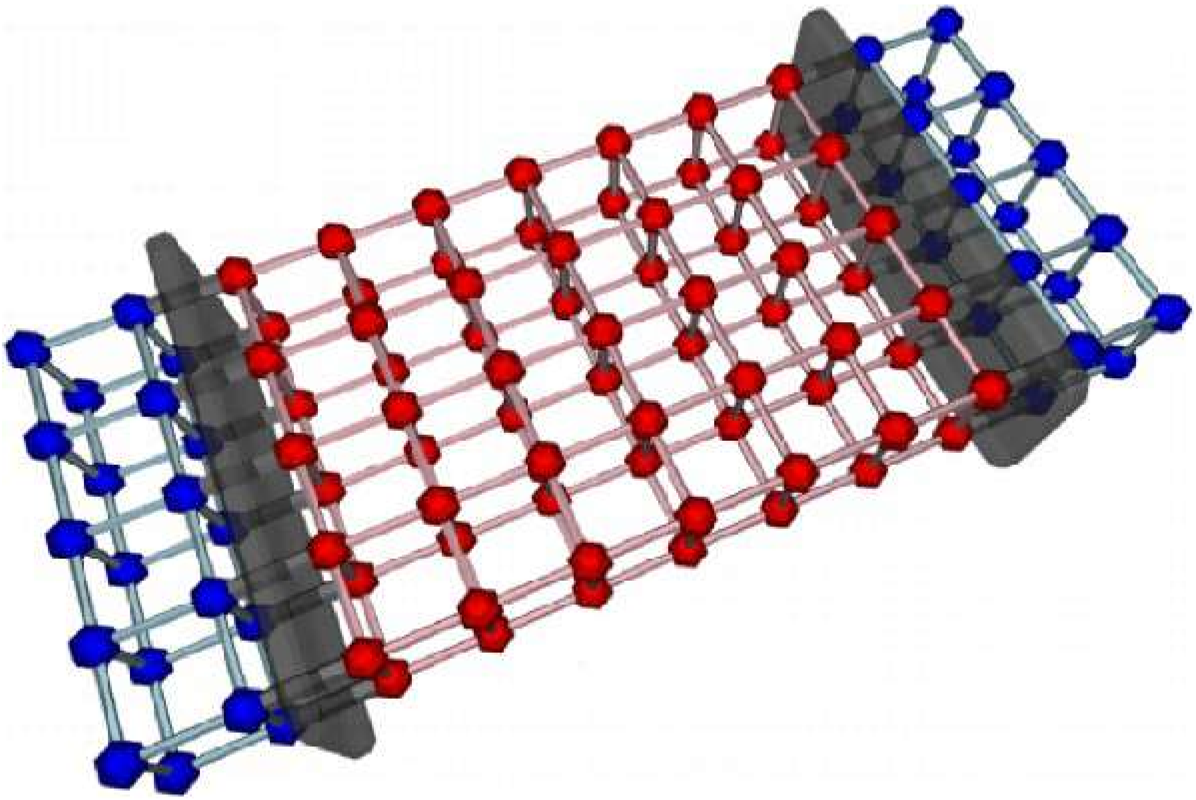
Modeling the magnetoresistance of disordered superconducting films

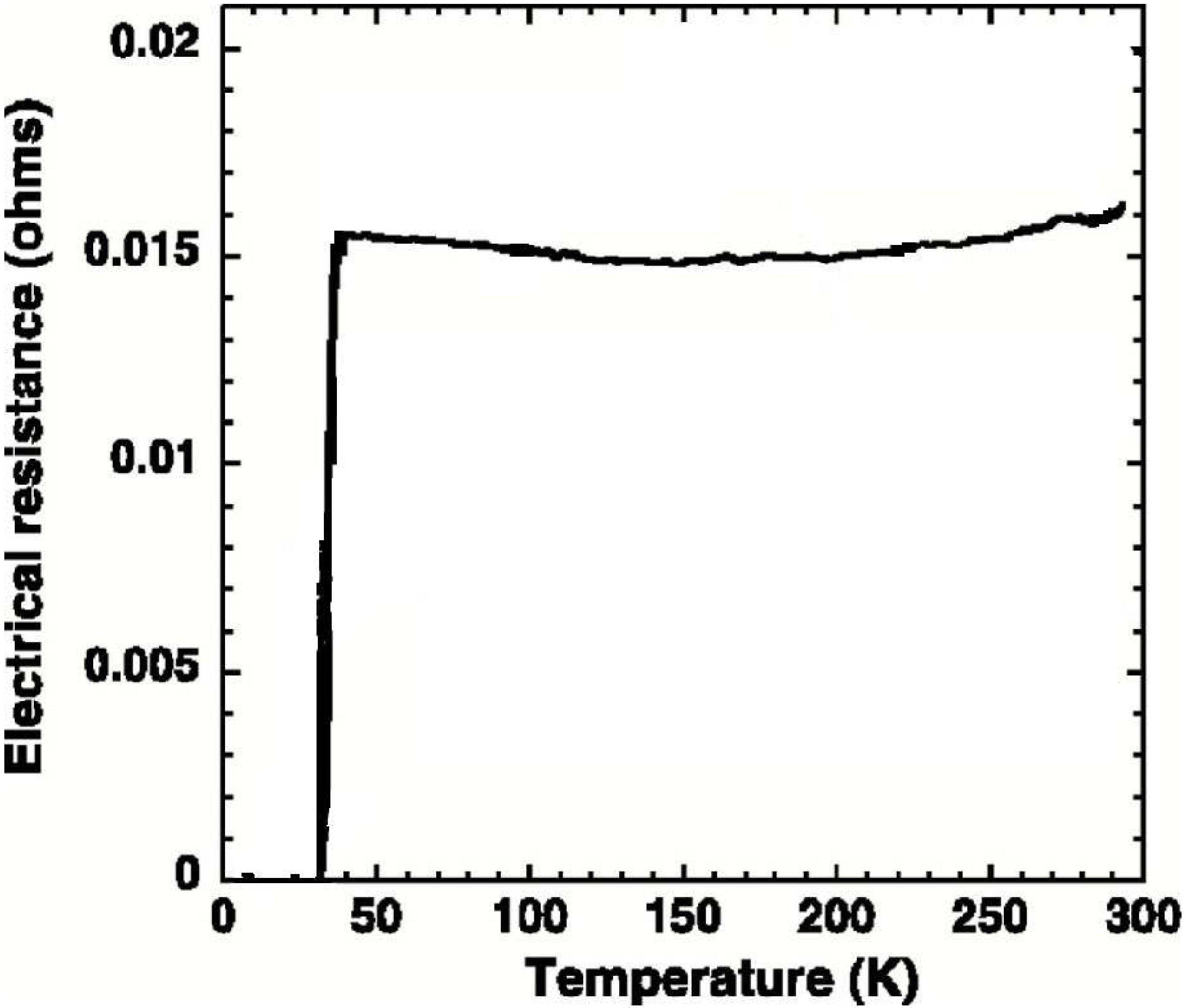


Gareth Conduit, Yigal Meir

University of Cambridge & Ben Gurion University

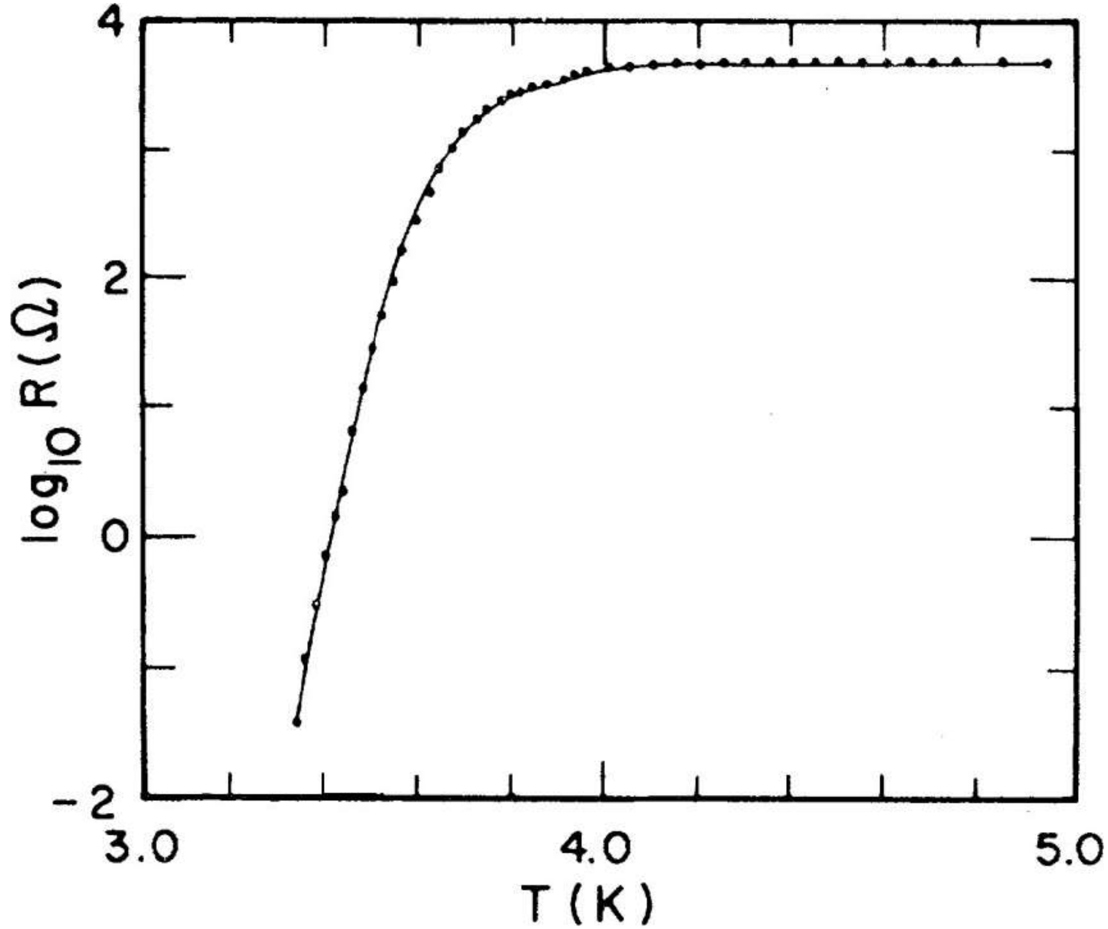
PRB **84**, 064513 (2011); PRL **108**, 159701 (2012)

BCS superconductivity in MgB_2



Monteverde *et al*., Science **292**, 75 (2001)

