

# Imprinting magnetic monopoles onto an atomic gas

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# Ways to realize monopoles

## Spin ice

Castelnovo *et al.* Nature 2008,  
Bramwell, Nature 2009

## Topological insulators

Qi *et al.* Science 2009

## Anomalous quantum Hall effect

Fang *et al.* Science 2003

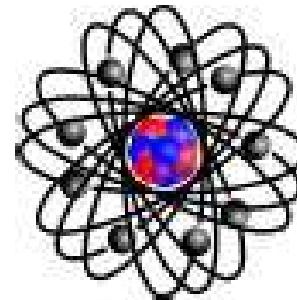
## Superfluid $^3\text{He}$

Blaha, PRL 1976

# Cold atom gas



Electrons  $\rightarrow$   $^{87}\text{Rb}$  atoms



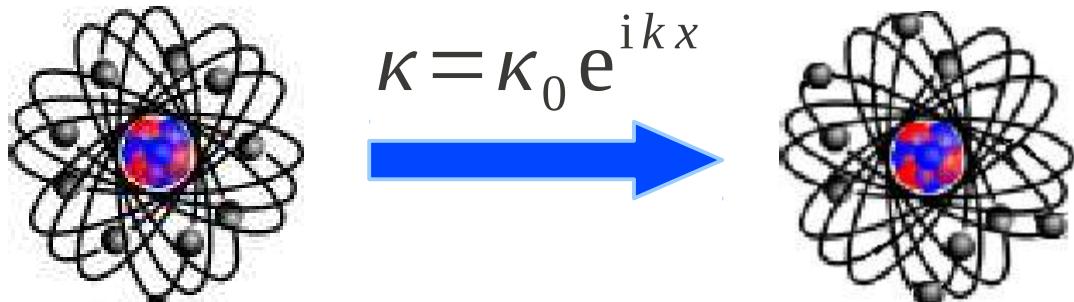
# How to imprint a uniform magnetic field

Desired Hamiltonian:

$$\hat{H} = \frac{(p - eA)^2}{2m}$$

$$B = \nabla \times A$$

Laser beam excites transition



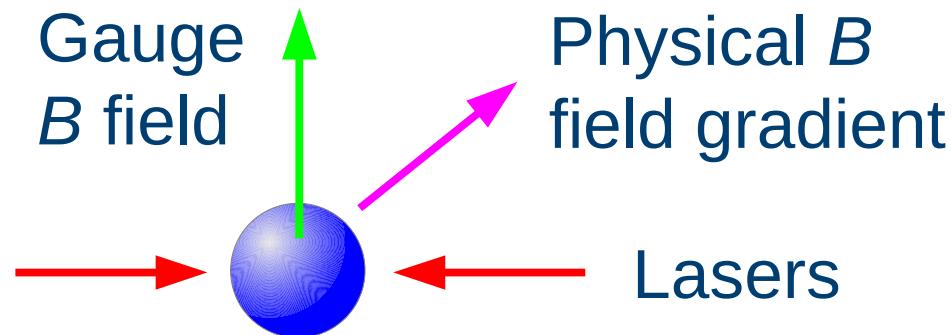
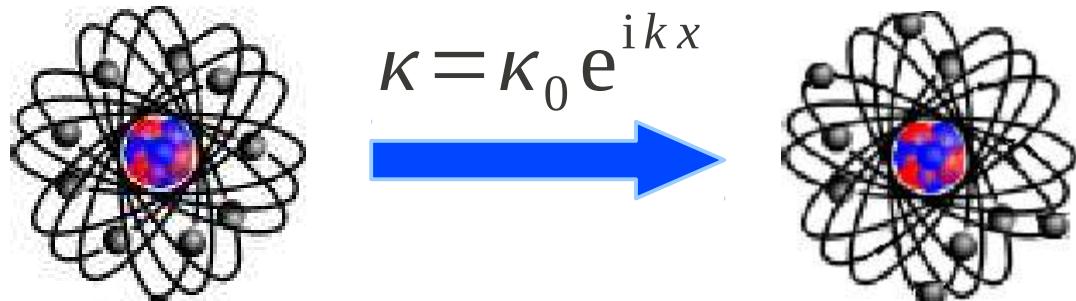
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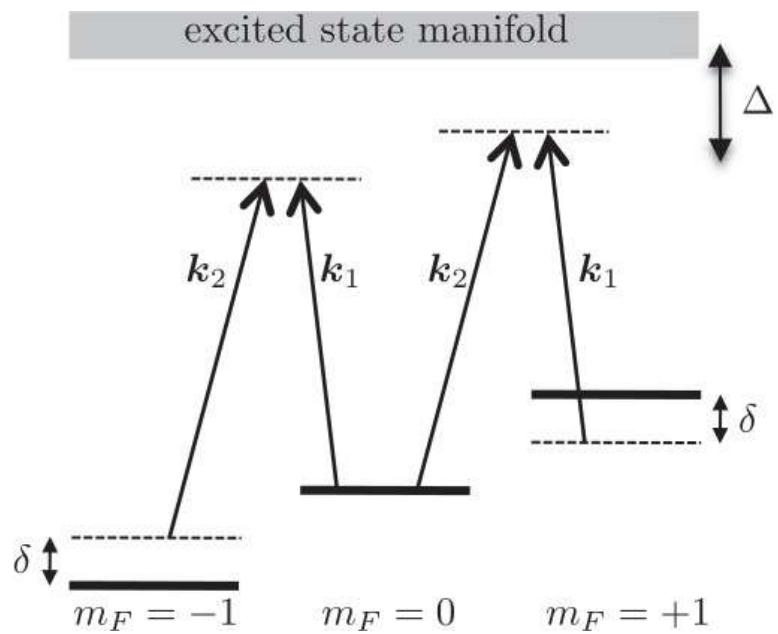
# How to imprint a uniform magnetic field

