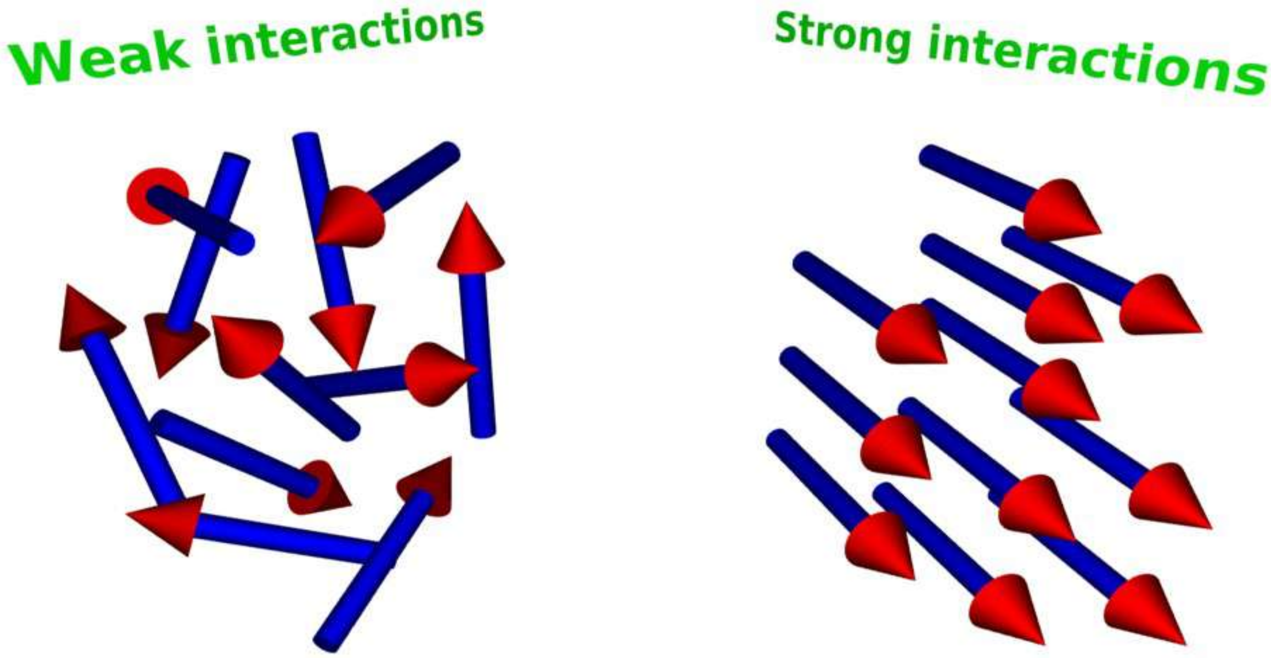
Perspectives on itinerant ferromagnetism in an atomic Fermi gas



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G.J. Conduit & B.D. Simons, Phys. Rev. A **79**, 053606 (2009)
G.J. Conduit, A.G. Green & B.D. Simons, Phys. Rev. Lett. **103**, 207201 (2009)
G.J. Conduit & B.D. Simons, Phys. Rev. Lett. **103**, 200403 (2009)
G.J Conduit & E. Altman, arXiv: 0911.2839

Strategy to analyze results

- Critically analyze the experimental results

1) Study atomic gas at equilibrium
2) Three-body loss damps quantum fluctuations and renormalizes interaction strength
3) Focus on new experimental protocol to minimize three-body loss

Free energy kernel

$$Z=\int D\psi \exp\left(-\iint d\tau\, d\boldsymbol{r} \sum_\sigma \bar{\psi}_\sigma (-i\hat{\omega}+\hat{\epsilon}-\mu)\psi_\sigma - g\,\bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow\right)$$

- Integrate over fluctuations in magnetization and density and develop a perturbation theory in interaction strength

$$F=F_0+\frac{1-g\nu}{2\nu}m^2+um^4+vm^6+g^2\left(r\,m^2+w\,m^4\ln|m|\right)$$

$$k_\mathrm{F}a_\mathrm{crit}=1.57$$

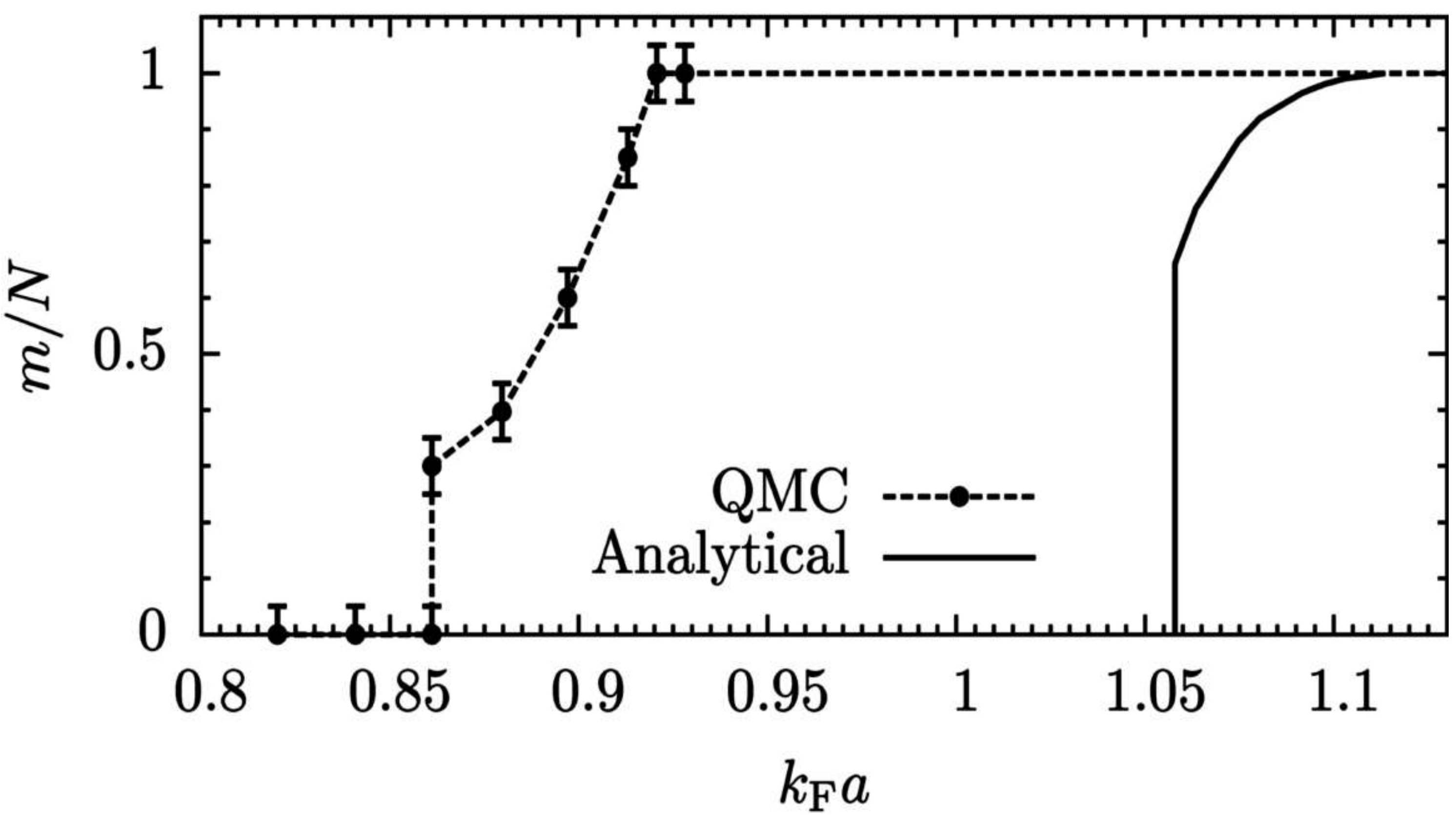
$$k_\mathrm{F}a_\mathrm{crit}=1.05$$

- First order transition[1] verified by *ab initio* Quantum Monte Carlo calculations[2]

[1]Abrikosov (1958), Duine & MacDonald (2005), Belitz *et al*. Z. Phys. B (1997) & Conduit & Simons (2009)
[2]Conduit & Simons, Phys. Rev. Lett. **103**, 207201 (2009)