KT transition conductivity



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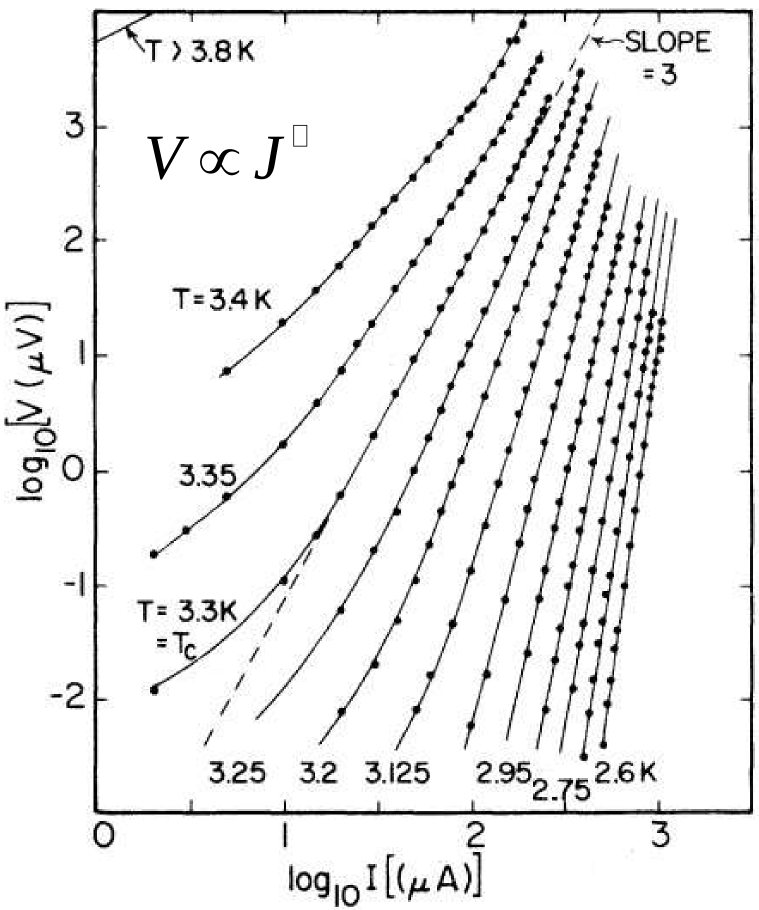
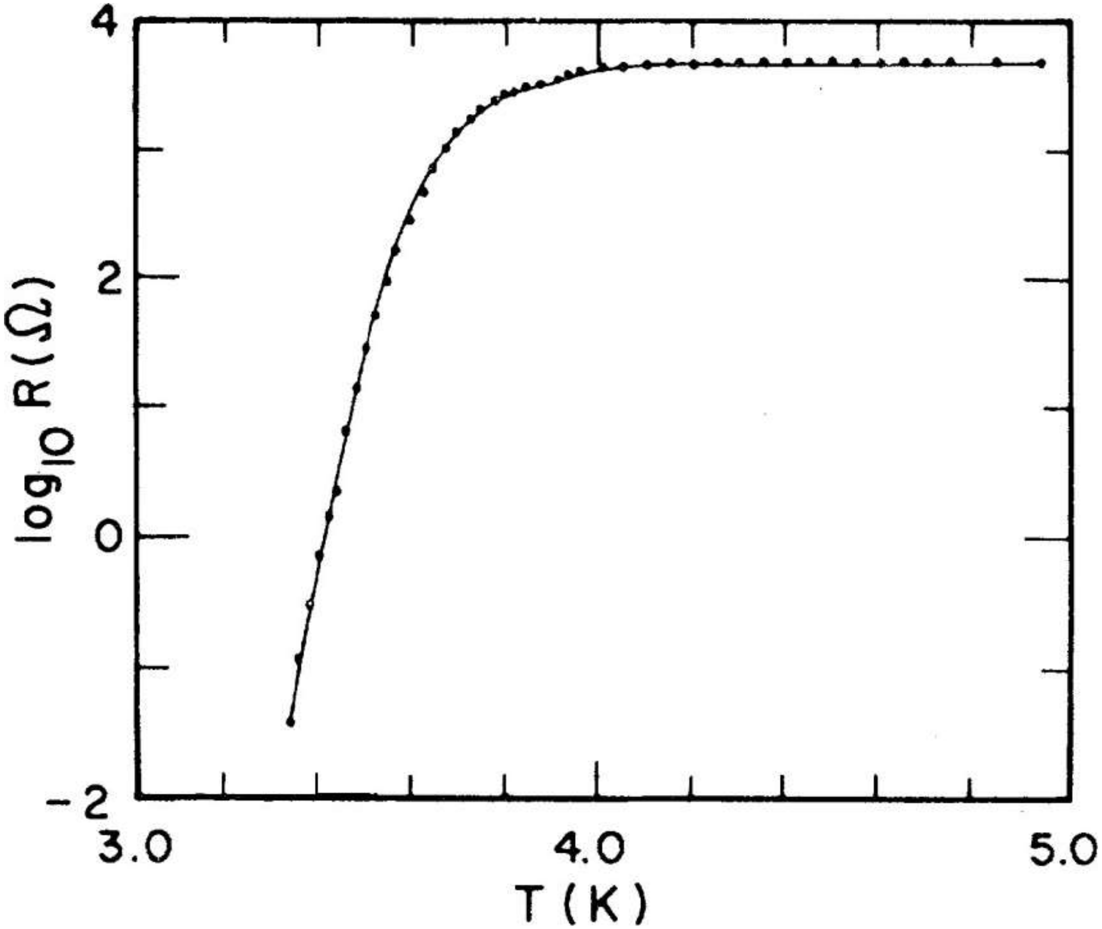
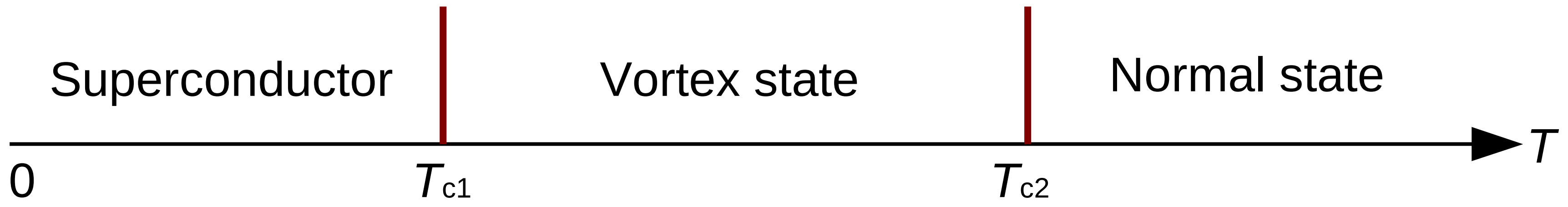
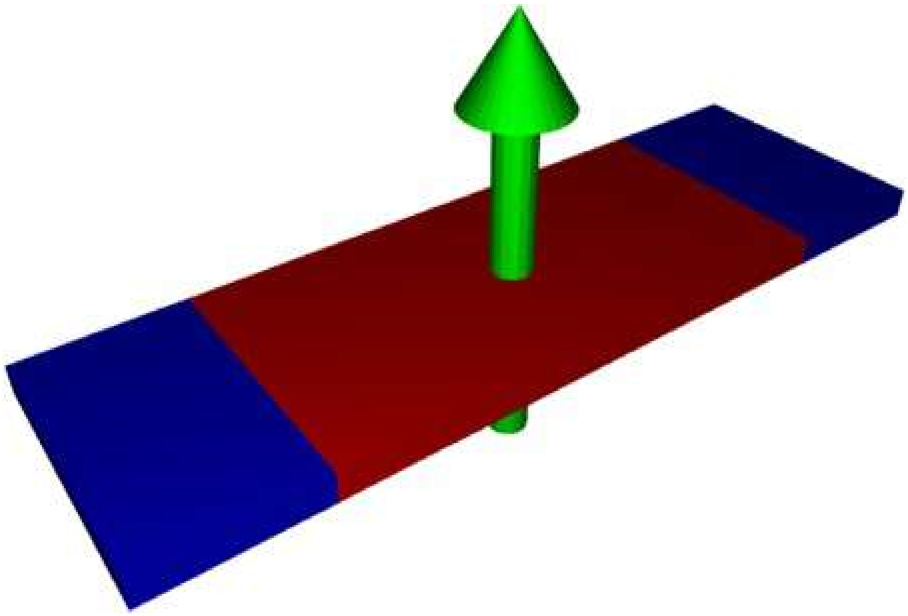


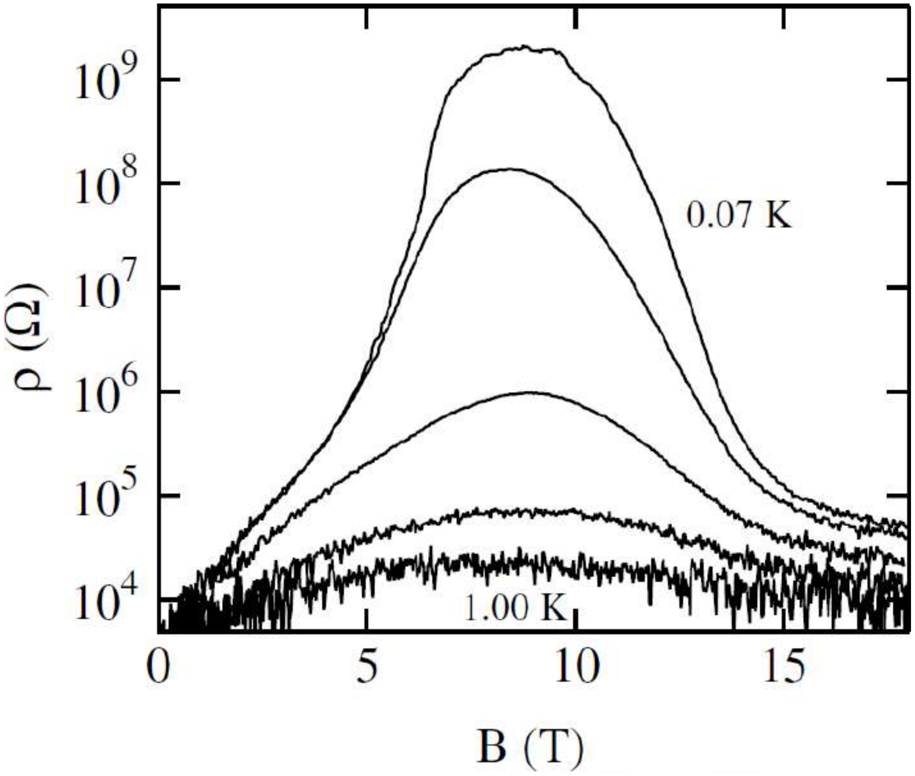




Transition in disordered systems

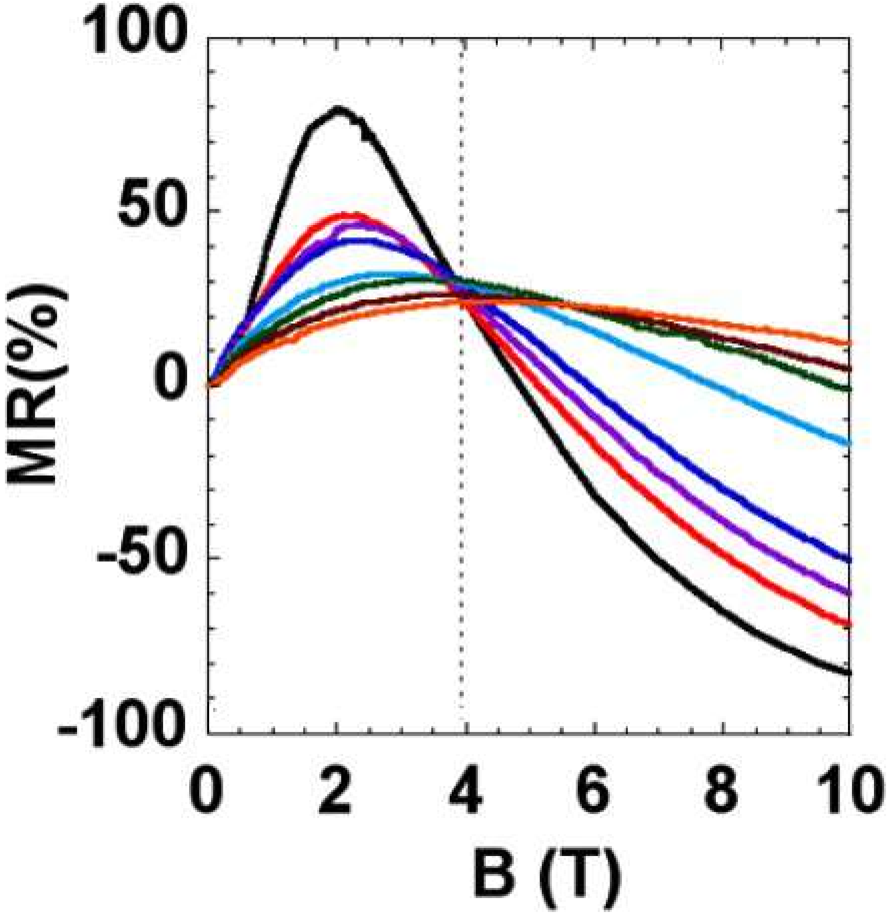
- Magnetoresistance peak [Sambandamurthy & Shahar, PRL 2004]