$$\hat{U} = \frac{\hbar}{2} \begin{pmatrix} -2\delta' y & \kappa_0 e^{ikx} & 0 \\ \kappa_0 e^{-ikx} & 0 & \kappa_0 e^{ikx} \\ 0 & \kappa_0 e^{-ikx} & 2\delta' y \end{pmatrix}$$

$$\chi = e^{ikx} \cos^2 \frac{\theta}{2} | -1 \rangle - \frac{\sin \theta}{\sqrt{2}} | 0 \rangle + e^{-ikx} \sin^2 \frac{\theta}{2} | 1 \rangle$$

$$\tan \theta = \frac{\kappa_0}{\sqrt{2}\delta'}$$

$$\mathbf{A} = \langle \chi | \nabla \chi \rangle = \frac{\delta' y}{\kappa_0} \hat{e}_x$$



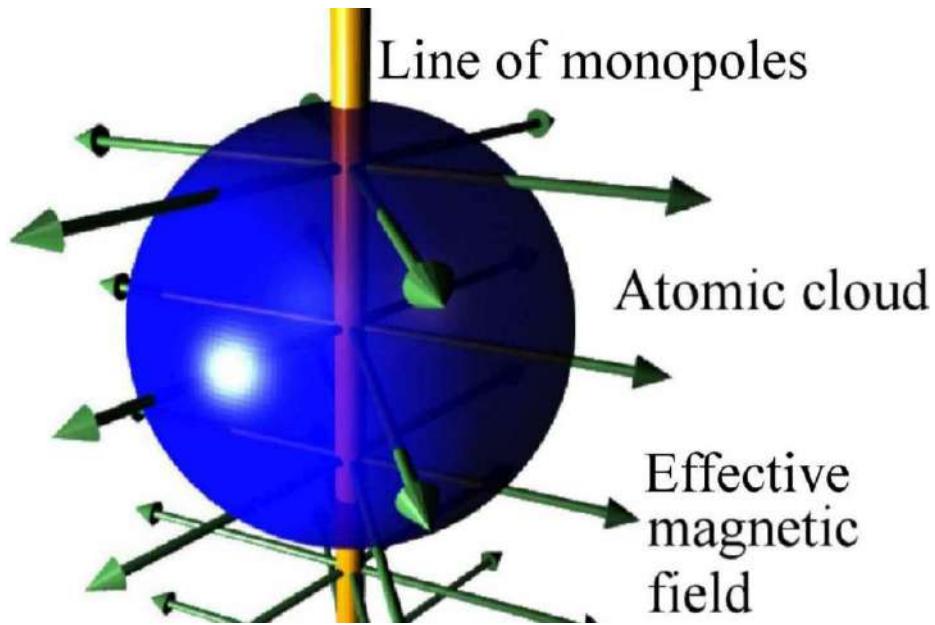
Spielman PRA (2009)