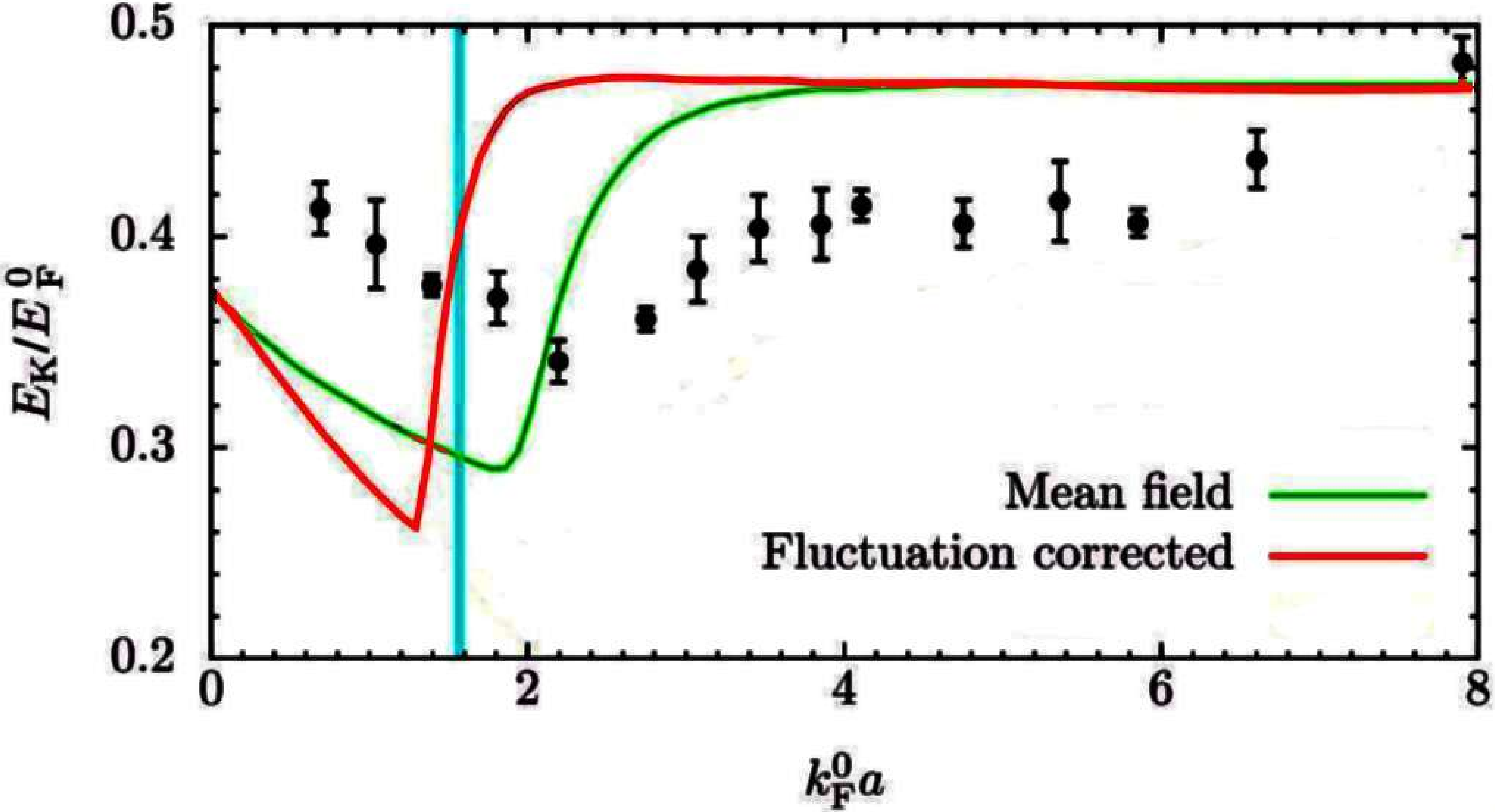
Quantum Monte Carlo verification

- Verified with Quantum Monte Carlo



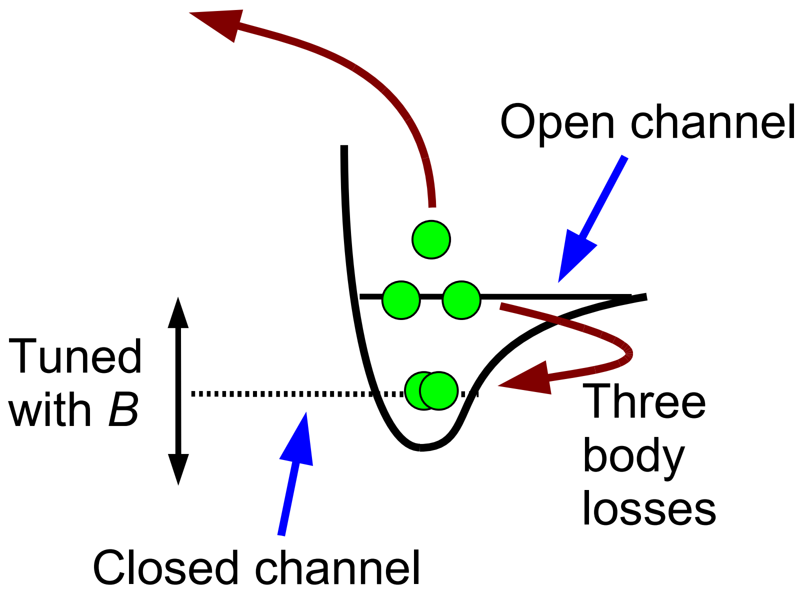
G.J. Conduit, A.G. Green & B.D. Simons, Phys. Rev. Lett. **103**, 207201 (2009)

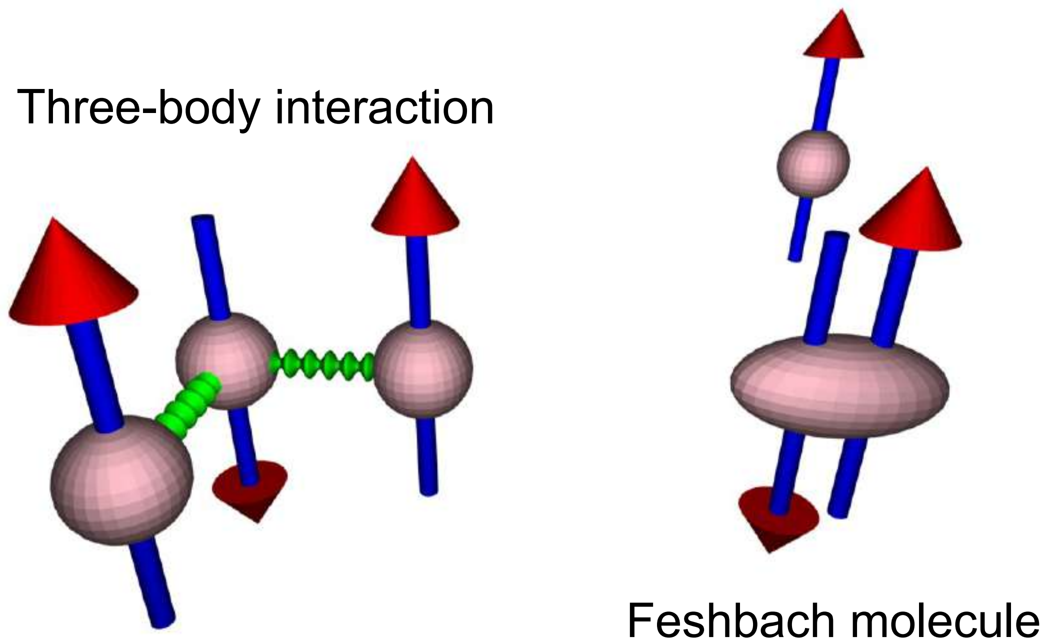
Theoretical prediction of the kinetic energy



Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

Three-body losses



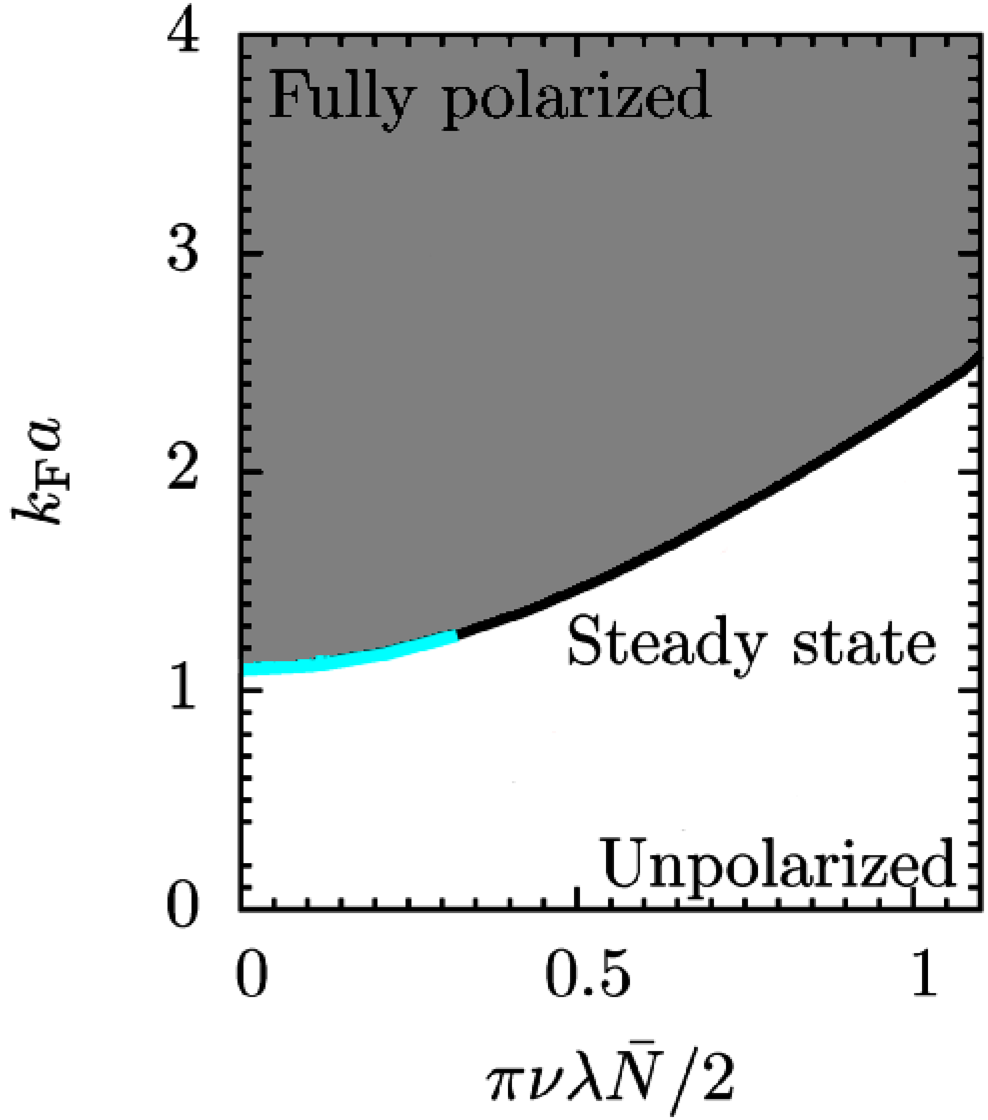


- In boson systems, three-body scattering drives the formation of a Tonks-Girardeau gas [Syassen *et al*., Science **320**, 1329 (2009)]
- Loss not only causes mean-field reduction in density but also damps quantum fluctuations

$$F = F_0 + \frac{1 - g\nu}{2\nu} m^2 + u m^4 + v m^6 + \left(g^2 - \lambda^2 \bar{N}^2\right)\left(r\, m^2 + w\, m^4 \ln|m|\right)$$

Phase boundary with atom loss

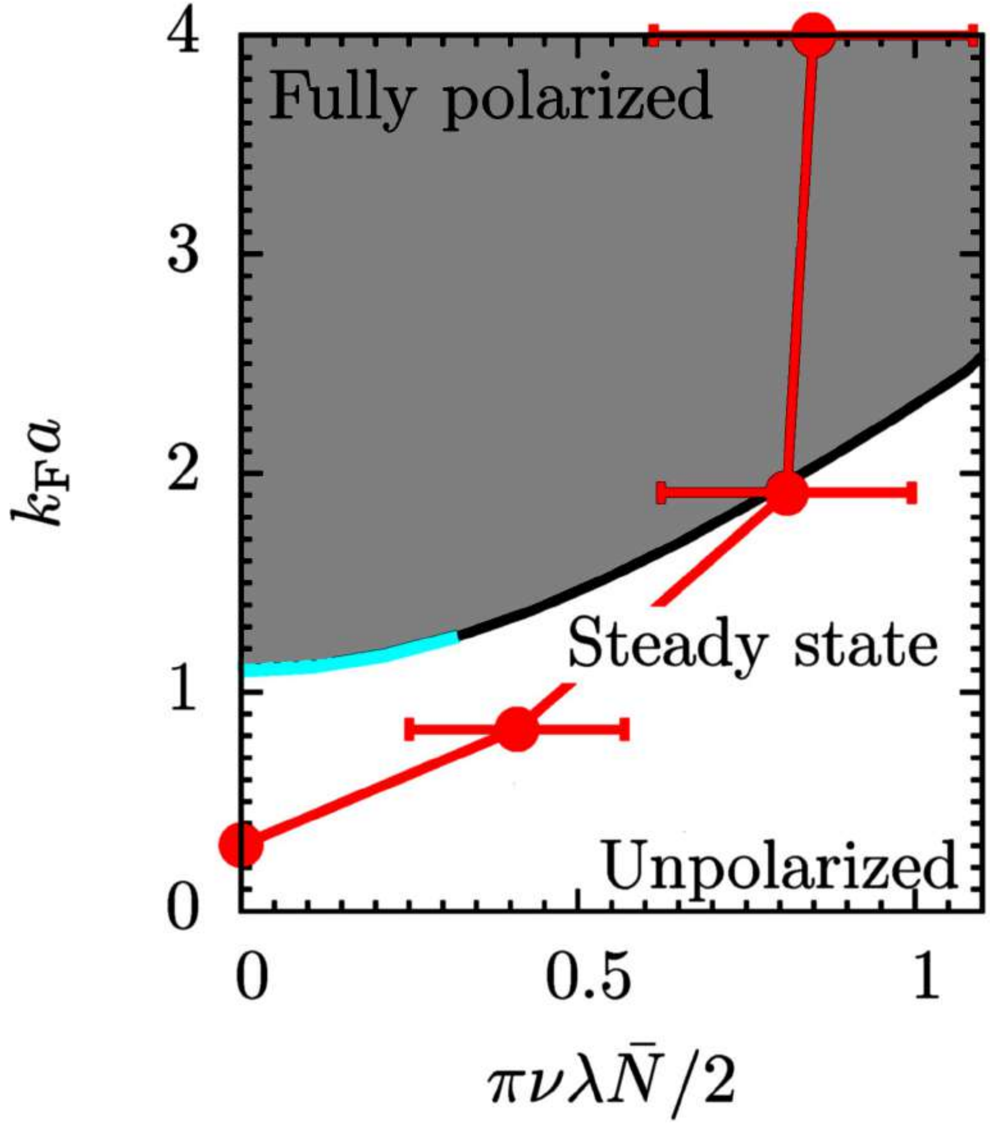
- Atom loss raises the interaction strength required for ferromagnetism



Conduit & Altman, arXiv: 0911.2839

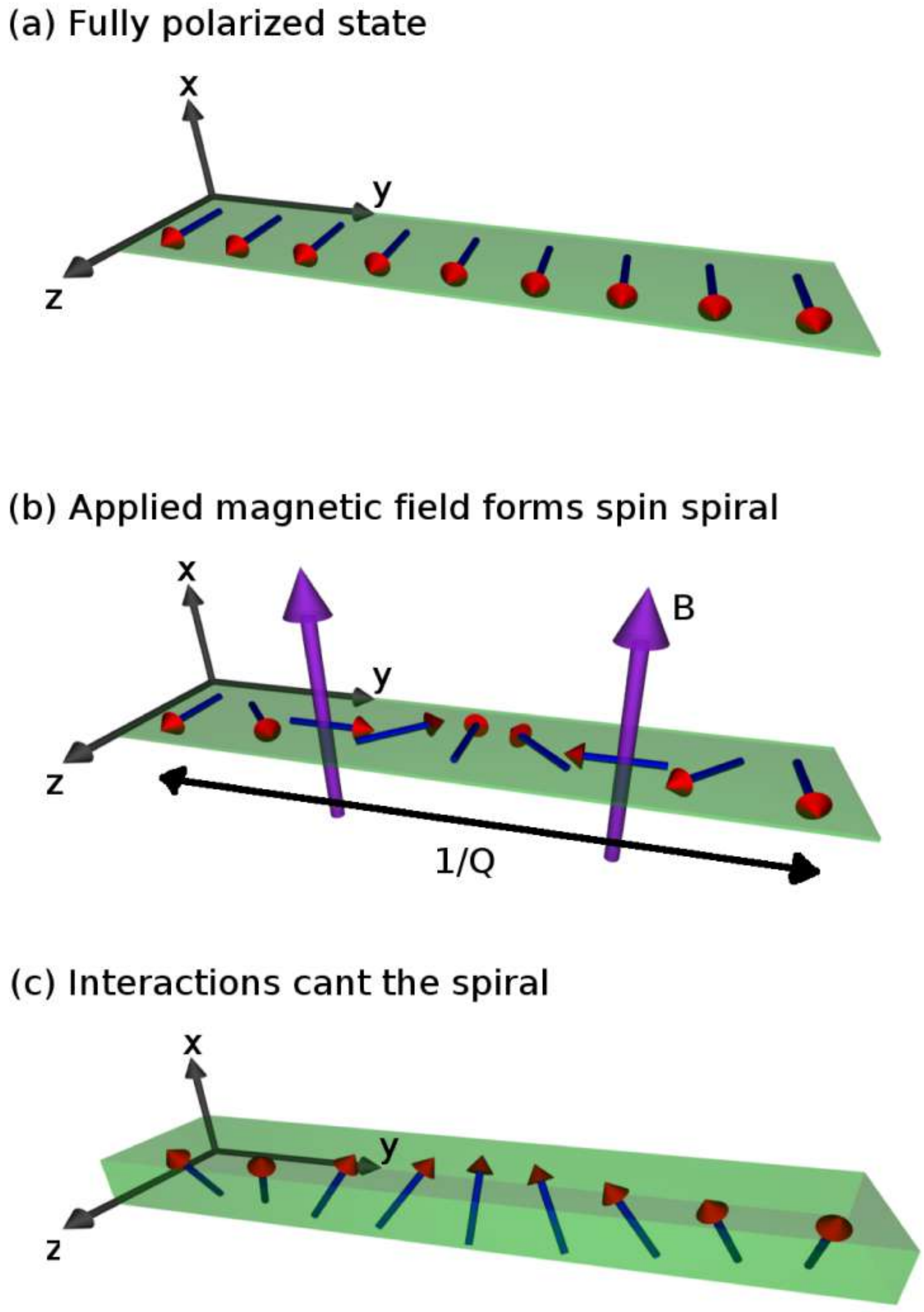
Interaction renormalization with atom loss