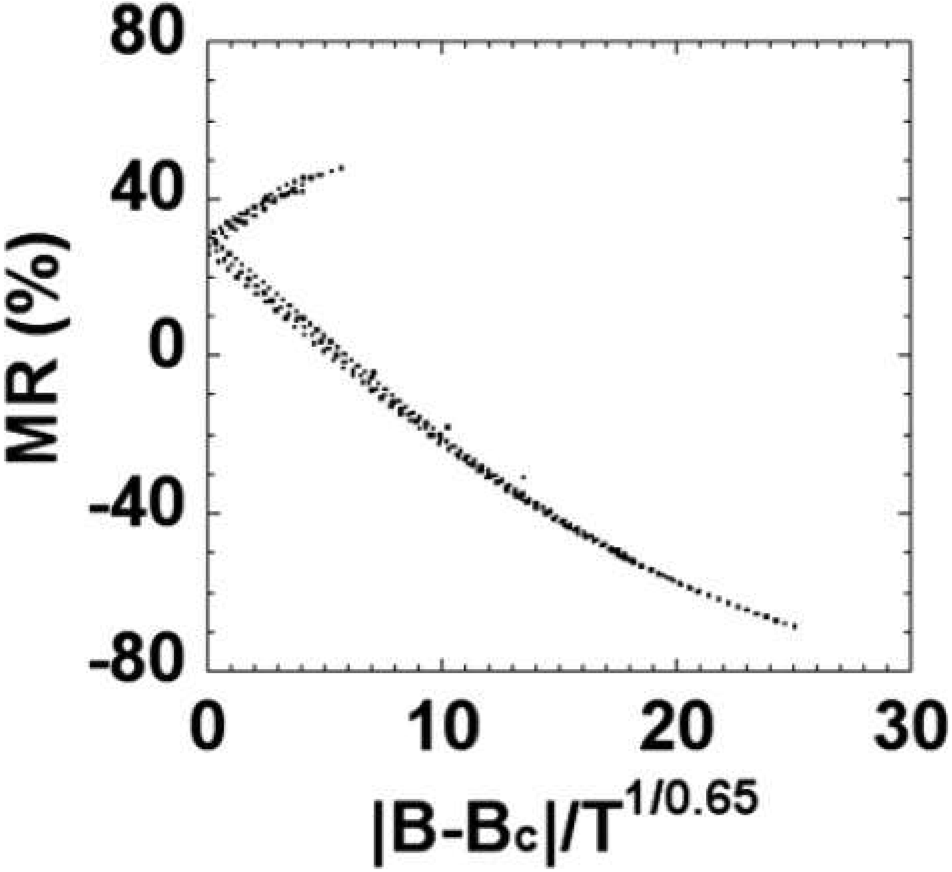




Transition in highly disordered systems

$$MR(B,T)=\frac{R(B,T)-R(0,T)}{R(0,T)}$$



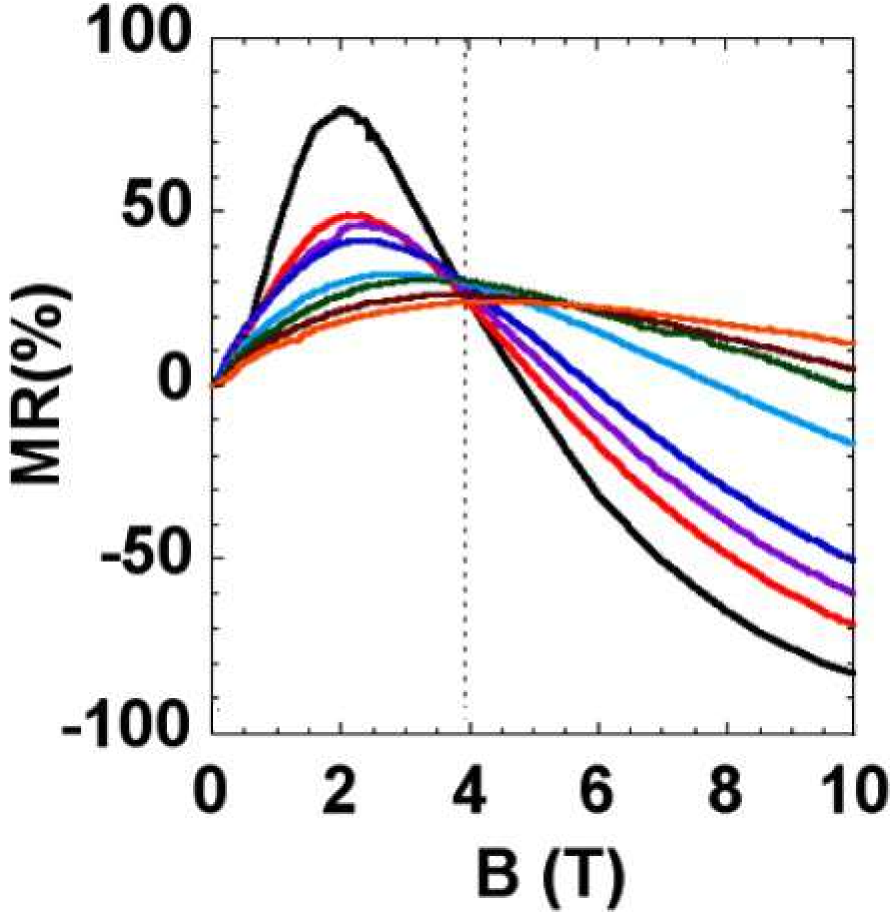


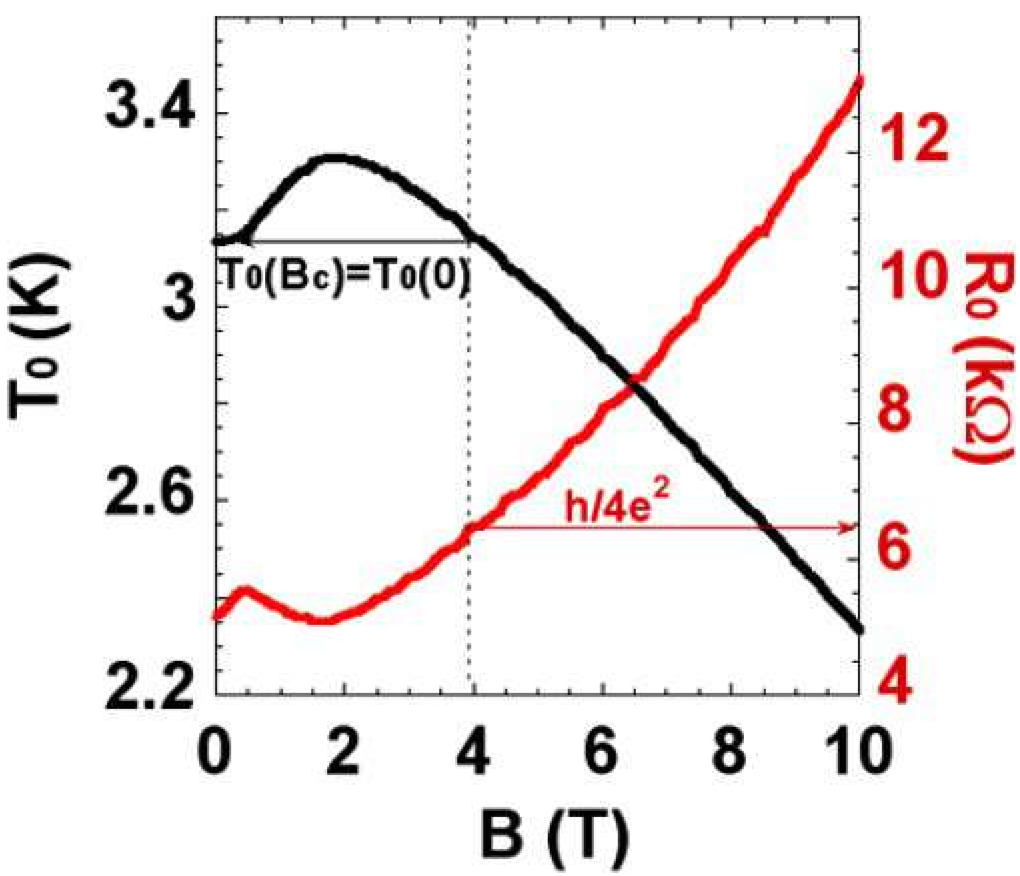
Lin & Goldman, PRL, (2011)

Transition in highly disordered systems

$$MR(B,T) = \frac{R(B,T) - R(0,T)}{R(0,T)}$$

$$R(B,T) = R_0(B)\, e^{T_A/T}$$





Lin & Goldman, PRL, (2011)

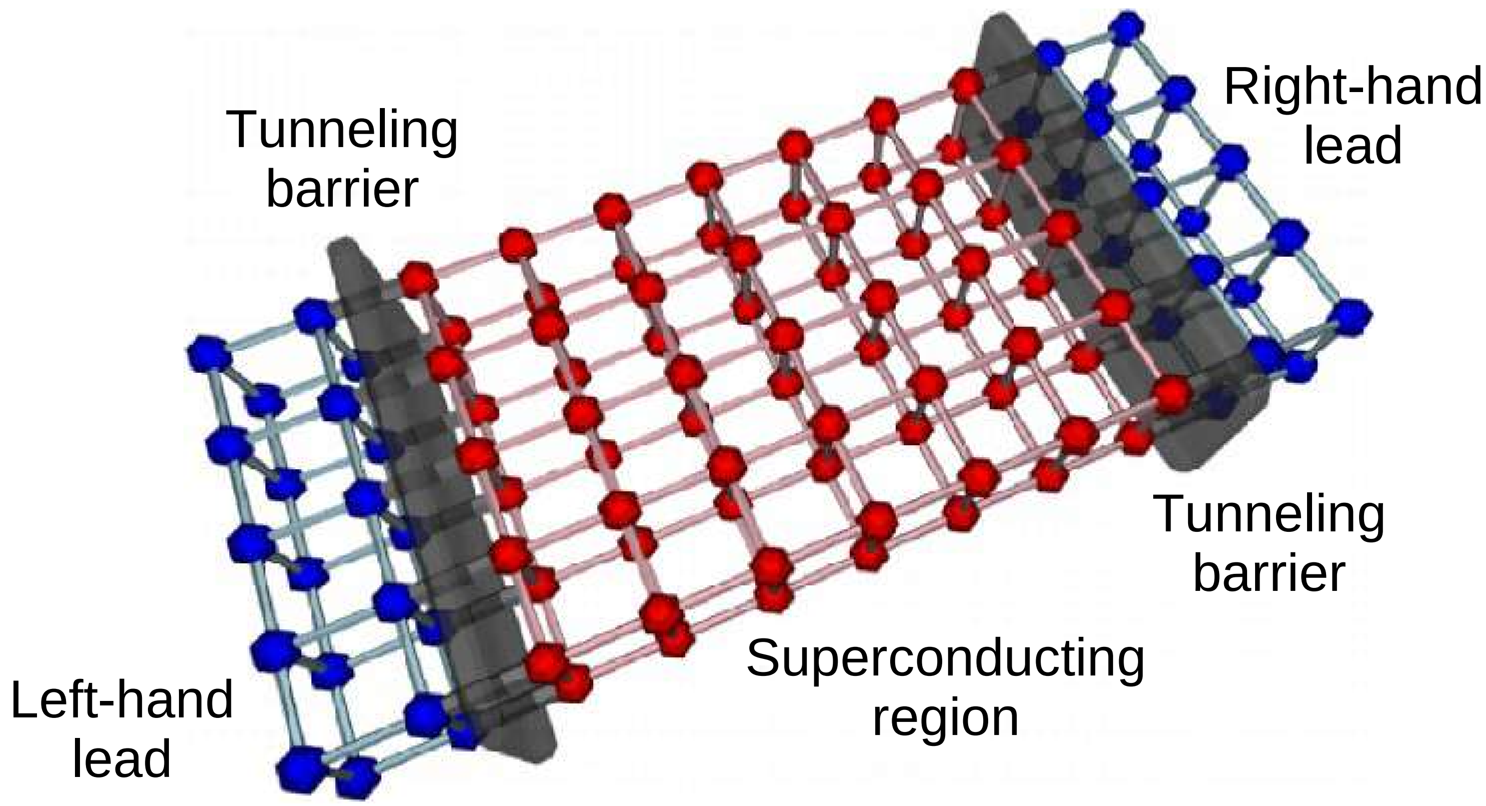
Strategy to study superconductors

- Develop new formalism to:
  - Calculate exact net current flow
  - Account for phase and amplitude fluctuations
  - Include disorder
  - Extract the microscopic current flow
  - Develop algorithm that permits access to large systems
- Test the formalism against a series of well-established results
- Study the magnetoresistance peak & putative quantum phase transition

