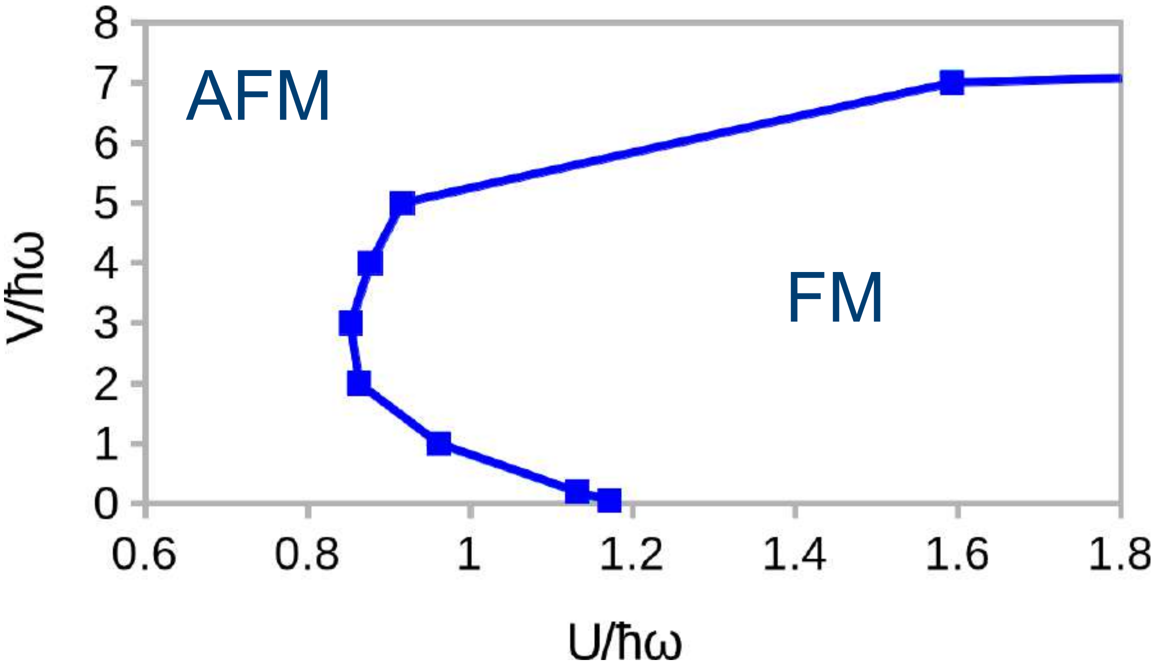
Direct exchange



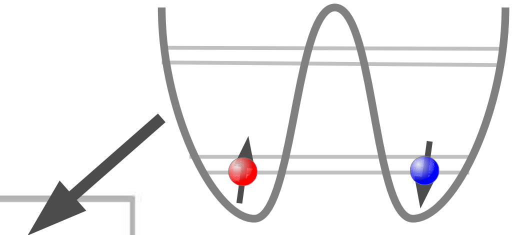
Direct exchange

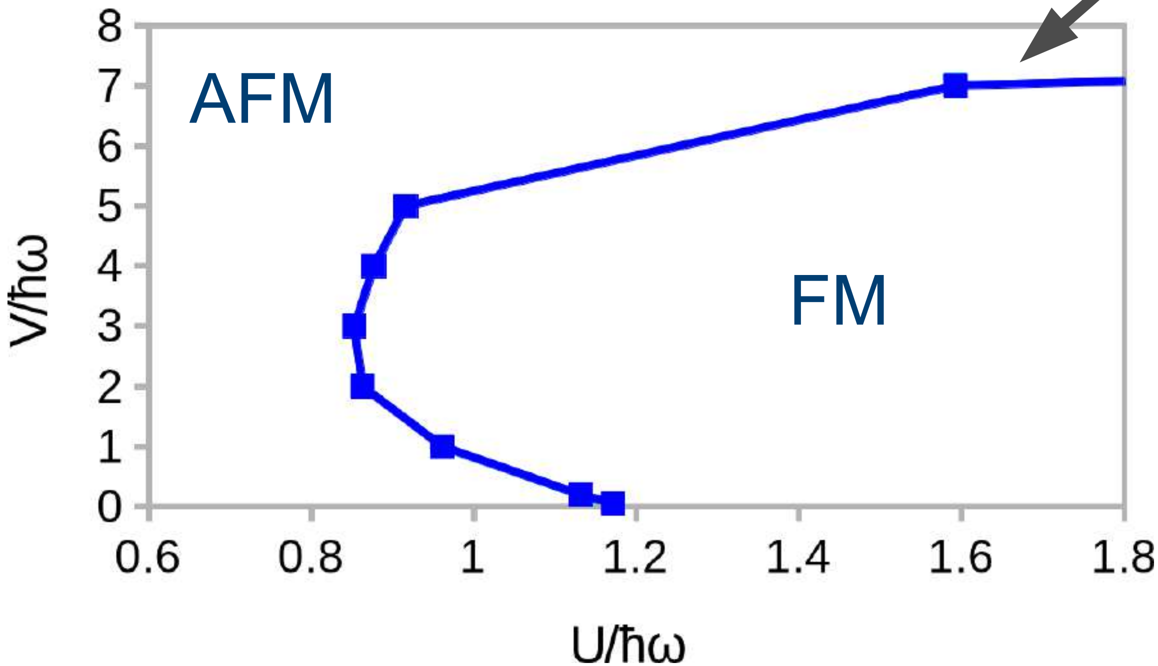


Direct exchange