- Comparing to experimental atom loss indicates transition at $k_F a \approx 2$



Conduit & Altman, arXiv: 0911.2839; Huckans *et al*. PRL **102**, 165302 (2009)

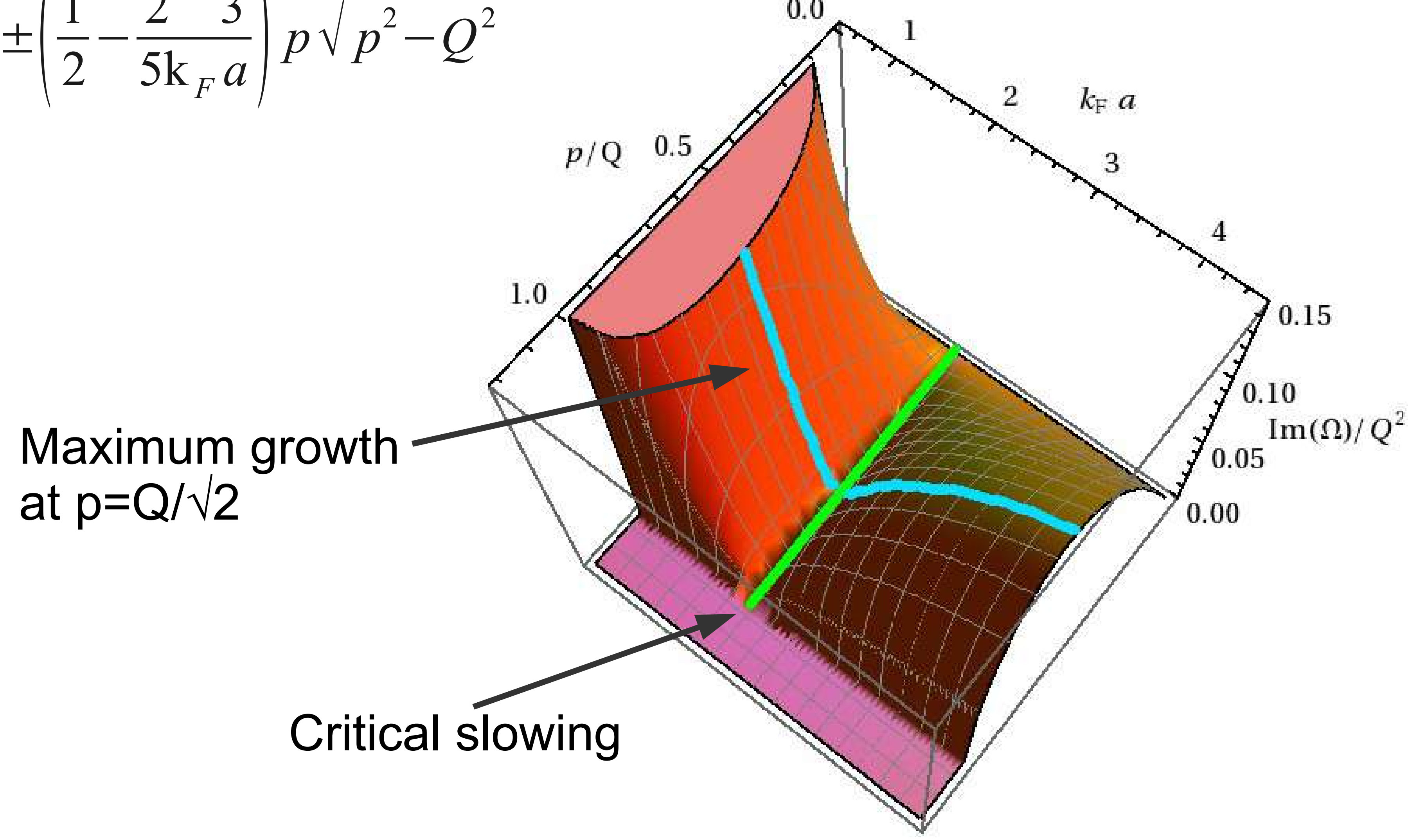
Alternative strategy: spin spiral



Spin spiral collective modes

- Exponentially growing collective modes if $p<Q$

$$\Omega(p)=\pm\left(\frac{1}{2}-\frac{2^{2/3}3}{5k_F a}\right)p\sqrt{p^2-Q^2}$$

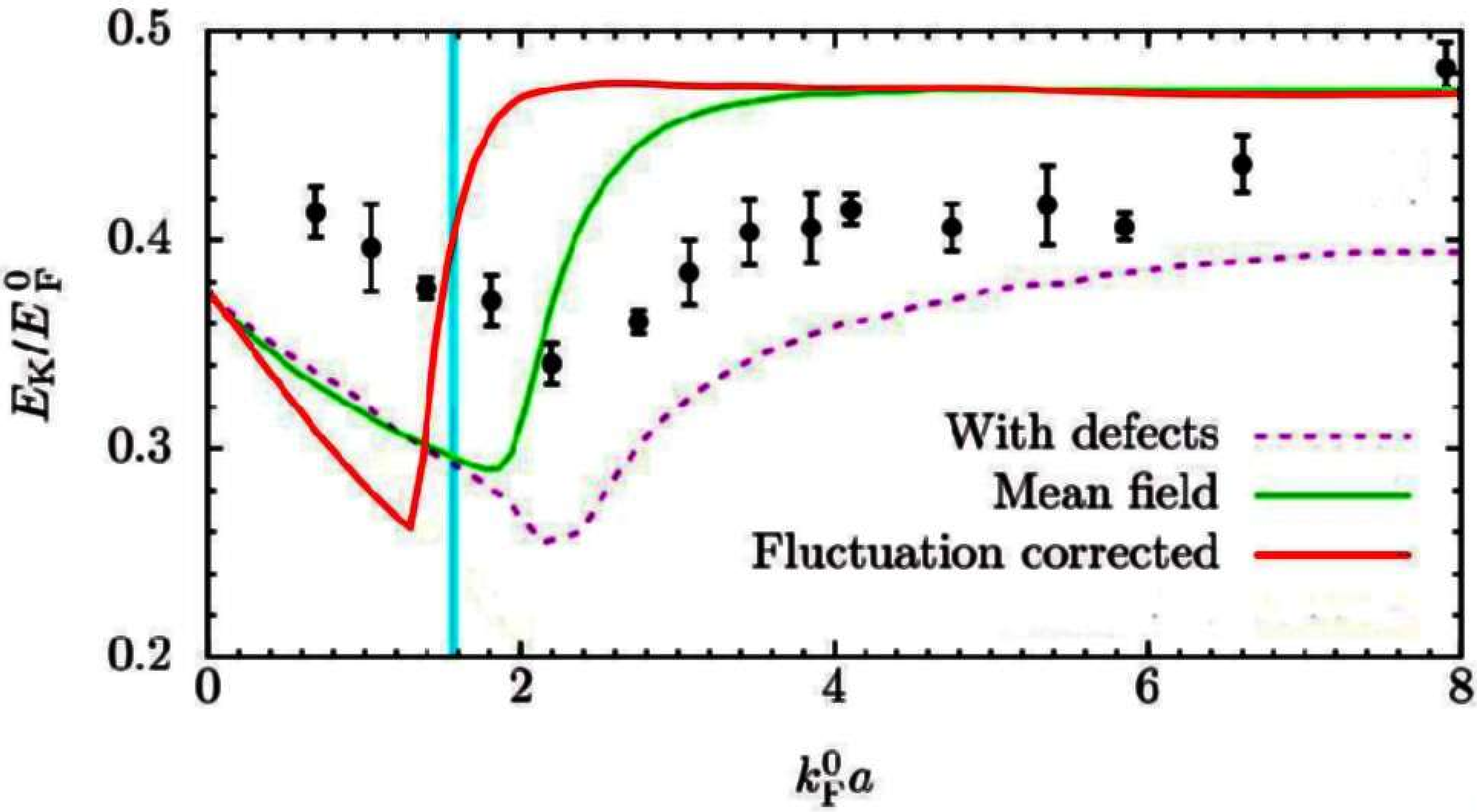


Summary

- Equilibrium theory provides a qualitative description of the ferromagnetic transition
- Three-body loss damps quantum fluctuations and renormalizes the interaction strength
- Atom loss can be avoided by starting the gas in a spin spiral state