How to calculate the current

- General expression for the current [Meir & Wingreen, PRL 1992]

$$J = \frac{\mathrm{i}e}{2h} \int \mathrm{d}\epsilon \Big[\mathrm{Tr}\left\{ \left(f_{\mathrm{L}}(\epsilon)\Gamma^{\mathrm{L}} - f_{\mathrm{R}}(\epsilon)\Gamma^{\mathrm{R}} \right) \left(G^{\mathrm{r}}_{\mathrm{e}\sigma} - G^{\mathrm{a}\sigma}_{\mathrm{e}} \right) \right\} + \mathrm{Tr}\left\{ (\Gamma^{\mathrm{L}} - \Gamma^{\mathrm{R}}) G^{<}_{\mathrm{e}\sigma} \right\} \Big]$$



Decoupling the interactions

- Negative *U* Hubbard model

$$\hat{H}_{\text{Hubbard}} = \sum_{i,\sigma} \epsilon_{i\sigma} c^{\dagger}_{i\sigma} c_{i\sigma} - \sum_{i} U_i c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow} c_{i\downarrow} c_{i\uparrow}$$

$$- \sum_{\langle i,j \rangle,\sigma} \left(t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + t^{*}_{ij} c^{\dagger}_{j\sigma} c_{i\sigma} \right)$$

- Decouple in density and Cooper pair channels

$$\rho_{i\sigma} = -|U_i| c^{\dagger}_{i\sigma} c_{i\sigma} \qquad \Delta_i = |U_i| \; c_{i\downarrow} c_{i\uparrow}$$

- Hamiltonian now contains single-body operators

$$\hat{\mathcal{H}}_{\text{BdG}} = \sum_{i,\sigma} (\epsilon_i + \rho_i) c^{\dagger}_{i\sigma} c_{i\sigma} - \sum_{\langle i,j \rangle,\sigma} \left(t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + t^{*}_{ij} c^{\dagger}_{j\sigma} c_{i\sigma} \right)$$

$$+ \sum_{i} \left(\Delta_i c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow} + \bar{\Delta}_i c_{i\downarrow} c_{i\uparrow} \right) + \sum_{i} \frac{|\Delta_i|^2 + \rho_i^2}{U_i}$$

Diagonalizing the Hamiltonian

- Hamiltonian now contains single-body operators

$$\hat{\mathcal{H}}_{\mathrm{BdG}} = \sum_{i,\sigma} (\epsilon_i + \rho_i) c^\dagger_{i\sigma} c_{i\sigma} - \sum_{\langle i,j \rangle,\sigma} \left(t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + t^*_{ij} c^\dagger_{j\sigma} c_{i\sigma} \right)$$

$$+ \sum_i \left(\Delta_i c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} + \bar{\Delta}_i c_{i\downarrow} c_{i\uparrow} \right) + \sum_i \frac{|\Delta_i|^2 + \rho_i^2}{U_i}$$

- Energy eigenstates can be found from diagonalization of

$$\hat{\mathcal{H}}_{\mathrm{BgG}} = \frac{|\Delta|^2 + \rho^2}{U} + \begin{pmatrix} c^\dagger_\uparrow & c_\downarrow \end{pmatrix} \begin{pmatrix} \epsilon + \rho & \Delta \\ \bar{\Delta} & -(\epsilon + \rho) \end{pmatrix} \begin{pmatrix} c_\uparrow \\ c^\dagger_\downarrow \end{pmatrix} + \epsilon + \rho$$

Accelerated Metropolis sampling

- To perform thermal sum calculate

$$\langle J \rangle = \sum_{\Delta,\rho} J[\Delta,\rho] \mathrm{e}^{-\beta(E[\Delta,\rho]-E_0)}$$

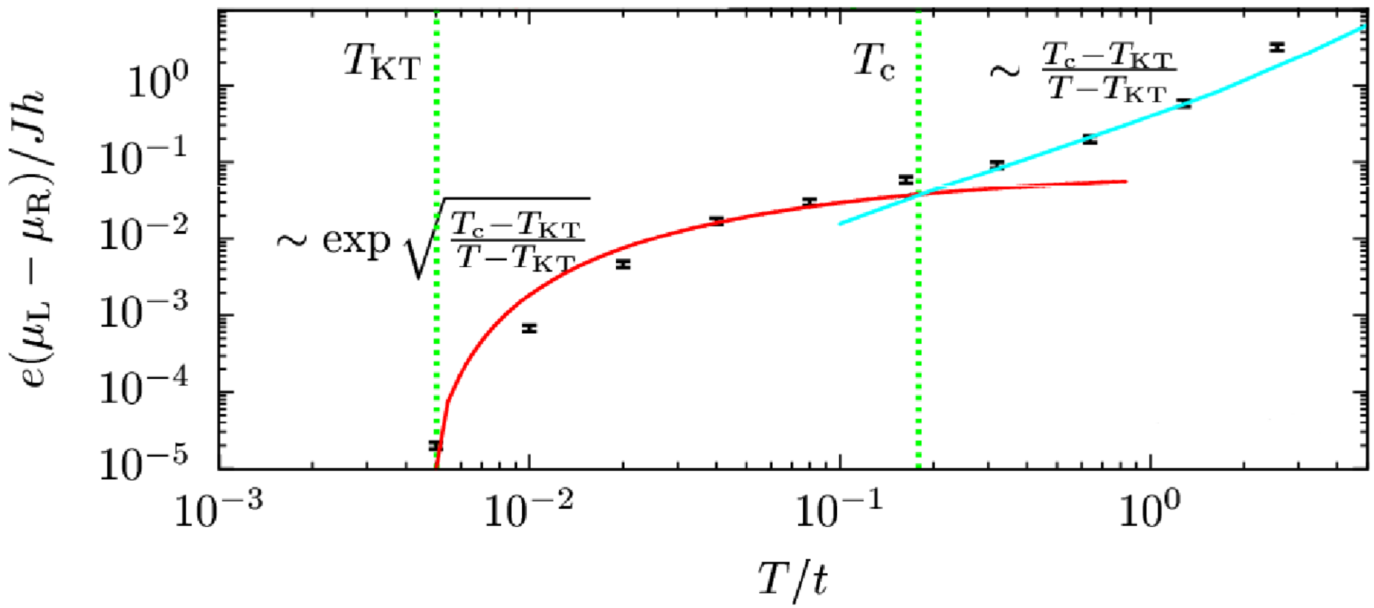
- Propose new configuration of Δ and ρ, accept with probability

$$\exp(\beta E[\Delta_{\mathrm{old}},\rho_{\mathrm{old}}]-\beta E[\Delta_{\mathrm{new}},\rho_{\mathrm{new}}])$$

- Calculating $E[\Delta,\rho]$ costs $O(N^3)$, where N is the number of sites
- New method calculates $E[\Delta,\rho]-E[\Delta+\delta\Delta,\rho+\delta\rho]$ using a Chebyshev expansion [Weisse 09] in $O(N^{1.56})$ time

Verification

- Resistivity at the Kosterlitz-Thouless transition
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect

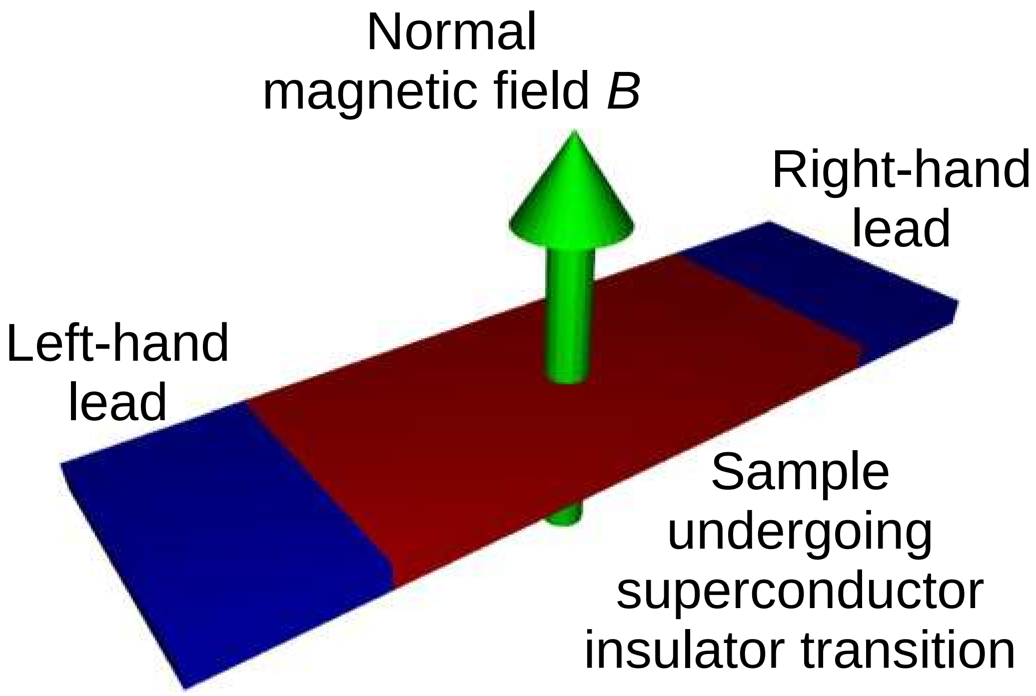


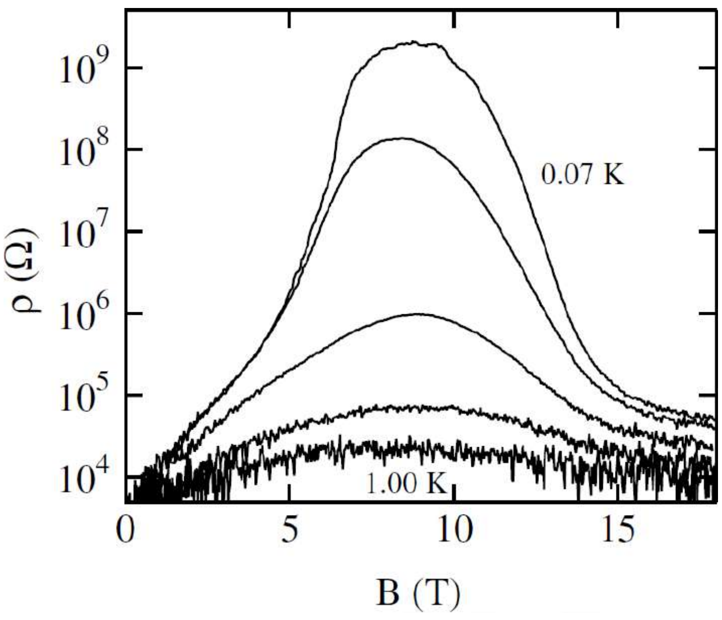
Halperin & Nelson, J. Low Temp. Phys (1979)

Ambegaokar *et al*., PRB (1980)

Magnetoresistance peak

- Study superconductor-insulator transition in dirty sample with perpendicular magnetic field