# Synthetic uniform magnetic field



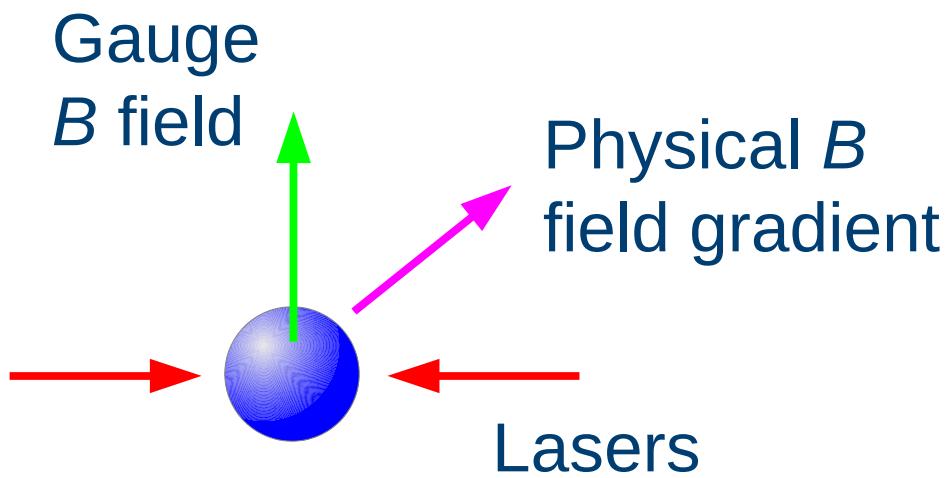
Lin *et al.* Nature 2009

# A line of magnetic monopoles

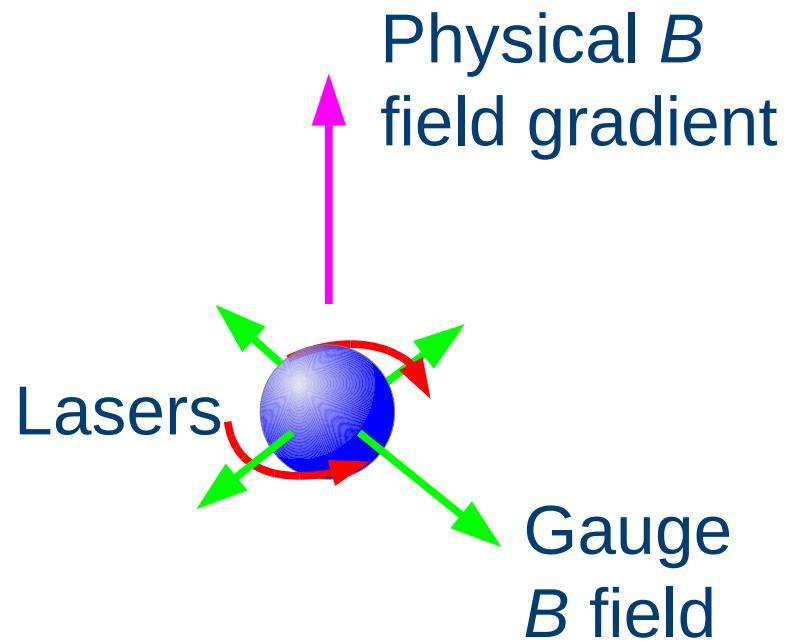


# A line of magnetic monopoles

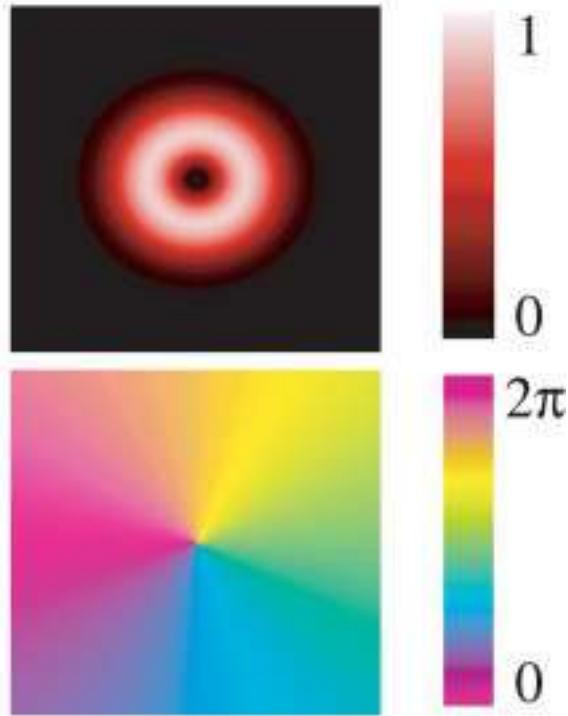
## Uniform $B$ field



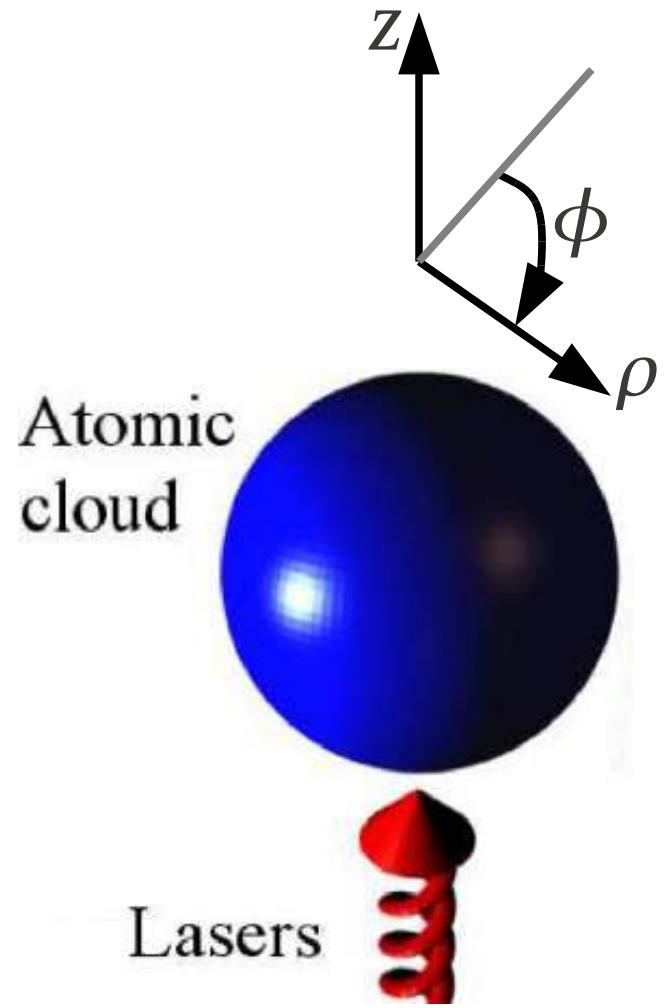
## Monopolar $B$ field



# How to imprint the magnetic monopoles



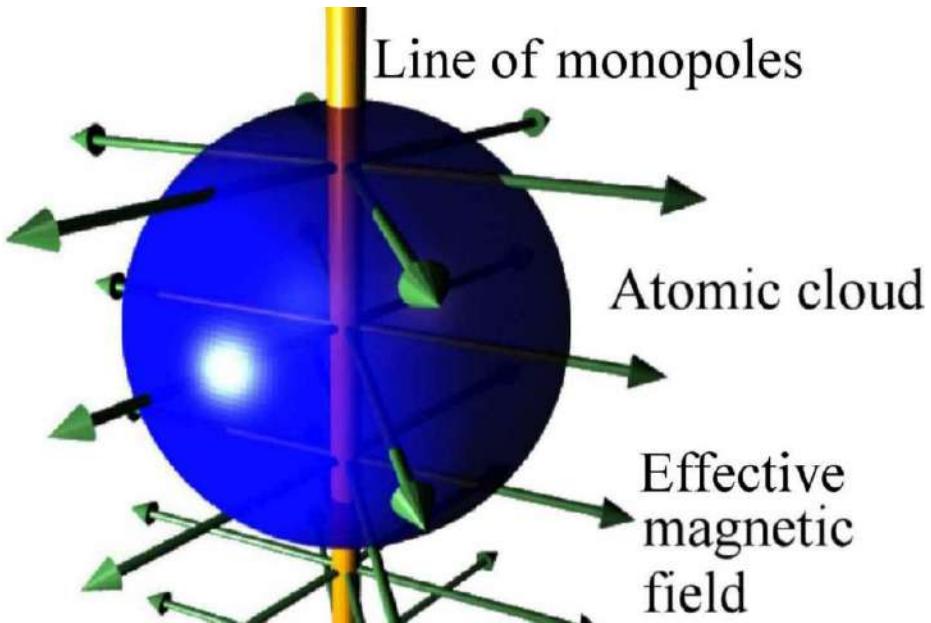
$$\kappa = \kappa_0 \rho e^{i\phi - \rho^2}$$