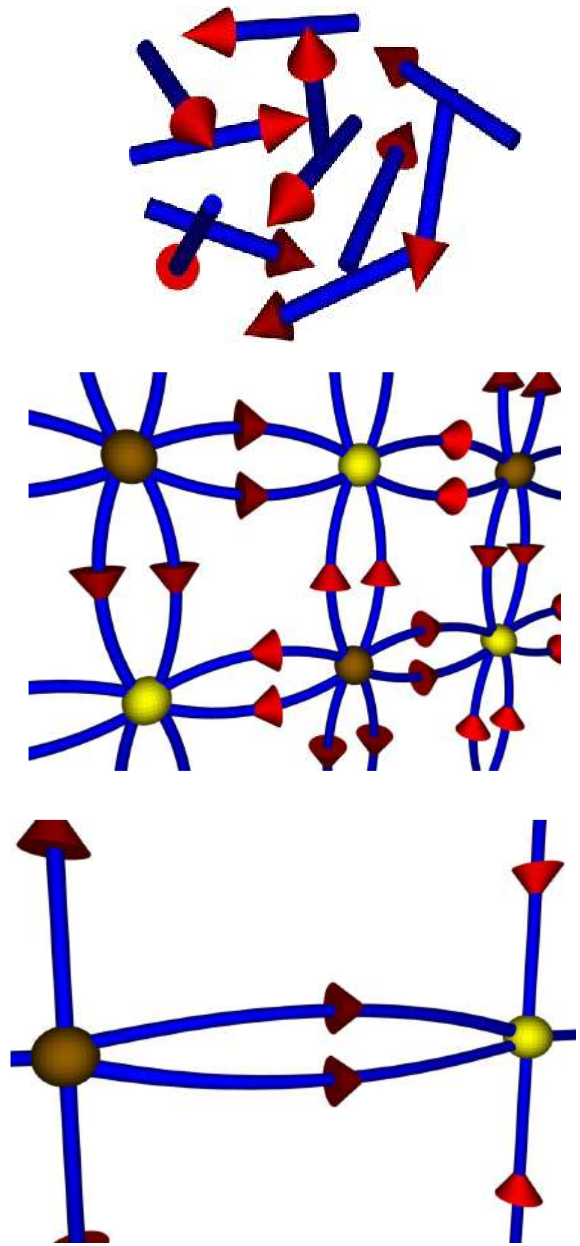
Consequences of defect annihilation

- Defect annihilation raises required interaction strength



Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009), Babadi *et al*. arXiv 0908.3482 (2009)

Condensation of topological defects



Ramp up interactions

Mutual annihilation of defects

- Defects freeze out from the paramagnetic state

- Defects grow as $L \sim t^{1/2}$
  [Bray, Adv. Phys. **43**, 357 (1994)]

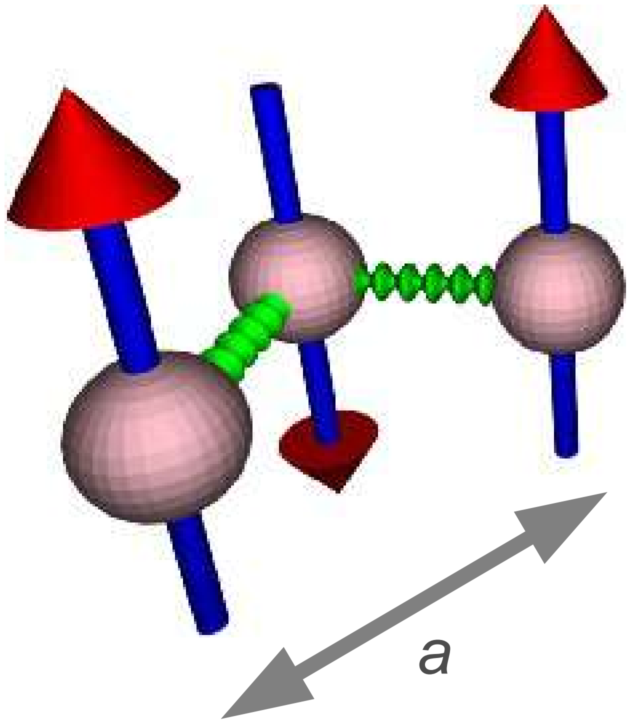
Damping of fluctuations by atom loss

- Atom loss rate $(k_F a)^6 n_\uparrow n_\downarrow (n_\uparrow + n_\downarrow)$ is

  $\lambda' \chi(\mathbf{r}-\mathbf{r}')[c_\uparrow^\dagger(\mathbf{r}')c_\uparrow(\mathbf{r}') + c_\downarrow^\dagger(\mathbf{r}')c_\downarrow(\mathbf{r}')]c_\uparrow^\dagger(\mathbf{r})c_\downarrow^\dagger(\mathbf{r})c_\downarrow(\mathbf{r})c_\uparrow(\mathbf{r})$

- A mean-field approximation, $\overline{N} = n_\uparrow(\mathbf{r}') + n_\downarrow(\mathbf{r}')$ places loss on same footing as interactions

  $S_{\text{int}} = (g + i\lambda\overline{N})c_\uparrow^\dagger(\mathbf{r})c_\downarrow^\dagger(\mathbf{r})c_\downarrow(\mathbf{r})c_\uparrow(\mathbf{r})$

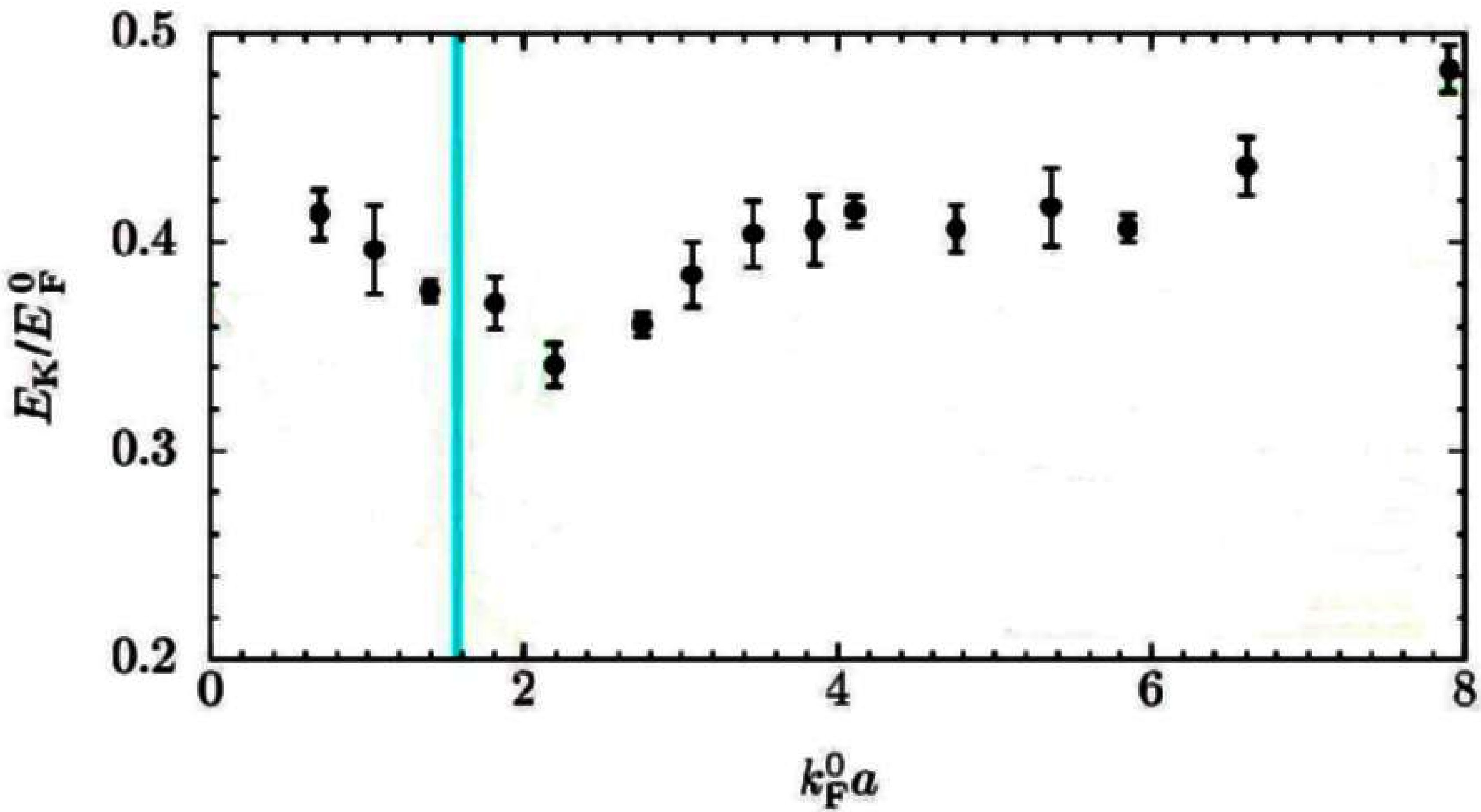


- Loss damps fluctuations so inhibits the transition

$$F = F_0 + \frac{1-g\nu}{2\nu} m^2 + u m^4 + v m^6 + \left(g^2 - \lambda^2 \overline{N}^2\right)\left(r m^2 + w m^4 \ln|m|\right)$$