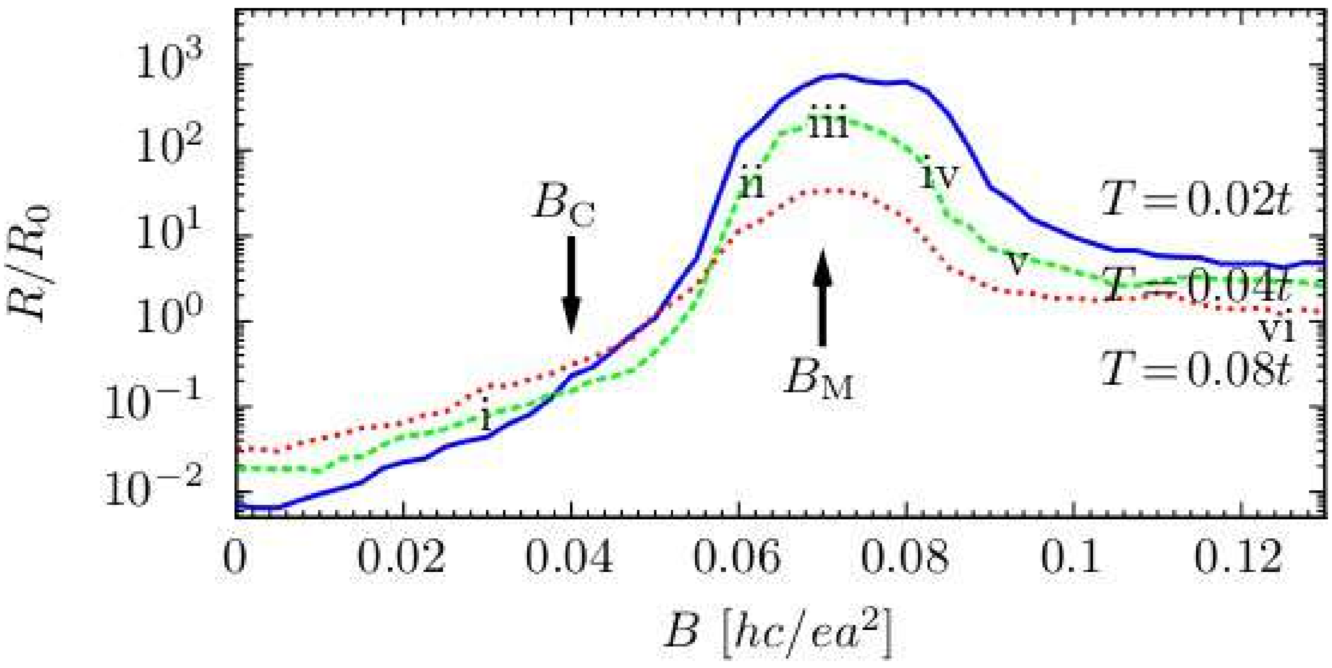


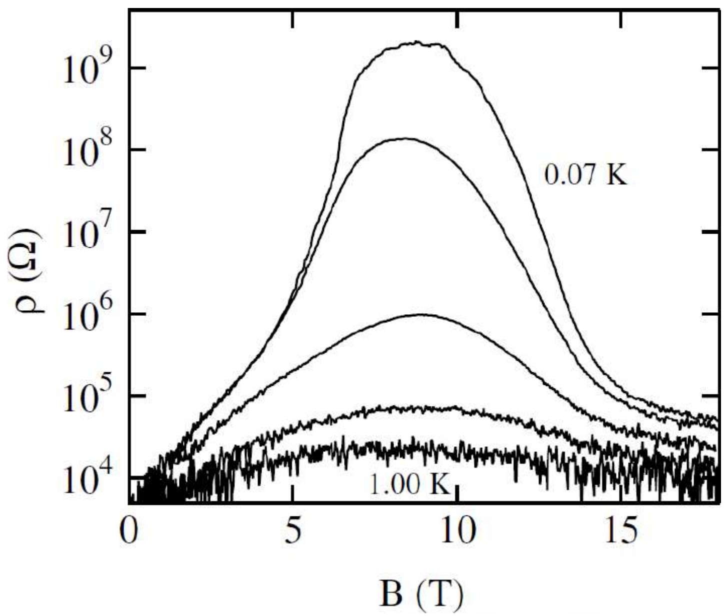


[Sambandamurthy & Shahar, PRL 2004]

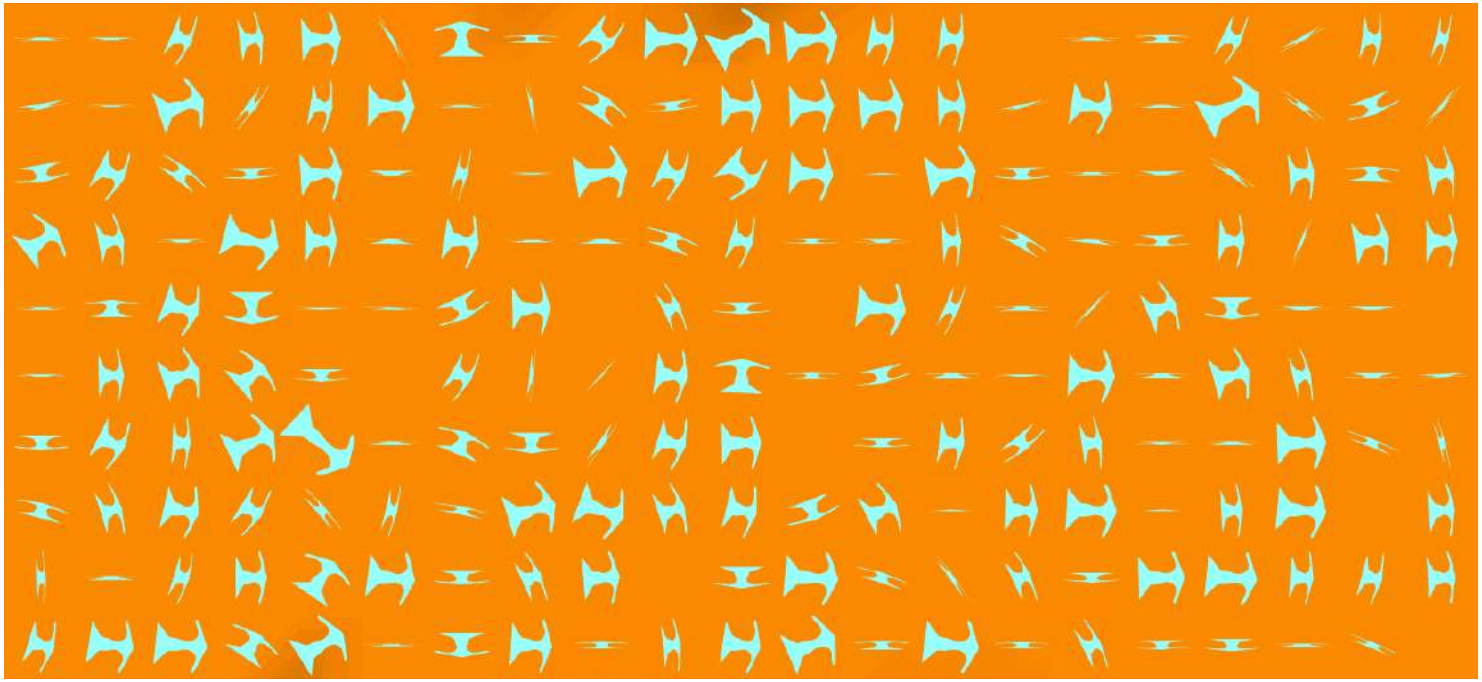
Magnetoresistance peak

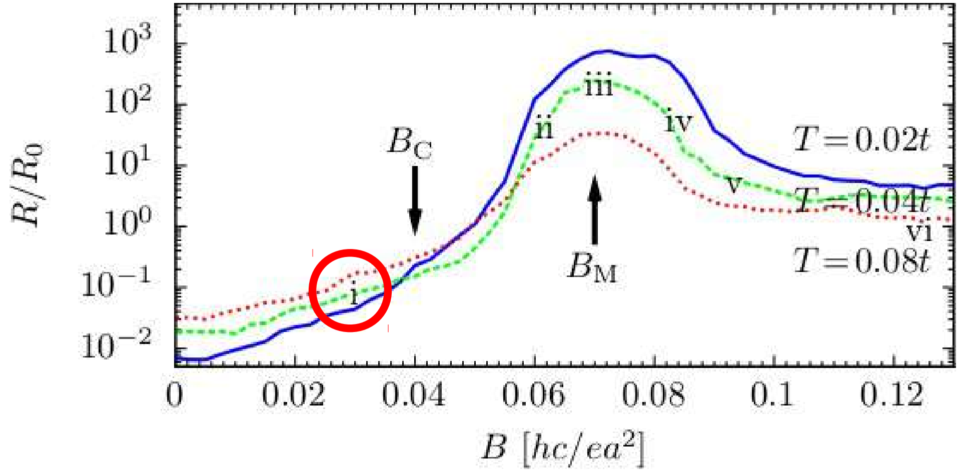
- Study superconductor-insulator transition in dirty sample with perpendicular magnetic field