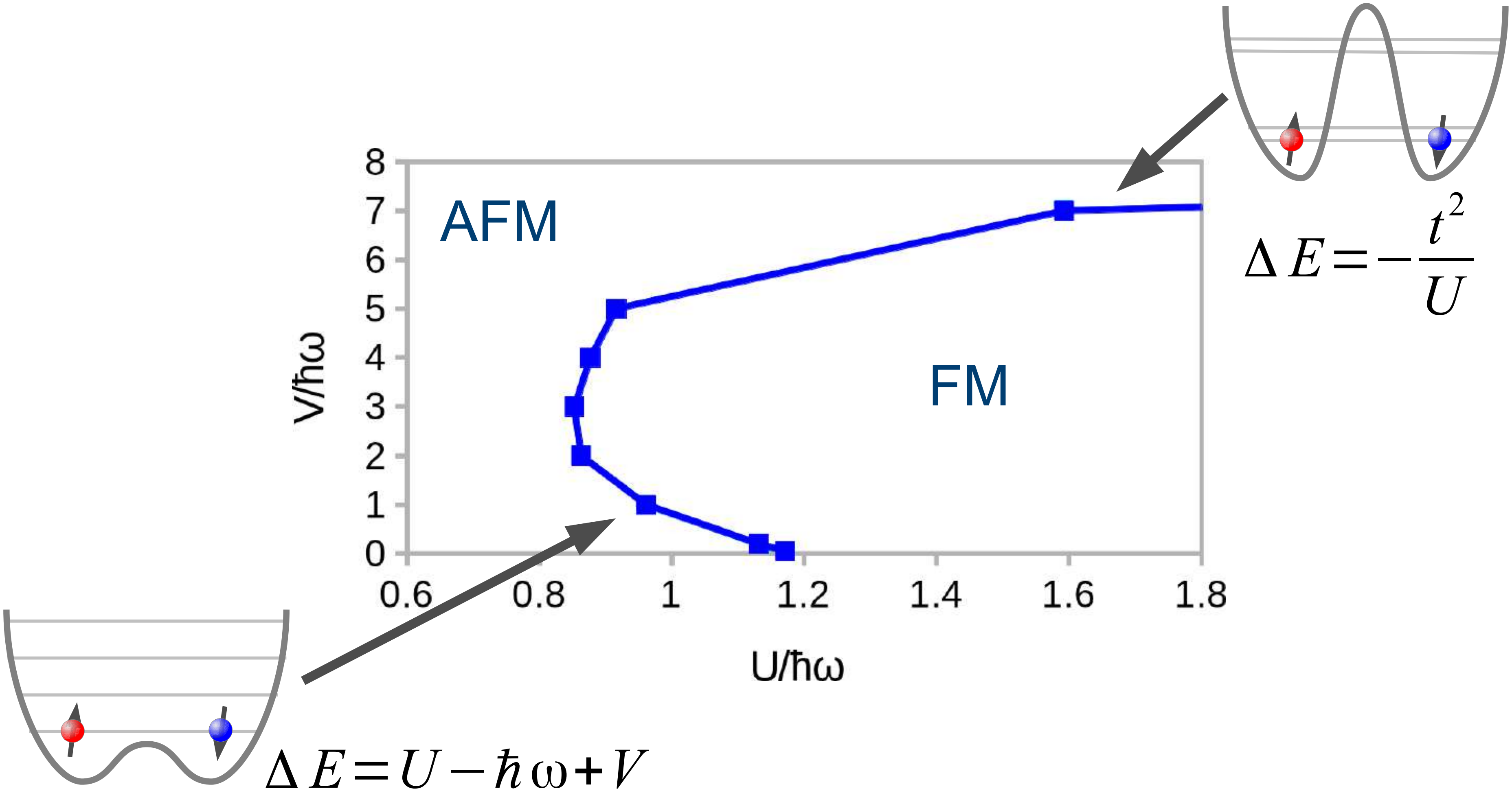
Direct exchange



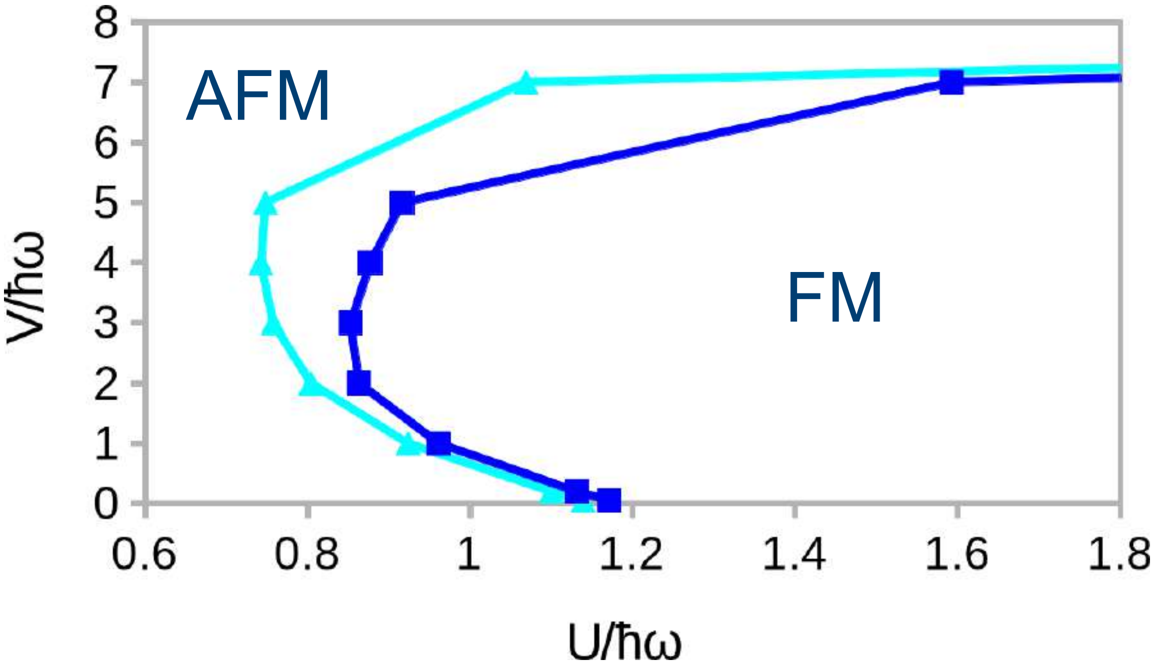


$$\Delta E = -\frac{t^2}{U}$$

Direct exchange



Direct exchange



Summary

A few-fermion system with repulsive interactions displays ferromagnetic correlations

Statistical tunneling measurement yields a probability of ½ due to degenerate triplet states