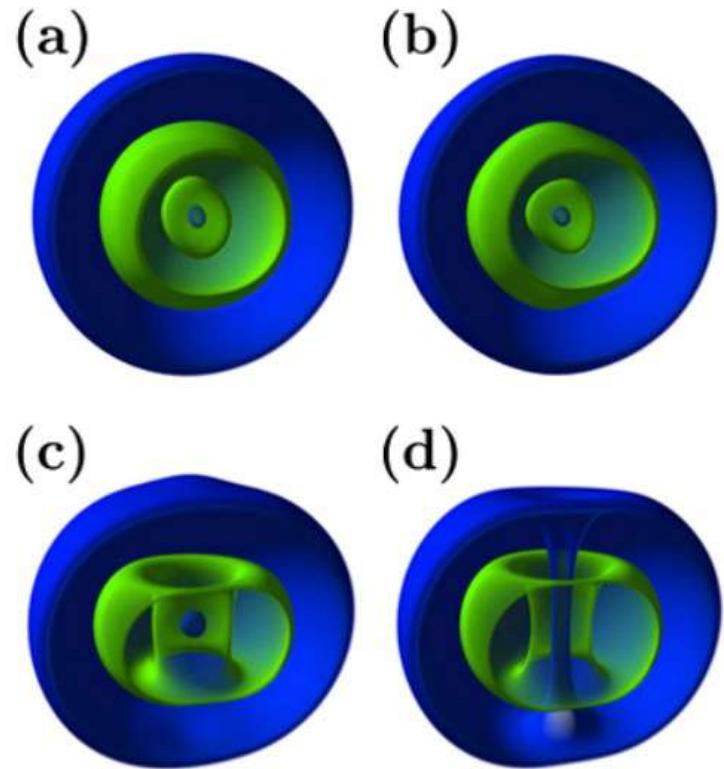
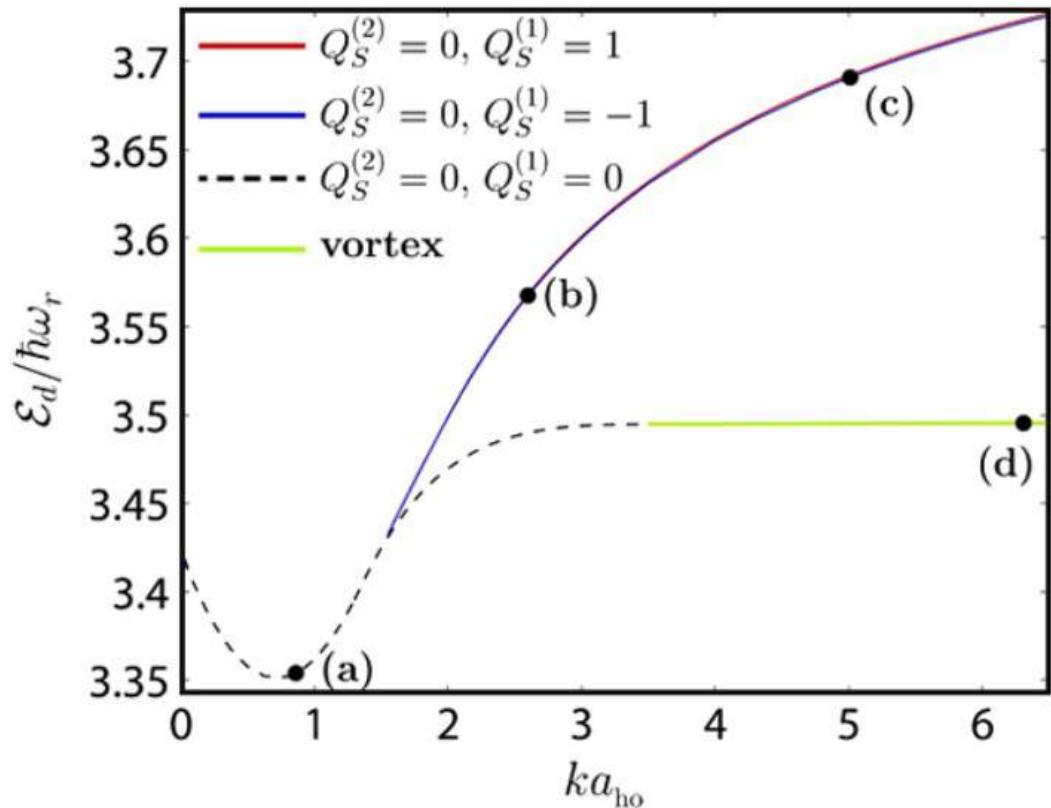
# How to imprint the magnetic monopoles

$$\hat{U} = \frac{\hbar}{2} \begin{pmatrix} -2\delta' z & \kappa_0 \rho e^{i\phi - \rho^2} & 0 \\ \kappa_0 \rho e^{-i\phi - \rho^2} & 0 & \kappa_0 \rho e^{i\phi - \rho^2} \\ 0 & \kappa_0 \rho e^{-i\phi - \rho^2} & 2\delta' z \end{pmatrix}$$

$$\begin{pmatrix} B_\rho \\ B_\phi \\ B_z \end{pmatrix} \approx \frac{\sqrt{2}\hbar\delta'}{\kappa_0\rho} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