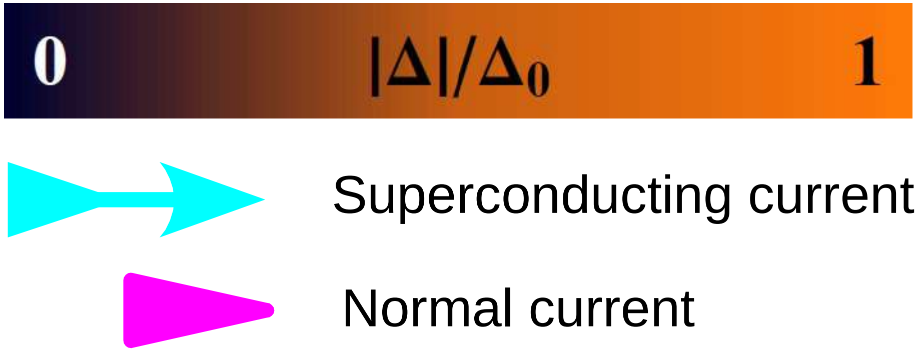




Clues: current maps







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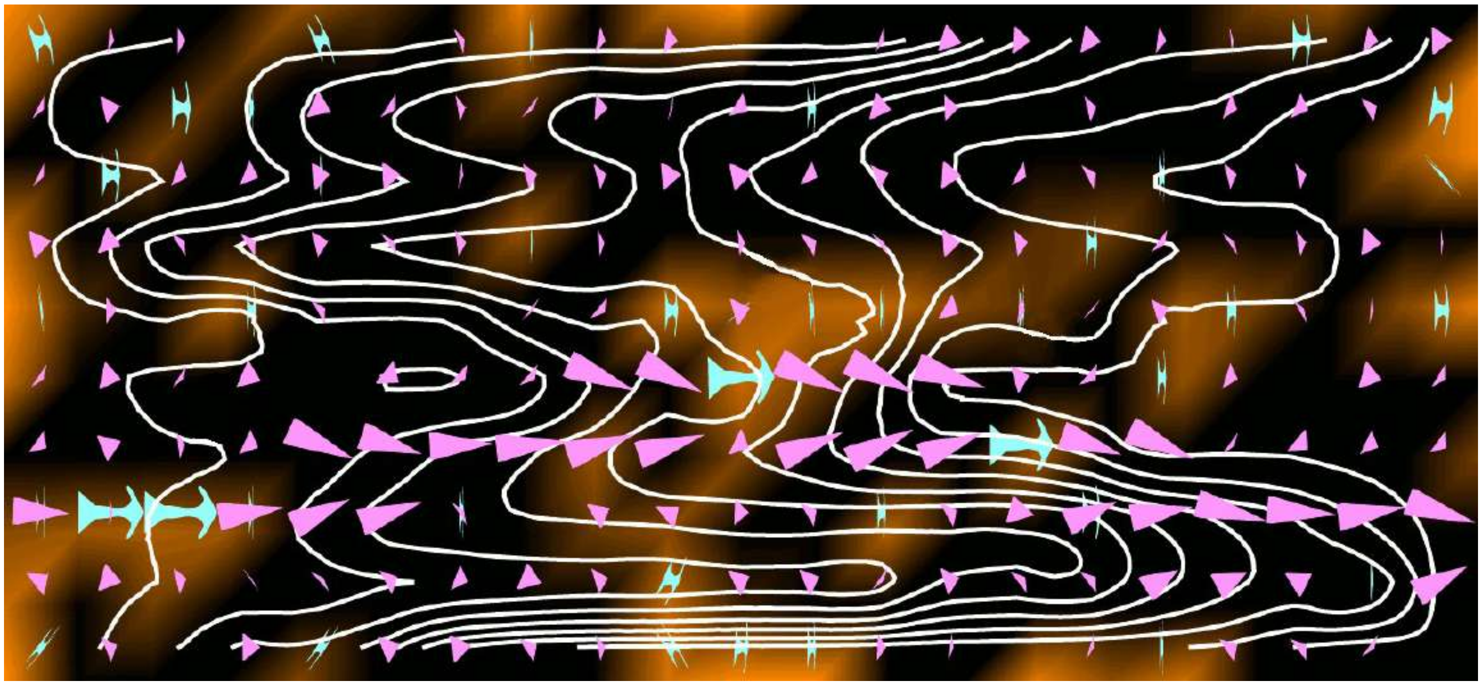
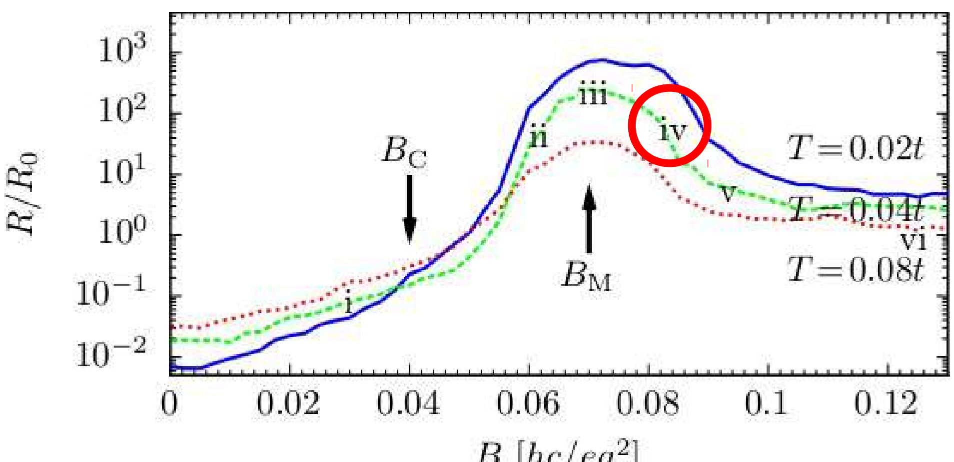
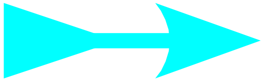
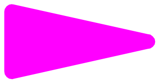
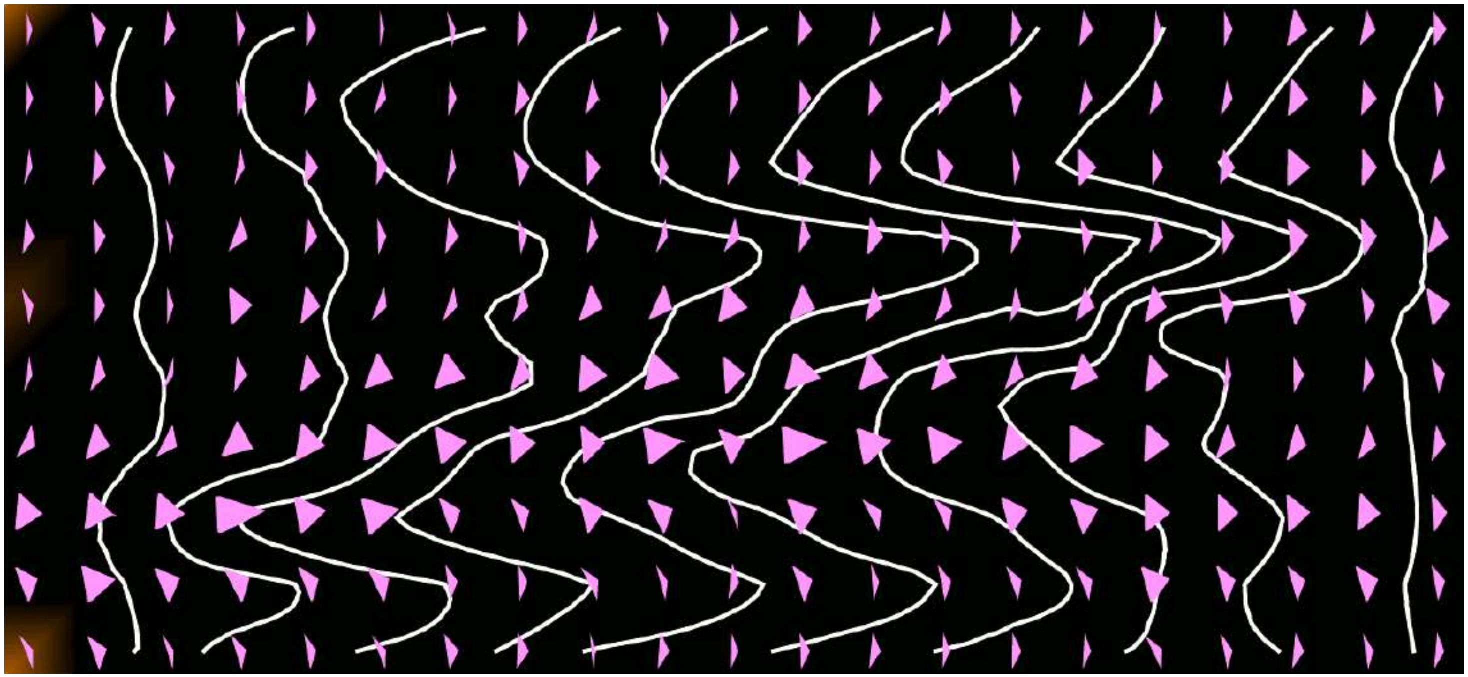