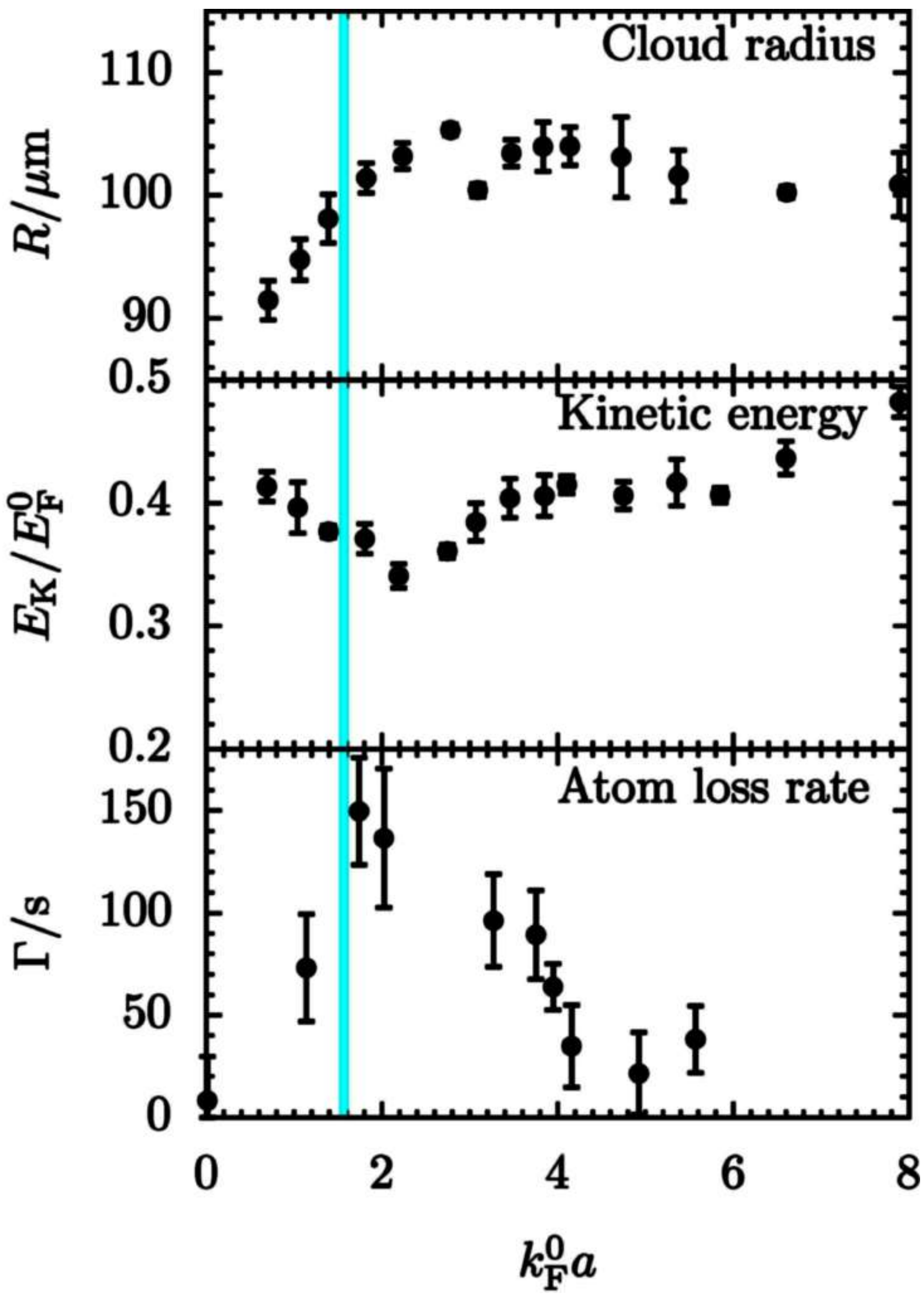
Experimental evidence for ferromagnetism

- Rise in kinetic energy at $k_F a \approx 2.2$



Jo, Lee, Choi, Christensen, Kim, Thywissen, Pritchard & Ketterle, Science **325**, 1521 (2009)

Further key experimental signatures



$E_K \propto n^{5/3}$

$\Gamma \propto (k_F a)^6 n_\uparrow n_\downarrow (n_\uparrow + n_\downarrow)$

Jo, Lee, Choi, Christensen, Kim, Thywissen, Pritchard & Ketterle, Science **325**, 1521 (2009)

Damping of fluctuations by atom loss

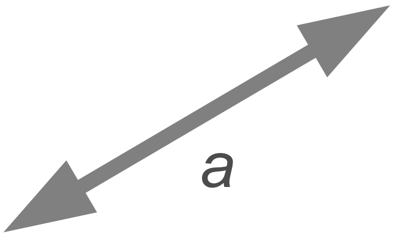
- Atom loss rate is

  $\lambda'\chi(\mathbf{r}-\mathbf{r}')[c_\uparrow^\dagger(\mathbf{r}')c_\uparrow(\mathbf{r}') + c_\downarrow^\dagger(\mathbf{r}')c_\downarrow(\mathbf{r}')]c_\uparrow^\dagger(\mathbf{r})c_\downarrow^\dagger(\mathbf{r})c_\downarrow(\mathbf{r})c_\uparrow(\mathbf{r})$

- A mean-field approximation, $\overline{N} = n_\uparrow(\mathbf{r}') + n_\downarrow(\mathbf{r}')$ places loss on same footing as interactions

  $S_{\mathrm{int}} = (g + i\lambda\overline{N})c_\uparrow^\dagger(\mathbf{r})c_\downarrow^\dagger(\mathbf{r})c_\downarrow(\mathbf{r})c_\uparrow(\mathbf{r})$

- Also include atom source $-i\gamma c_\sigma^\dagger c_\sigma$ to ensure gas remains at equilibrium



- Loss damps fluctuations so inhibits the transition

$$F = F_0 + \frac{1-g\,v}{2v} m^2 + um^4 + vm^6 + \left(g^2 - \lambda^2 \overline{N}^2\right)\left(r\,m^2 + w\,m^4 \ln|m|\right)$$