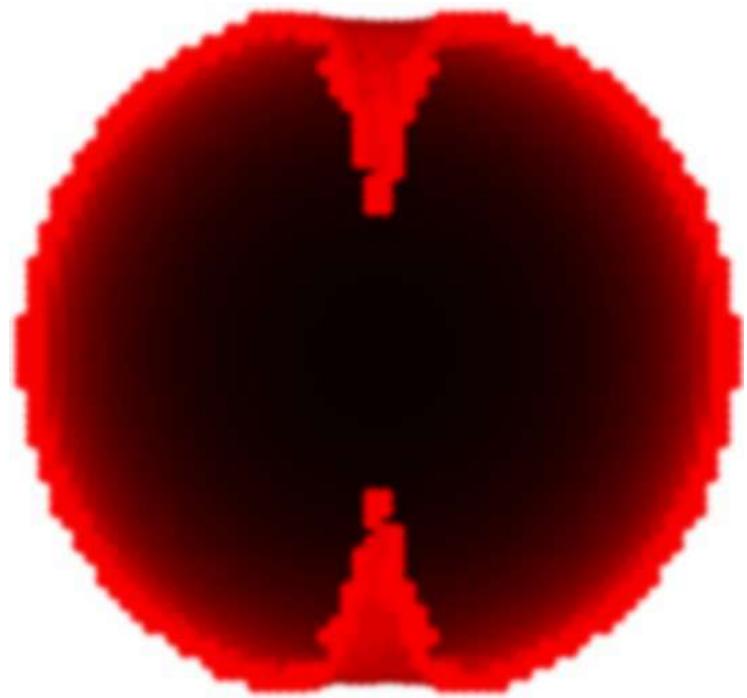


# Single magnetic monopole

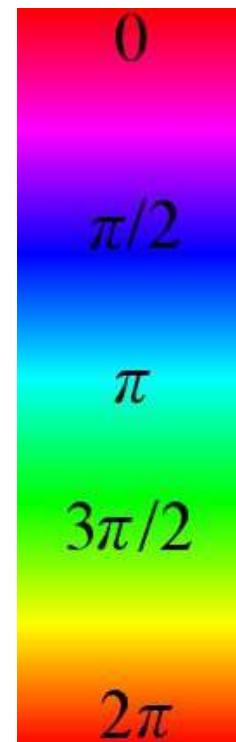
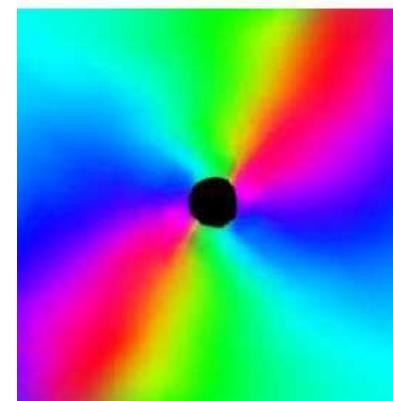
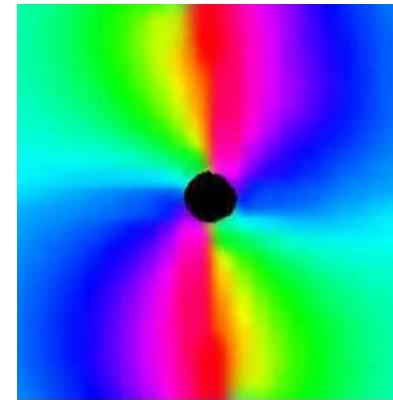


Pietilä & Möttönen PRL (2009)

# 4 monopoles embedded



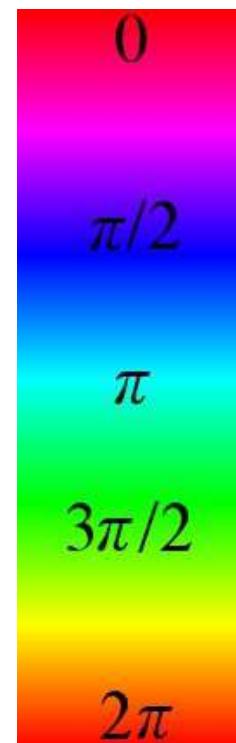
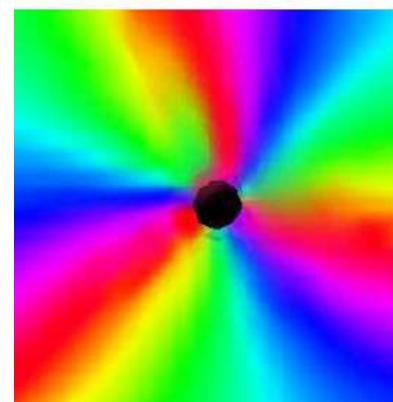
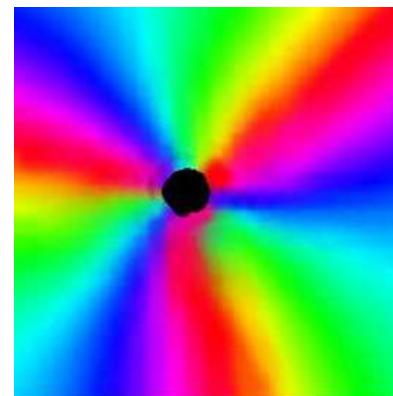
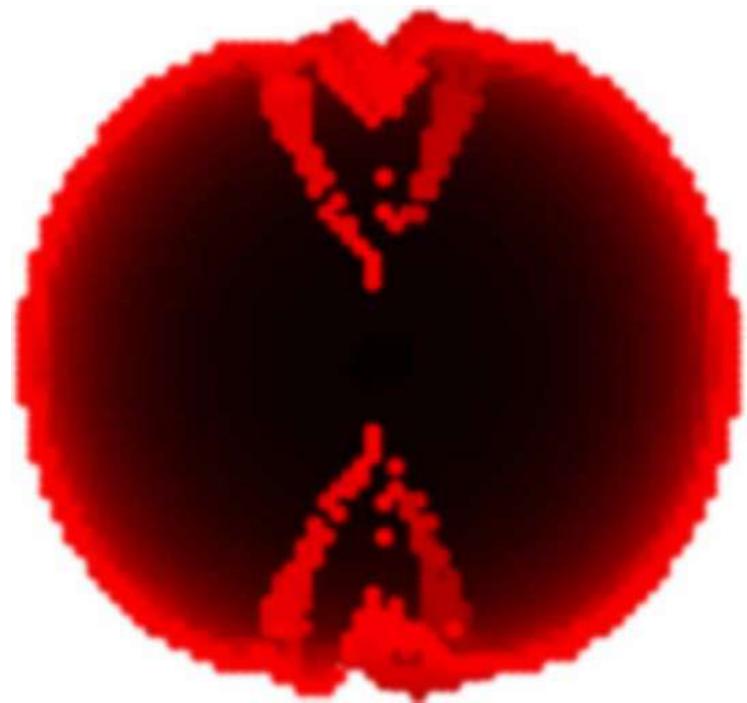
# 4 monopoles embedded