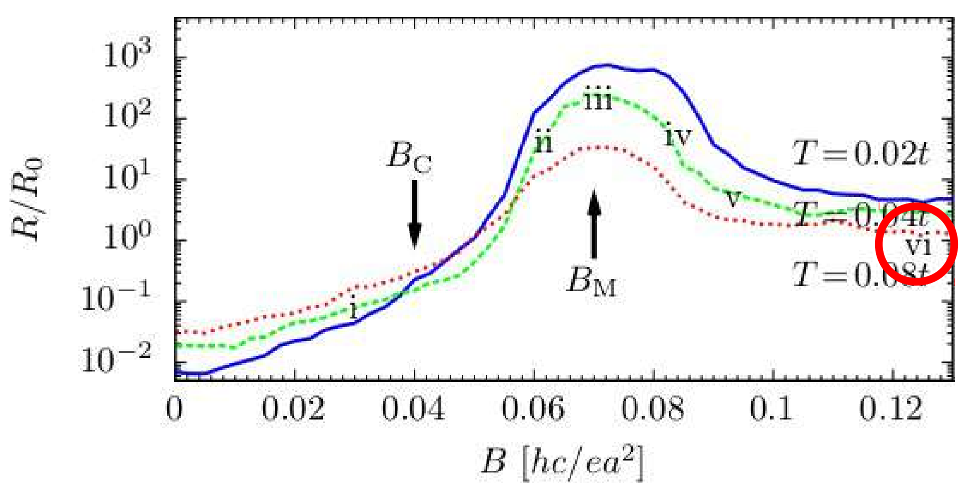


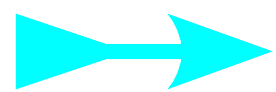

Superconducting current


Normal current

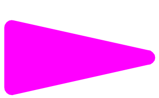
Clues: current maps







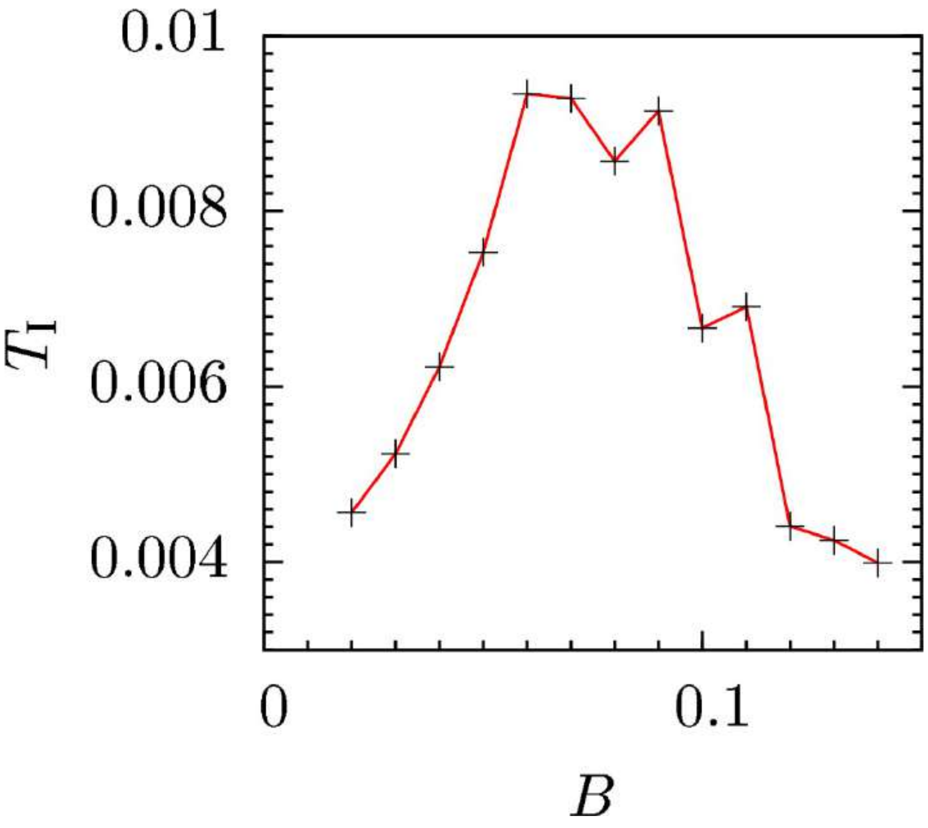
Superconducting current

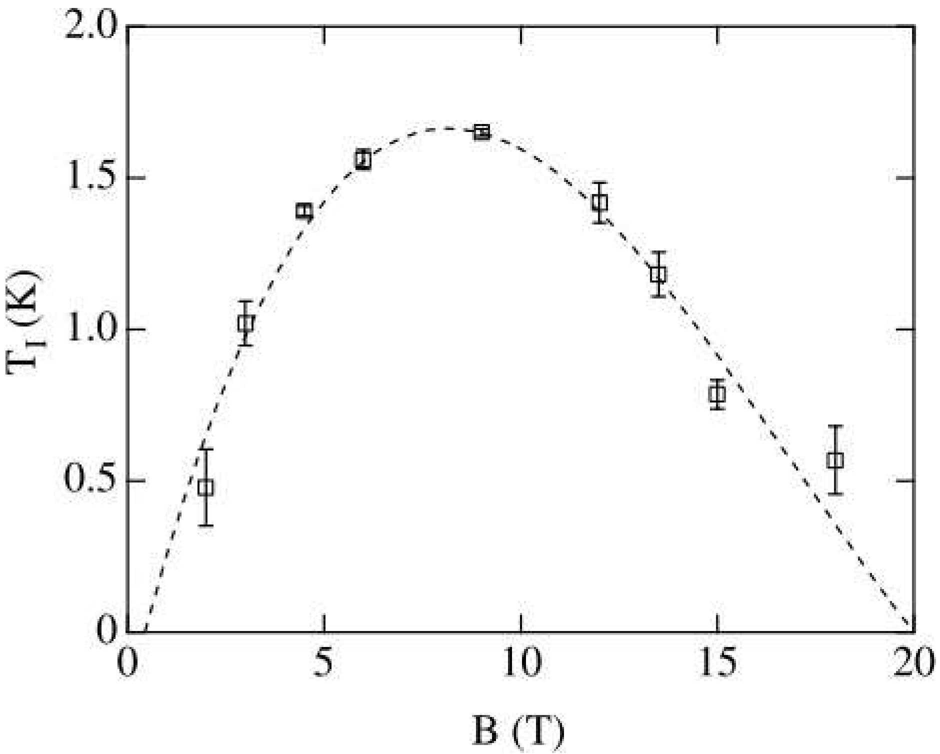


Normal current

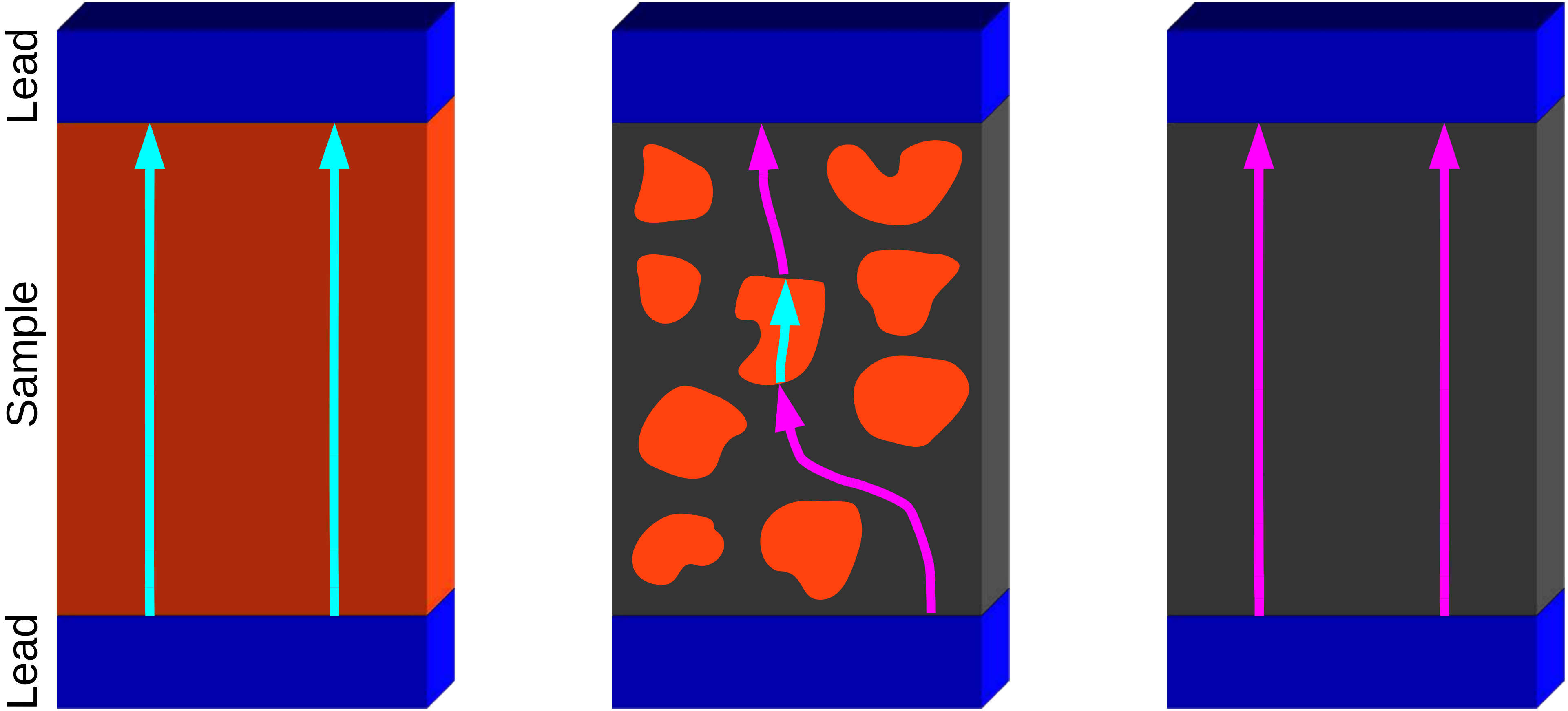
Clues: activated transport

- Activated transport $\rho = \rho_0 e^{T_I/T}$





Proposed mechanism



Sample entirely superconducting

Superconducting puddles have a charging energy and a tunneling barrier

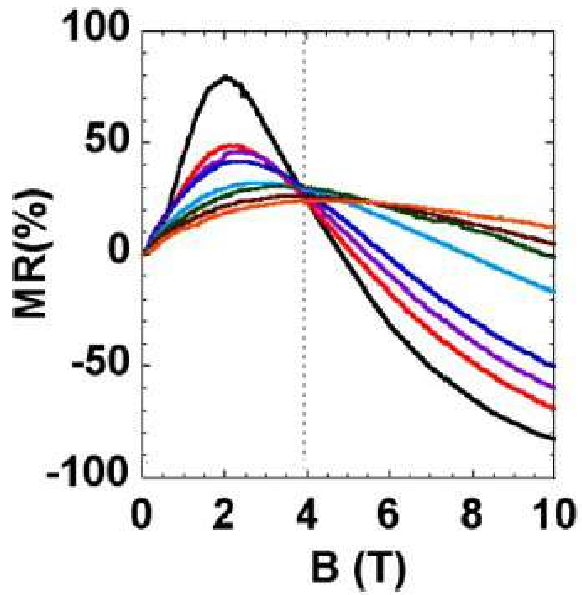
Sample entirely normal

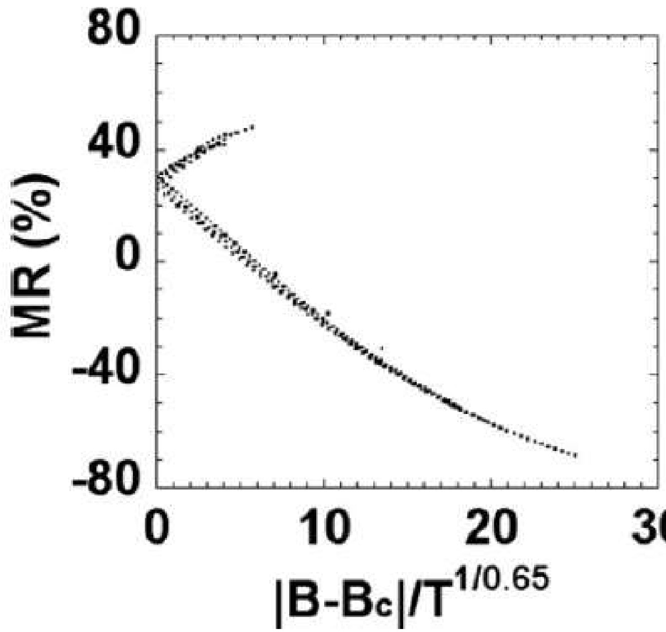
Highly disordered films

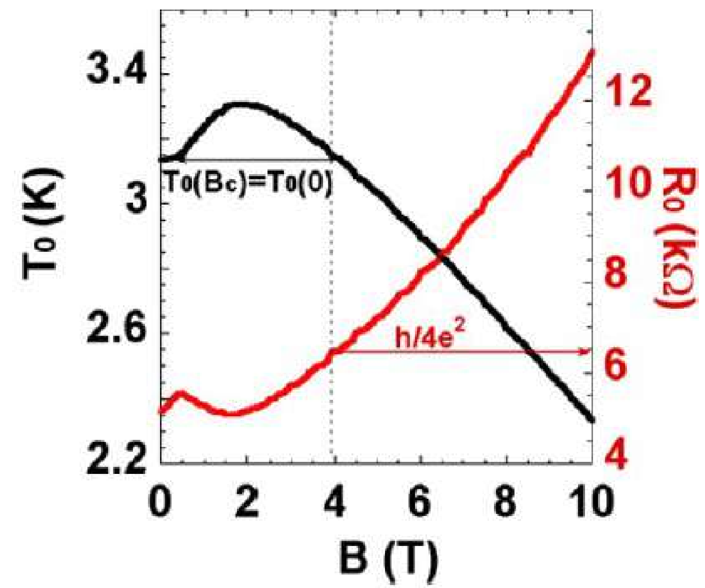
$$MR(B,T)=\frac{R(B,T)-R(0,T)}{R(0,T)}$$

$$R(B,T)=R_0(B)e^{T_A/T}$$

$$T_A(0)=T_A(B_C)$$







Lin & Goldman, PRL, (2011)

Highly disordered films

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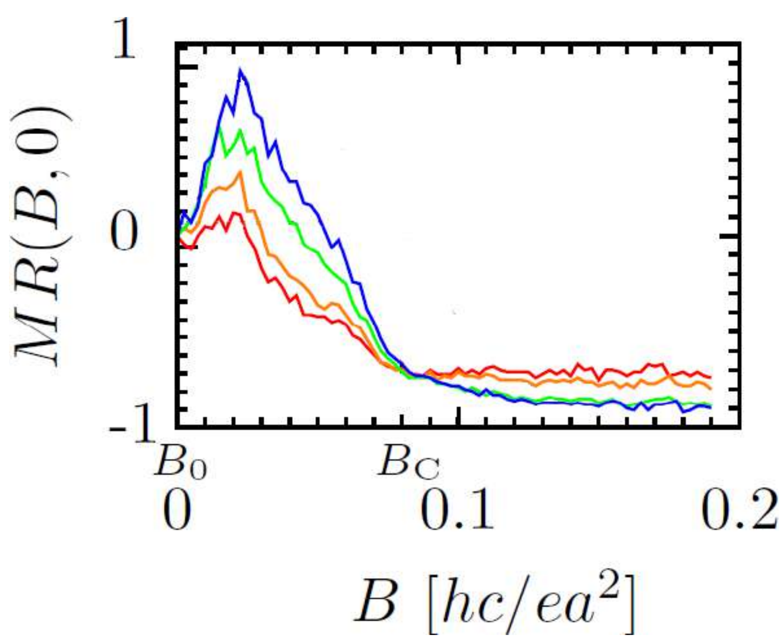
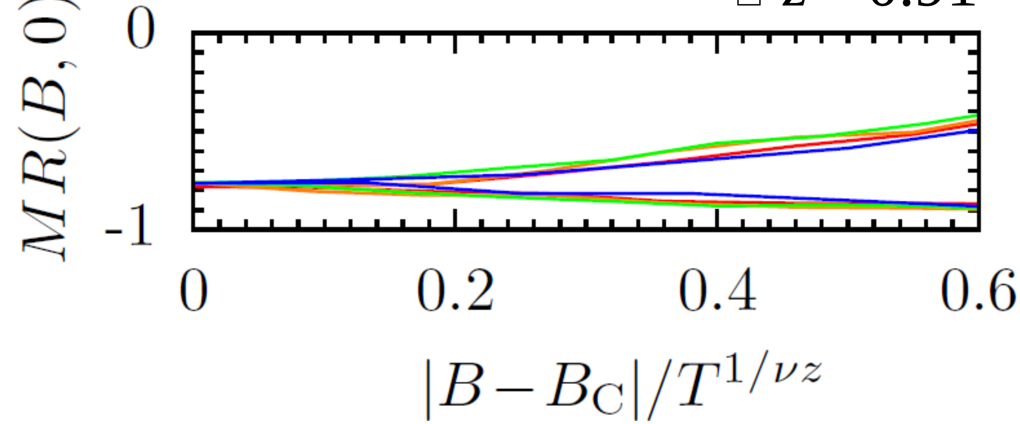