Losses to the bound state occur in narrow range of interaction strengths

Opportunity to probe direct and double exchange

Computational analysis: Quantum Monte Carlo

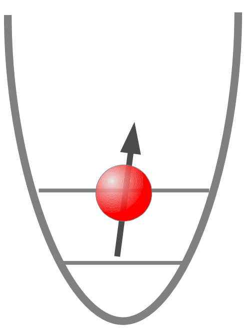
$$E = \int \mathrm{d}\boldsymbol{r} \, \frac{\langle \psi | \hat{H} | \psi \rangle}{\langle \psi | \psi \rangle}$$

$$|\psi\rangle = \mathrm{e}^{-J} D$$

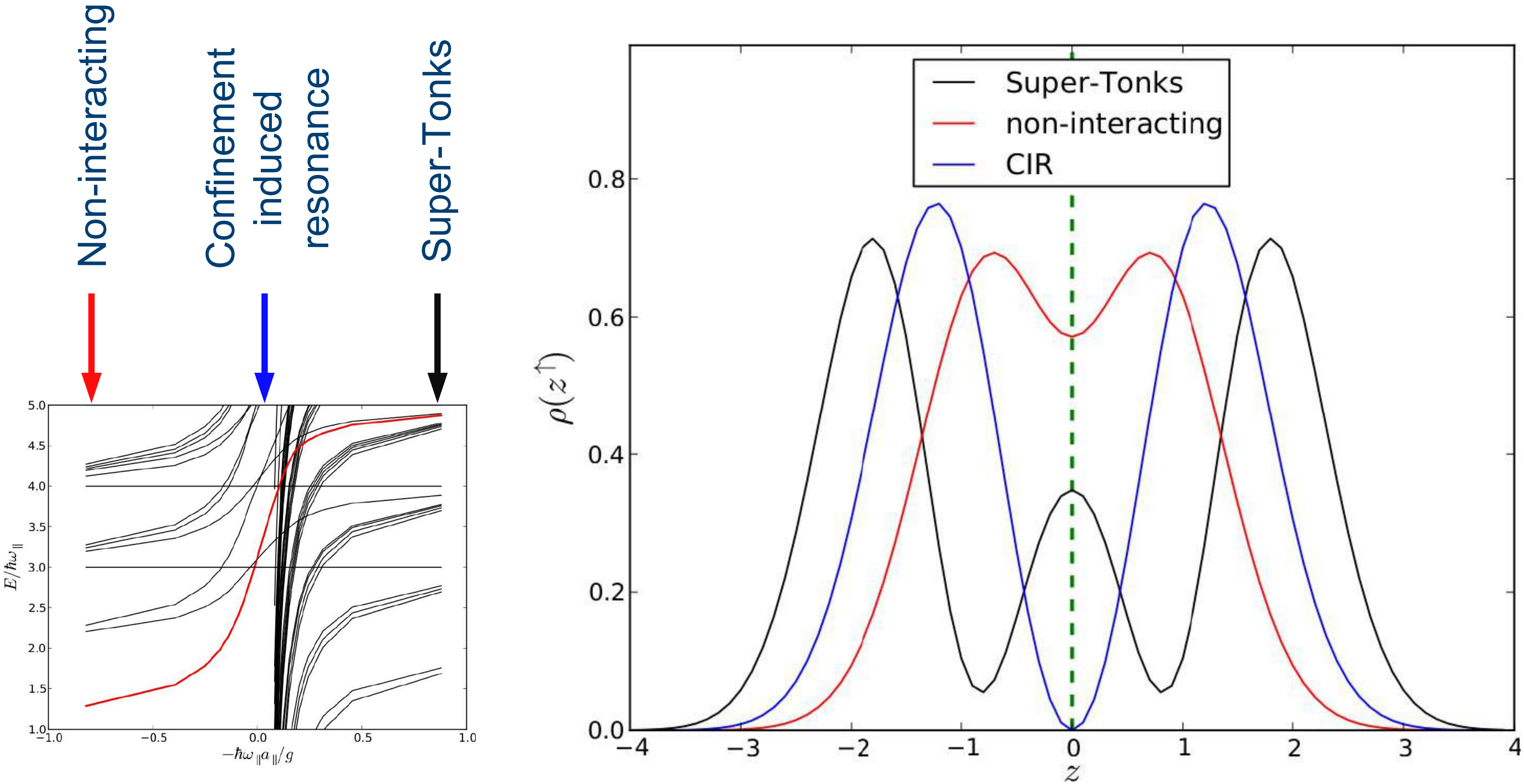
$$D = \det\left(\{\varphi_{n\uparrow}\}\right) \det\left(\{\varphi_{n\downarrow}\}\right)$$

$$J = \sum_{n,i,j,\sigma,\sigma'} u_n \left(\boldsymbol{r}_{i\sigma} - \boldsymbol{r}_{j\sigma'}\right)^n + \cdots$$