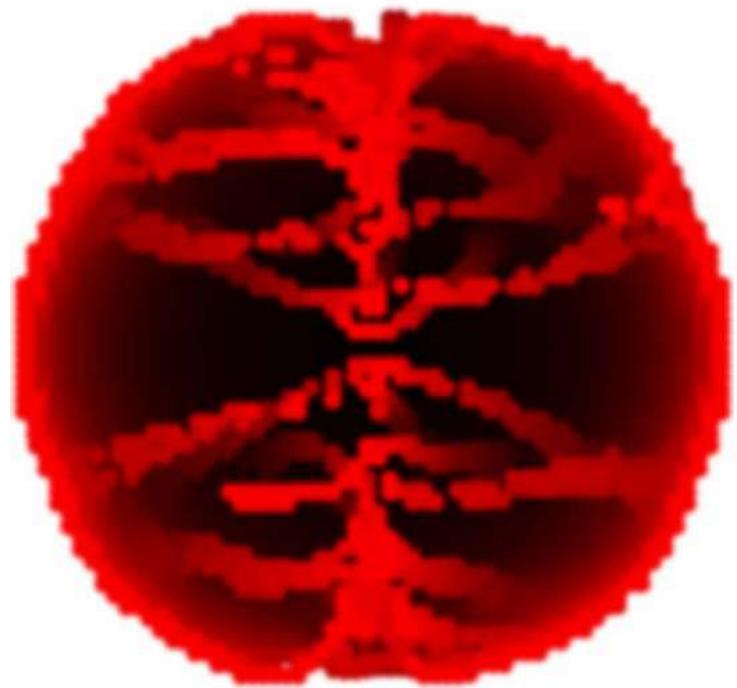
# 12 monopoles embedded



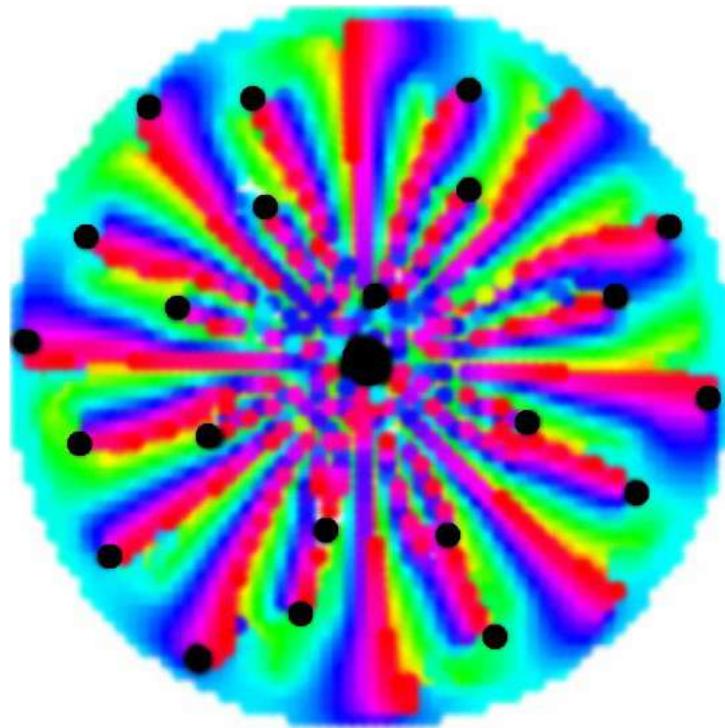
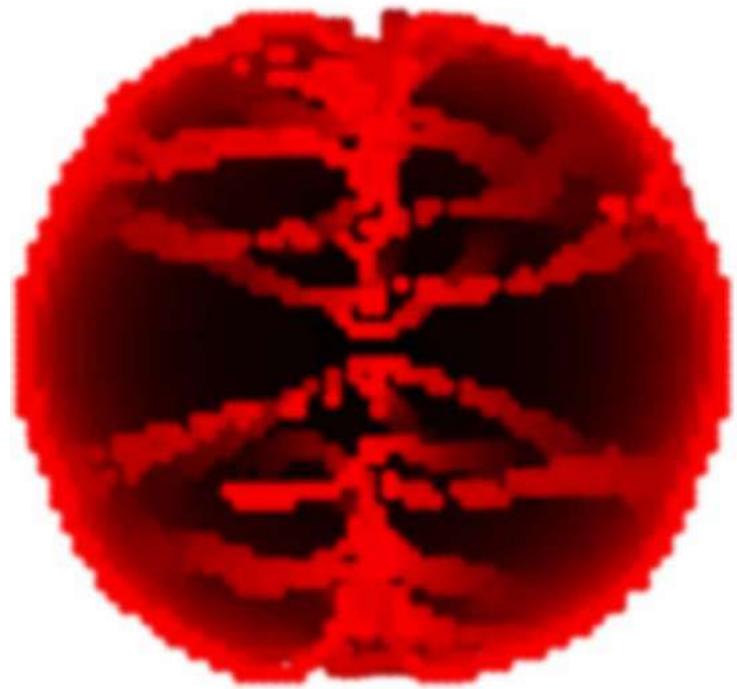
# 12 monopoles embedded