$\nu z = 0.91$



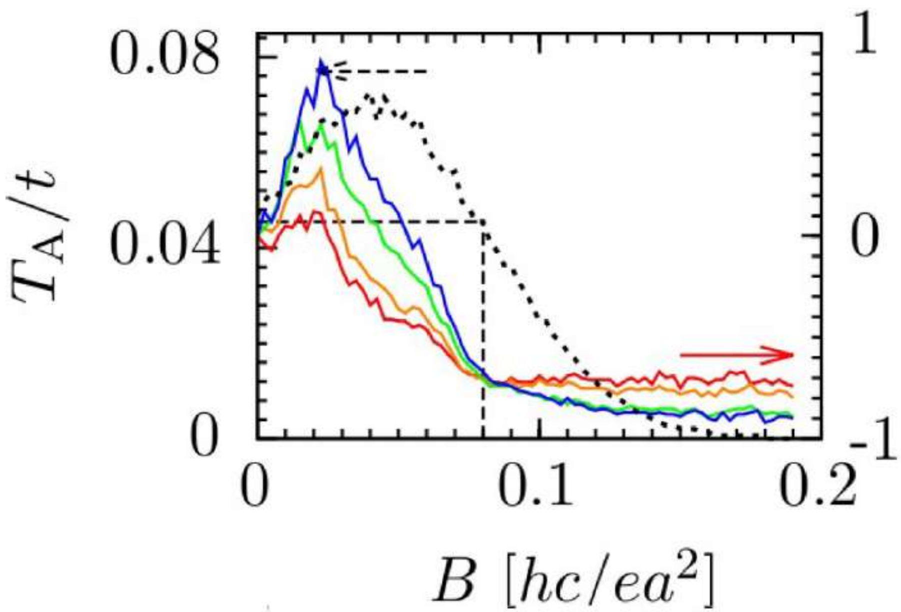
Highly disordered films

$$MR(B,T)=\frac{R(B,T)-R(0,T)}{R(0,T)}$$

$$MR(B,T)=\frac{R_0(B)}{R_0(0)}\left(1+\frac{T'_{\mathrm{A}}(B_{\mathrm{C}})(B-B_{\mathrm{C}})}{T}\right)-1$$

$$R(B,T)=R_0(B)\,\mathrm{e}^{T_{\mathrm{A}}/T}$$

$$T_{\mathrm{A}}(0)=T_{\mathrm{A}}(B_{\mathrm{C}})$$



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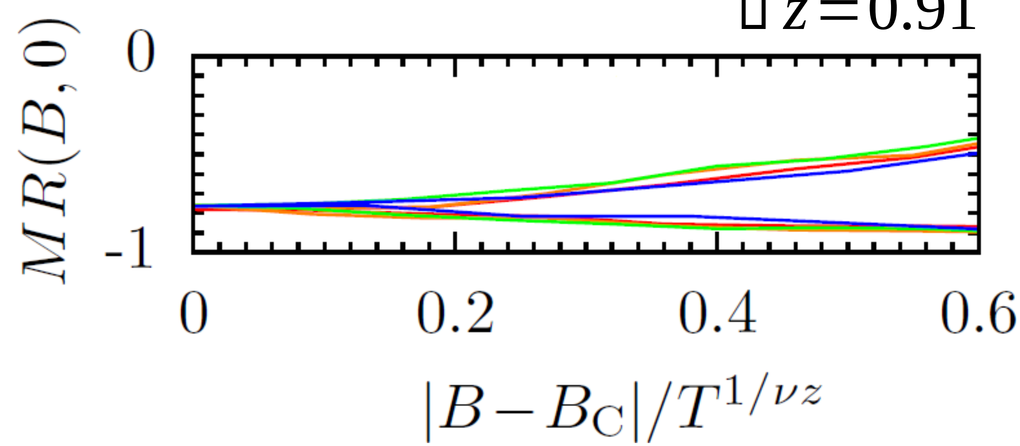

Highly disordered films

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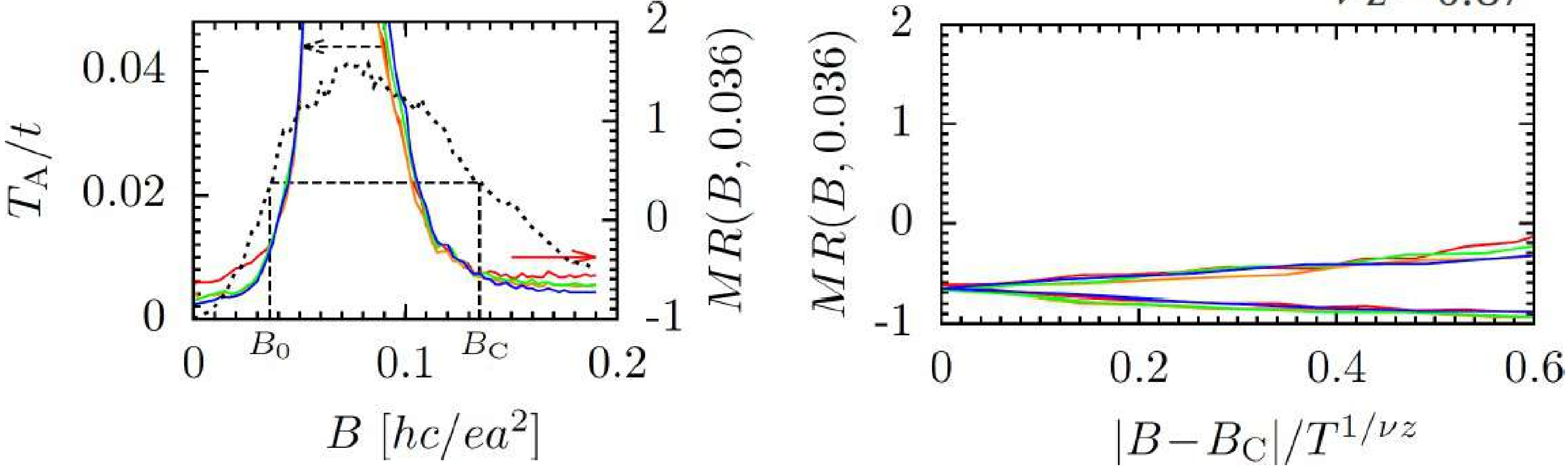
$$R(B,T)=R_0(B)\,\mathrm{e}^{T_A/T}$$

$$T_A(B_0)=T_A(B_C)$$

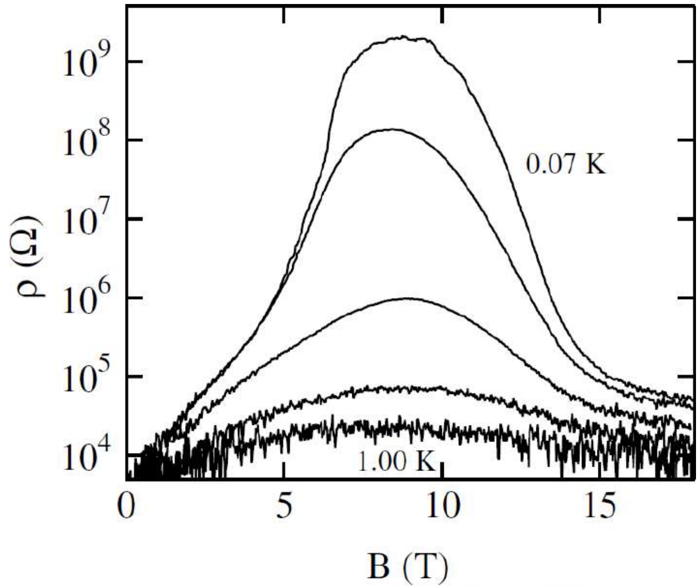
$$MR(B,T)=\frac{R_0(B)}{R_0(B_0)}\left(1+\frac{T'_A(B_C)(B-B_C)}{T}\right)-1$$

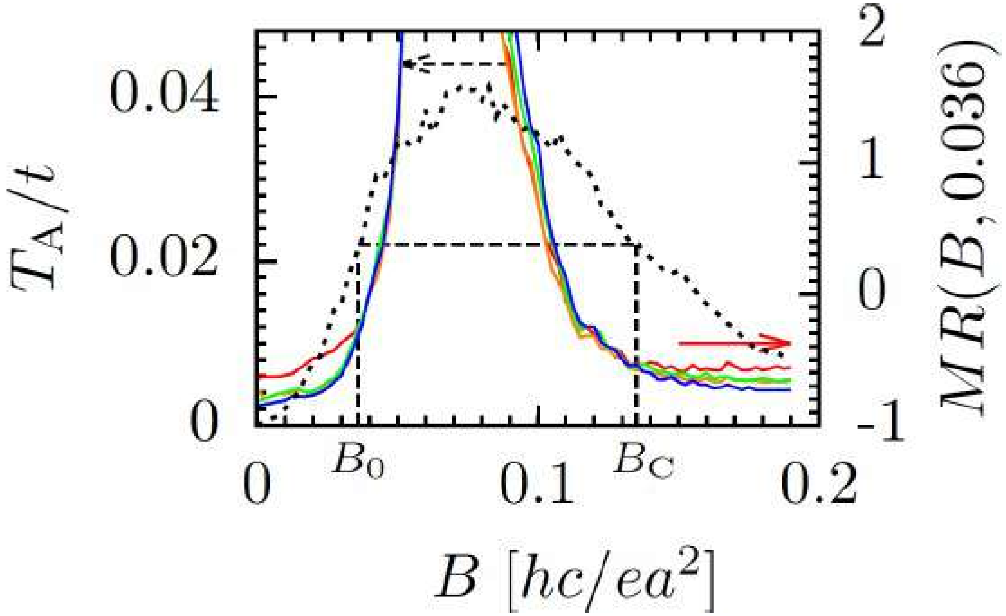
$\nu z = 0.87$

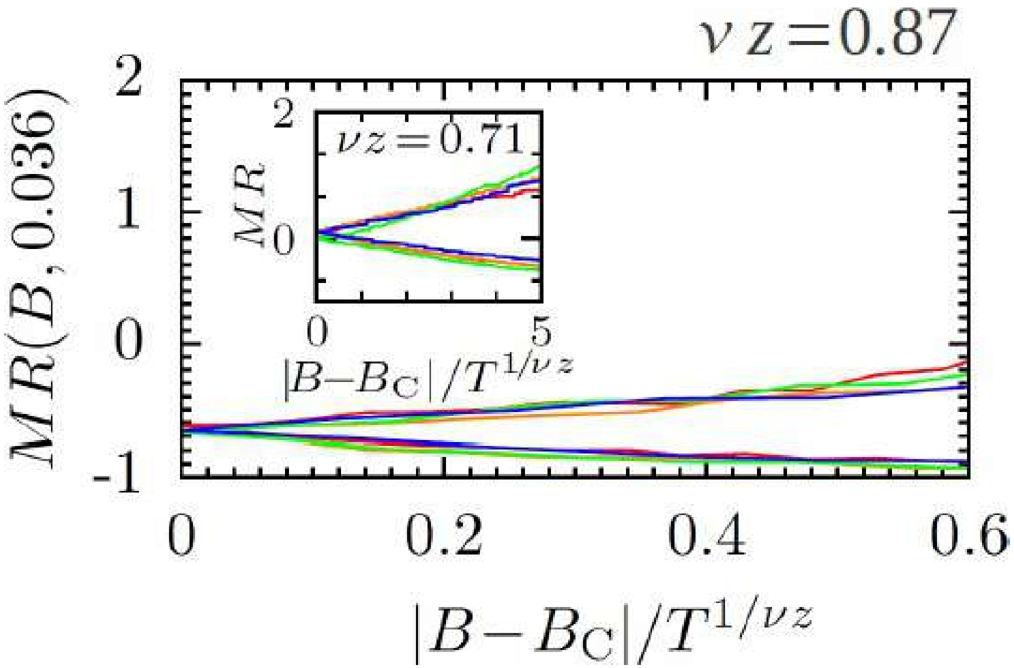


Highly disordered films

Sambandamurthy & Shahar, PRL 2004







Summary & future prospects

- Developed new formalism that includes thermal phase fluctuations to calculate and probe transport in superconductor

- Magnetoresistance peak may be driven by condensation of superconducting puddles

- Activated transport explains results of Goldman group on highly disordered superconductors

- Flexibility allows us to study wide range of unexplained effects