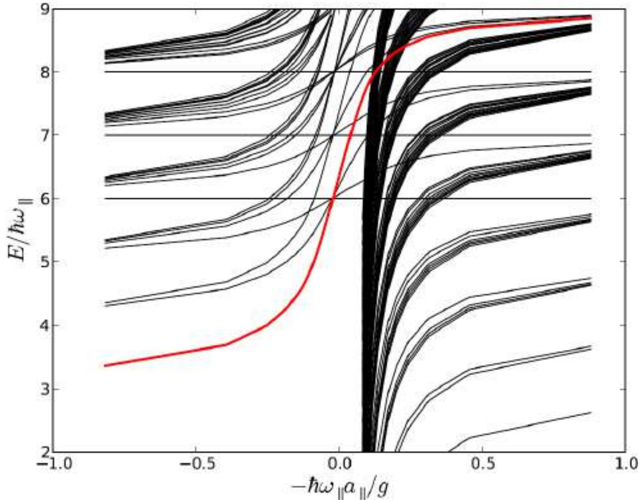
$$|\varphi_{2\uparrow}\rangle =$$



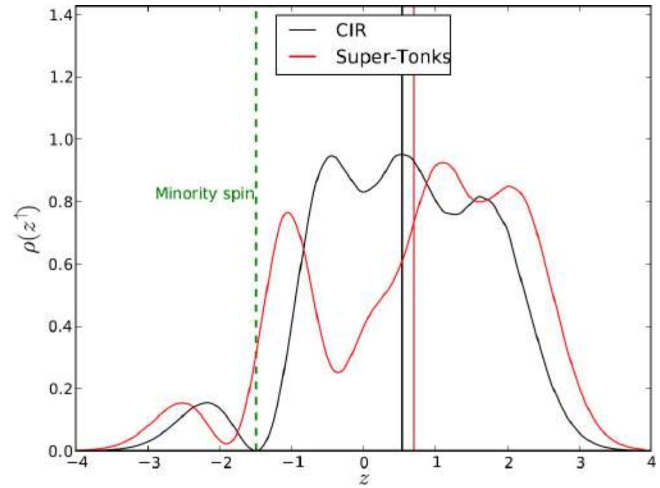
Density profiles

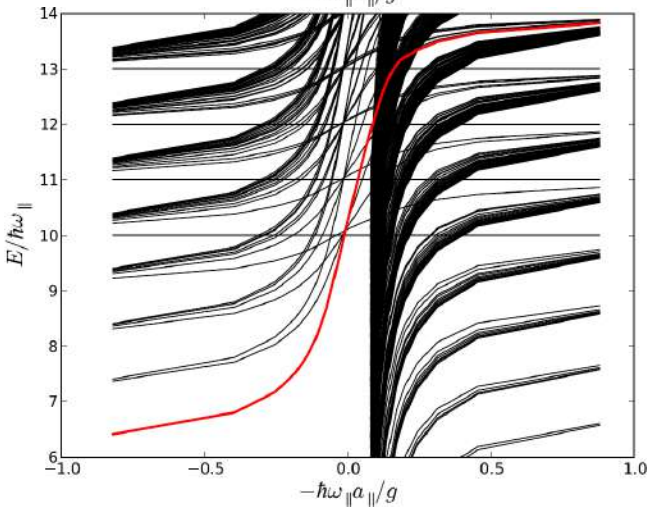