Equilibrium study of ferromagnetism

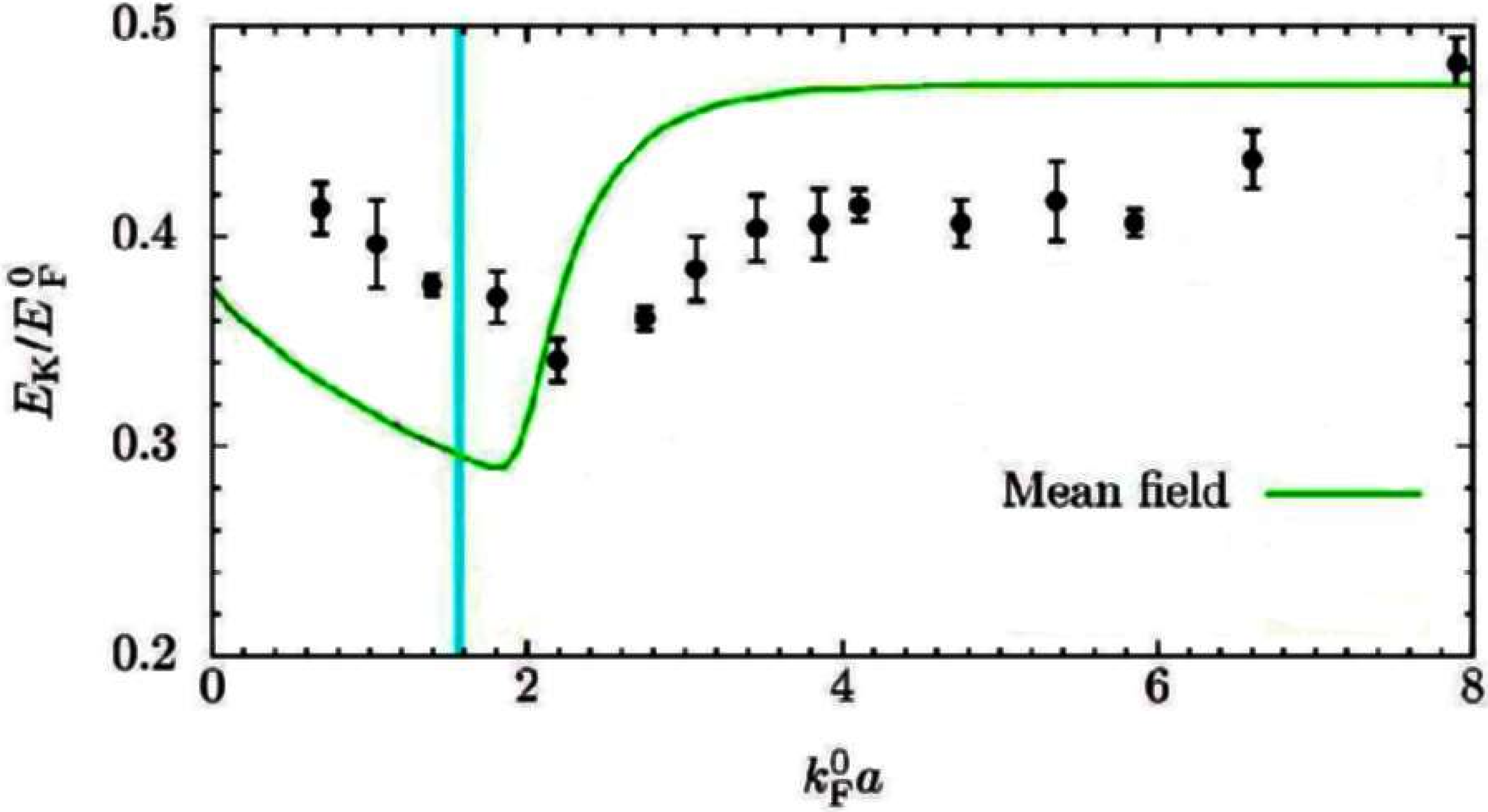
$$Z=\int D\psi \exp\left(-\iint d\tau\, d\boldsymbol{r} \sum_{\sigma} \bar{\psi}_{\sigma}(-i\hat{\omega}+\hat{\epsilon}-\mu)\psi_{\sigma}-g\,\bar{\psi}_{\uparrow}\bar{\psi}_{\downarrow}\psi_{\downarrow}\psi_{\uparrow}\right)$$

- Decouple with the average magnetisation *m* gives the Stoner criterion

$$F=F_0+\frac{1-g\nu}{2\nu}m^2+um^4+vm^6$$

Mean-field analysis & consequences of trap

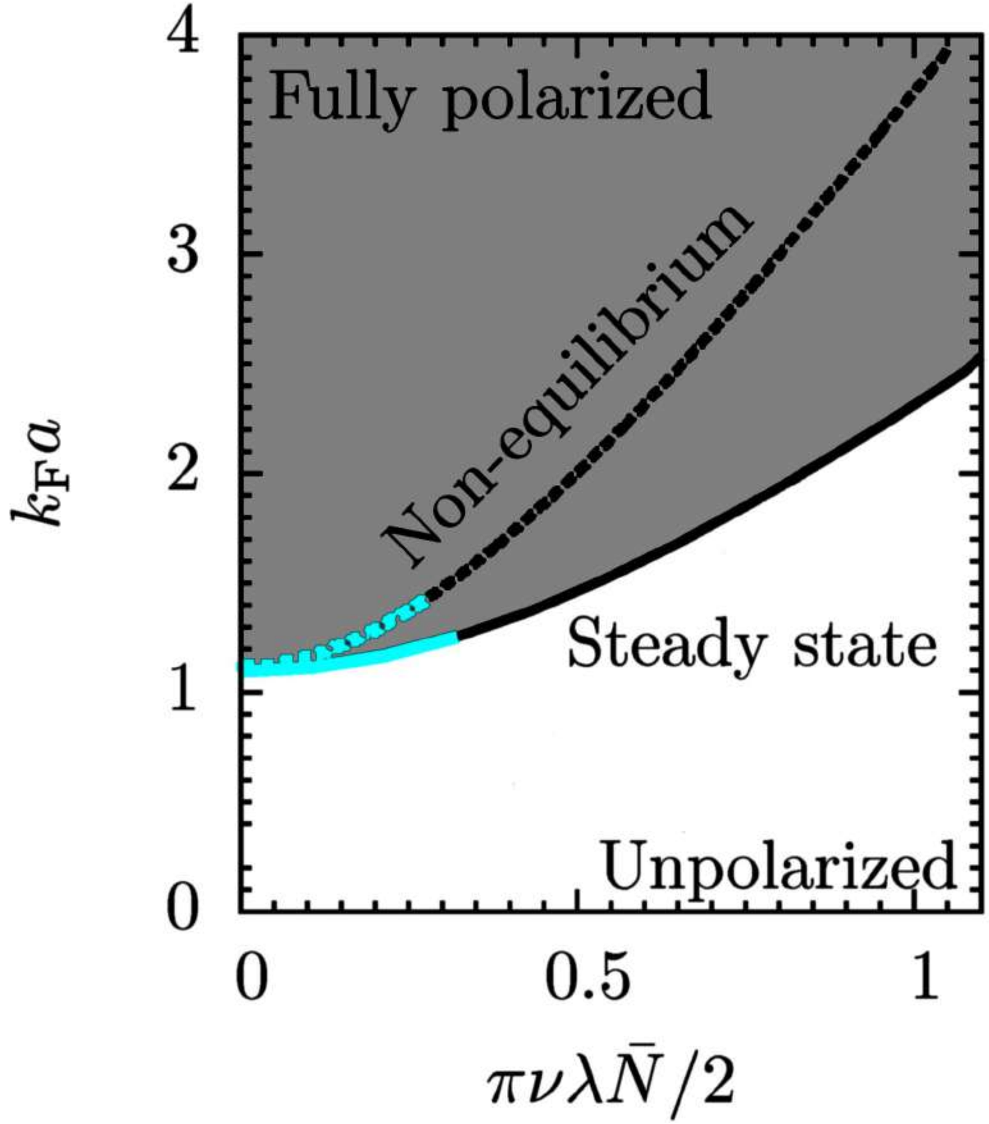
- Recovers qualitative behavior[1] but transition at $k_F a=1.8$ instead of $k_F a=2.2$



[1]LeBlanc, Thywissen, Burkov & Paramekanti, Phys. Rev. A **80**, 013607 (2009) & Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