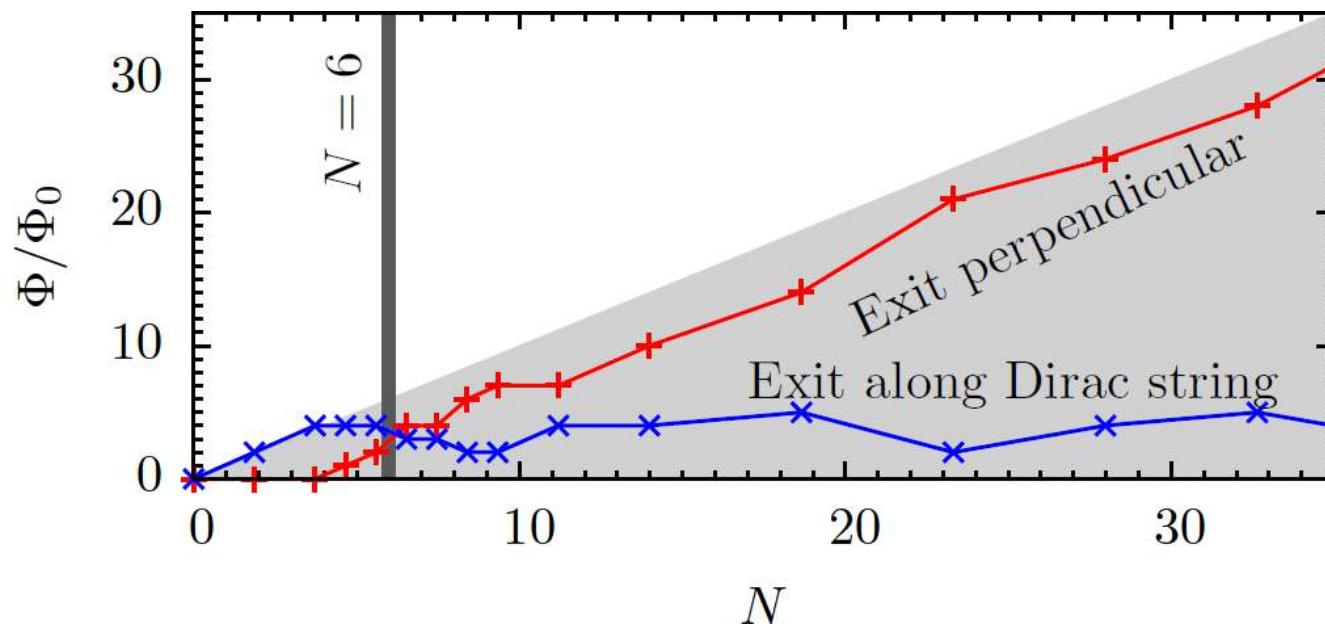
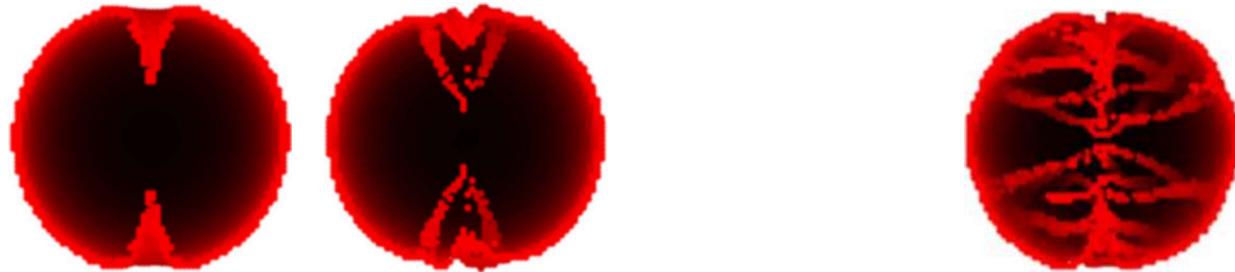
# 55 monopoles embedded