Density profiles

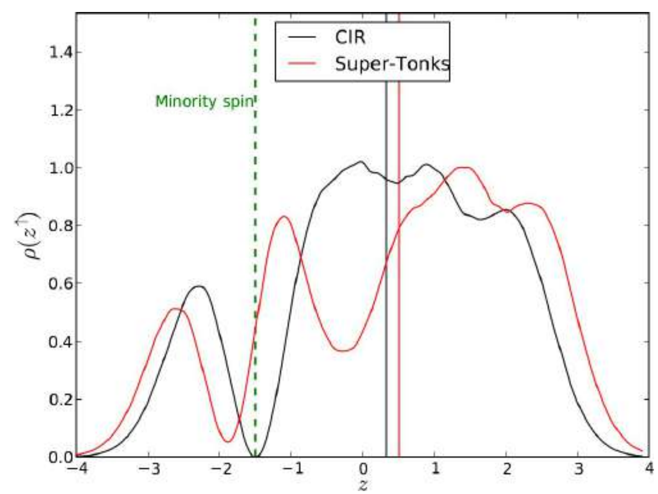


3 up
1 down

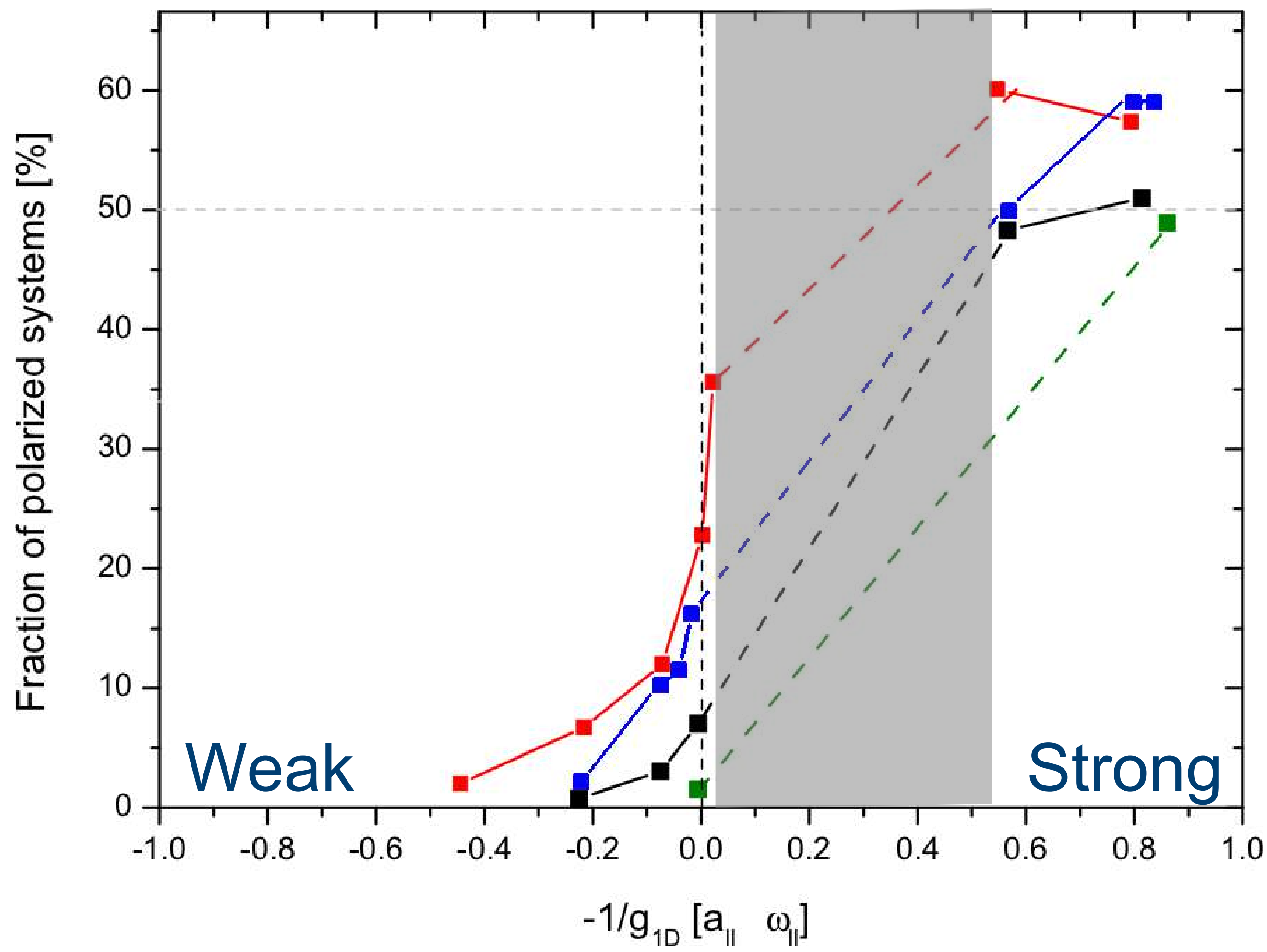




4 up
1 down

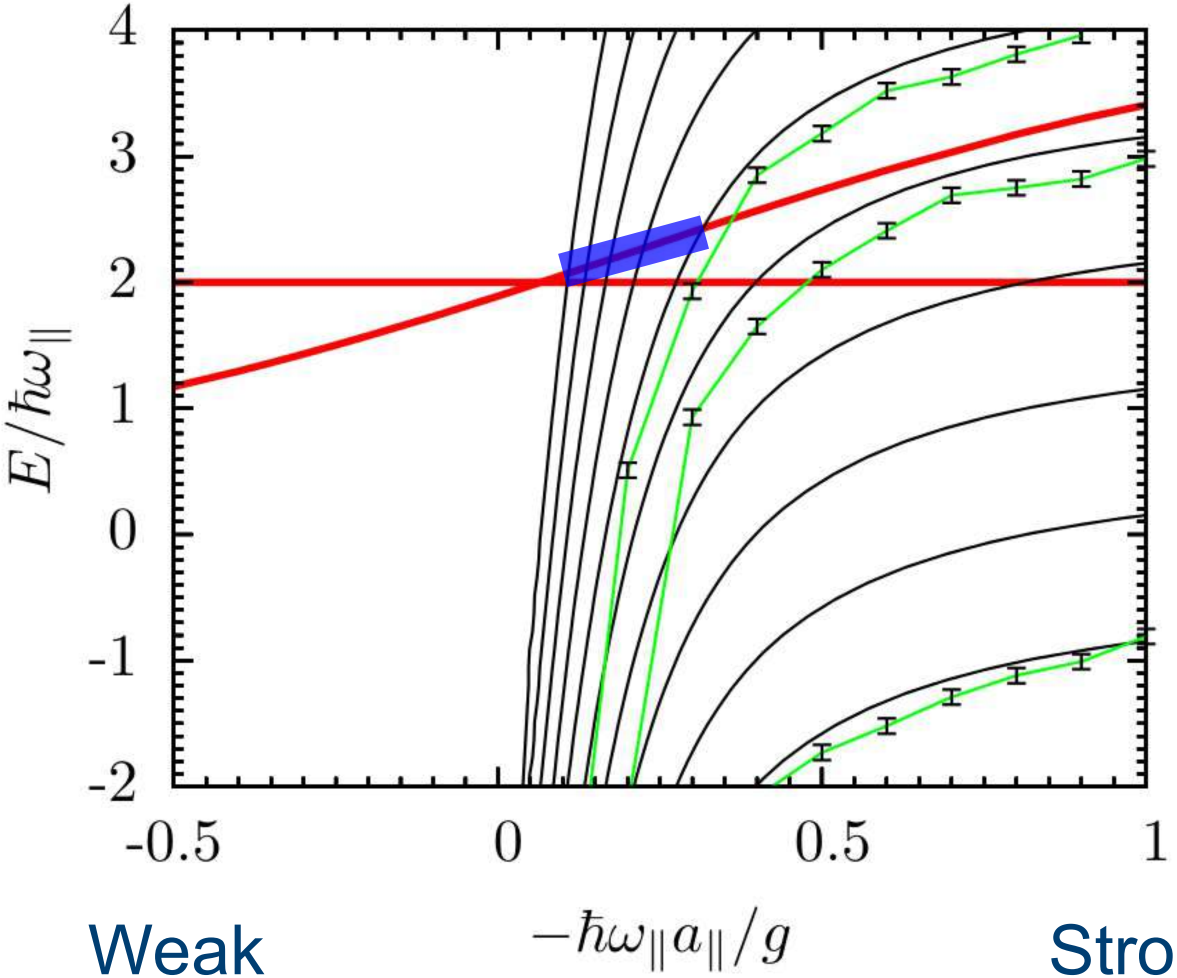


Losses



Fraction of polarized systems [%]
60
50
40
30
20
10
0
-1.0
-0.8
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
-1/g1D [a|| ω||]
Weak
Strong

Three-atom bound state



Weak

Strong