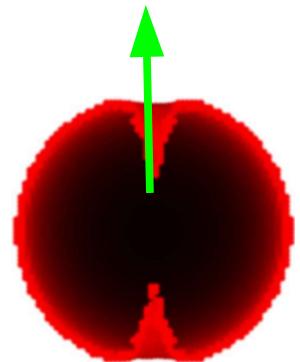
# 55 monopoles embedded



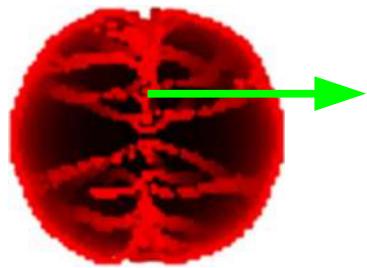
# Phase behavior



# Energetics of flux escape



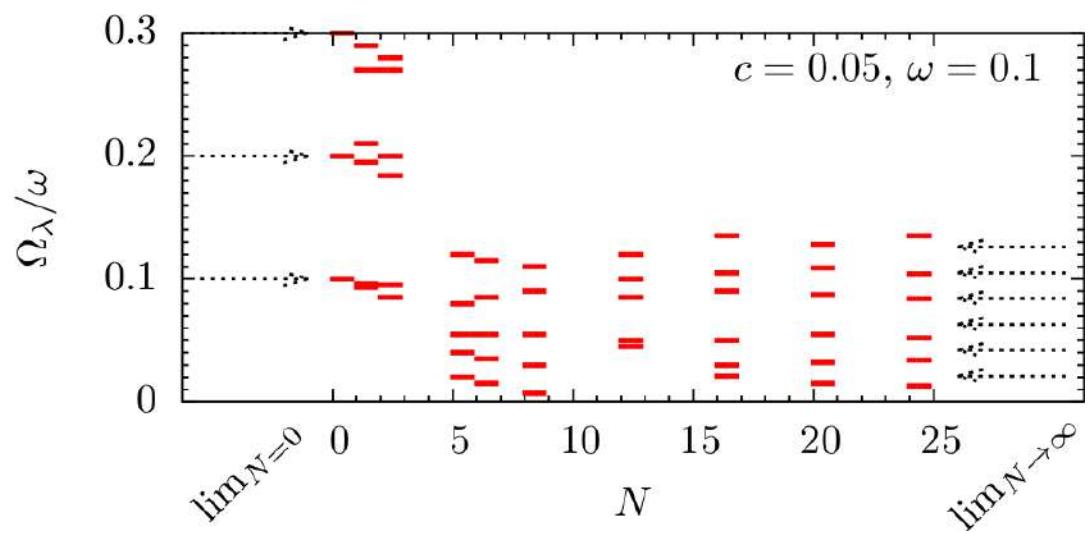
$$E = \frac{C}{6} N^2 L$$



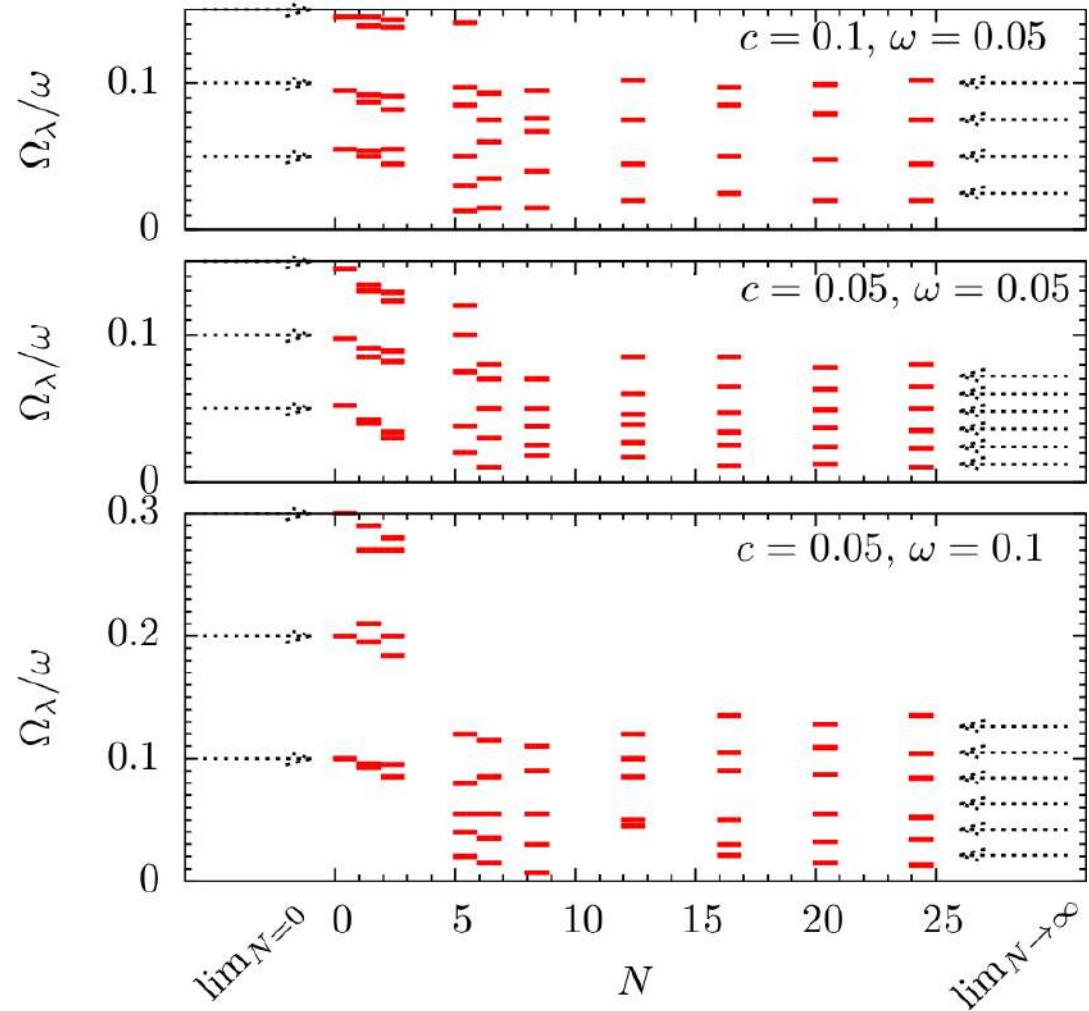
$$E = C N L$$

Crossover at  $N=6$

# Collective modes



# Collective modes



# Summary

Proposed an experimental prescription to embed a line of magnetic monopoles onto an atomic gas

The atomic gas undergoes a transition when 6 monopoles are trapped