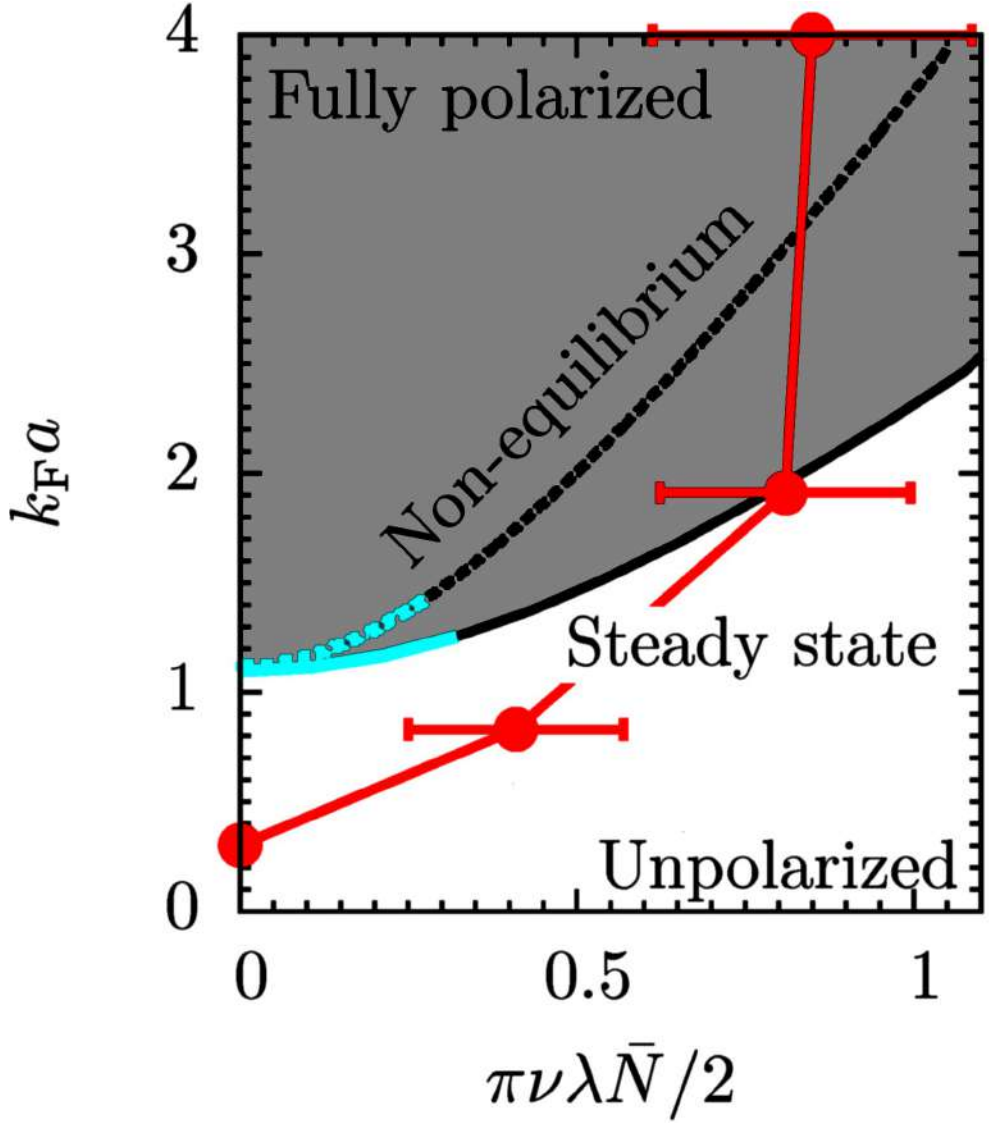
Phase boundary with atom loss

- Atom loss raises the interaction strength required for ferromagnetism



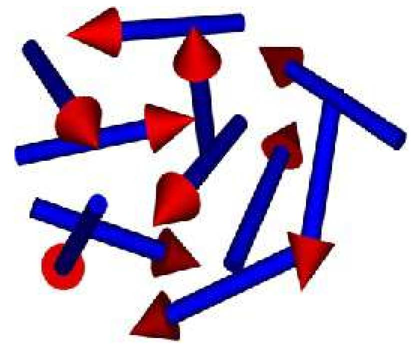
Conduit & Altman, arXiv: 0911.2839

Interaction renormalization with atom loss

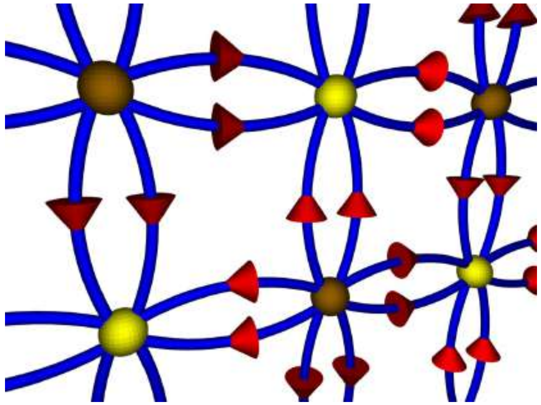


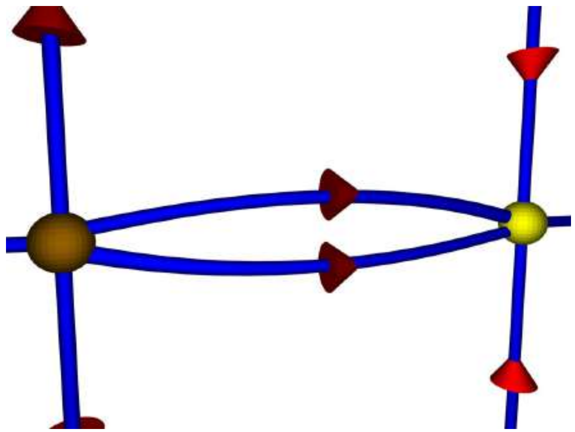
Conduit & Altman, arXiv: 0911.2839; Huckans *et al*. PRL **102**, 165302 (2009)

Condensation of topological defects



Start in paramagnetic state and ramp up interaction strength

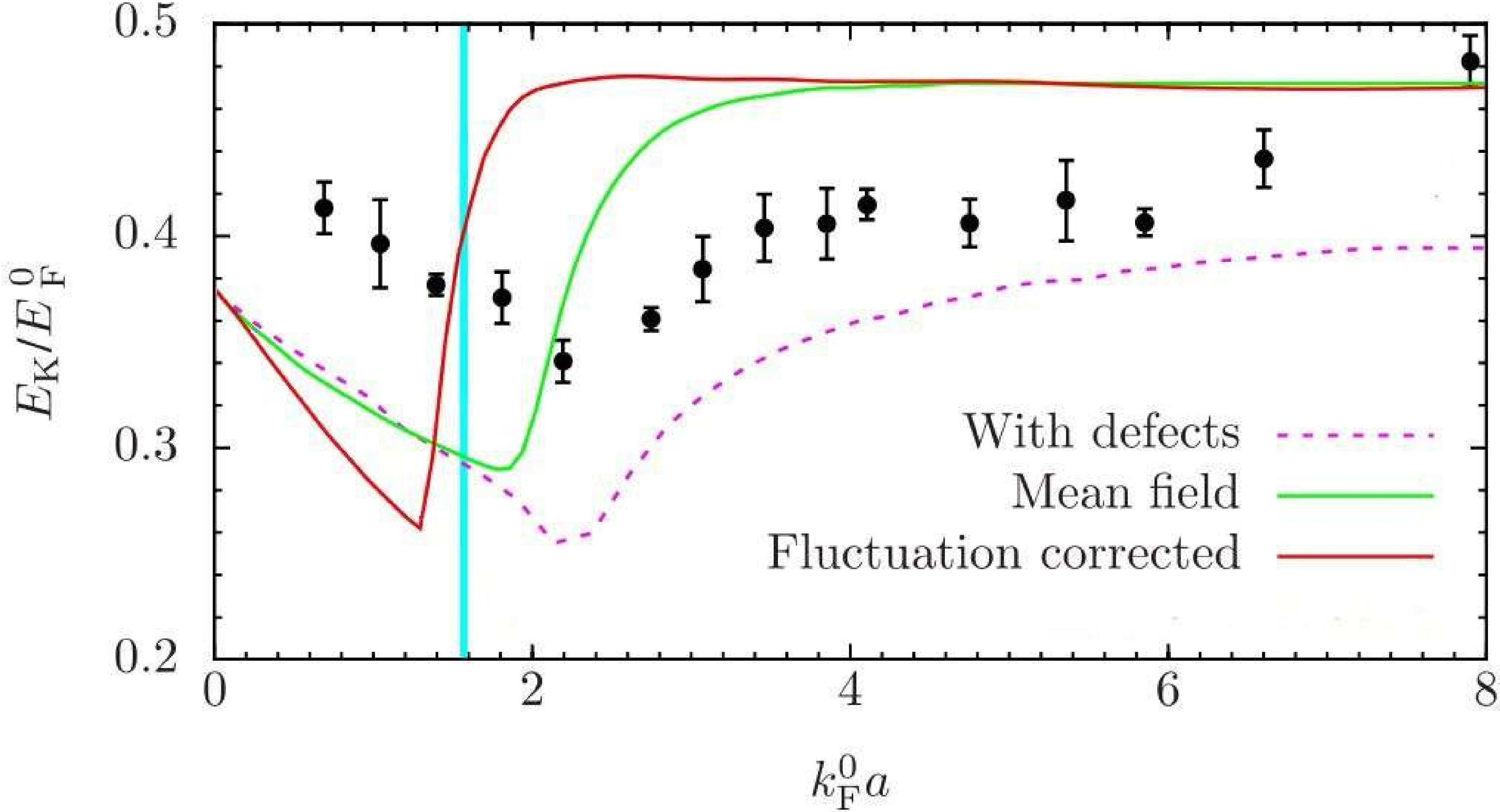




- Defects freeze out from disordered state
- Defect annihilation hinders the formation of the ferromagnetic phase thus raising the required interaction strength
- Defect radius $L \sim t^{1/2}$ [Bray, Adv. Phys. **43**, 357 (1994)]

Condensation of topological defects

- Condensation of defects inhibits the transition



Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

First order phase transition and Quantum Monte Carlo verification

- First order transition into uniform phase with TCP

- QMC also sees first order transition

Summary of equilibrium results

Momentum distribution

New approach to fluctuation corrections

$$Z=\int D\psi \exp\left(-\int \sum_{\sigma} \bar{\psi}_{\sigma}(-i\omega+\epsilon-\mu)\psi_{\sigma}-g\int \bar{\psi}_{\uparrow}\bar{\psi}_{\downarrow}\psi_{\downarrow}\psi_{\uparrow}\right)$$

- Analytic strategy:

  1) Decouple in both the density and spin channels (previous approaches employ only spin)

  2) Integrate out electrons

  3) Expand about uniform magnetisation

  4) Expand density and magnetisation fluctuations to second order

  5) Integrate out density and magnetisation fluctuations
- Aim to uncover connection to second order perturbation theory
- Unravel origin of logarithmic divergence in energy and relation to tricritical point structure

Analytical method

- System free energy $F=-k_B T \ln Z$ is found via the partition function

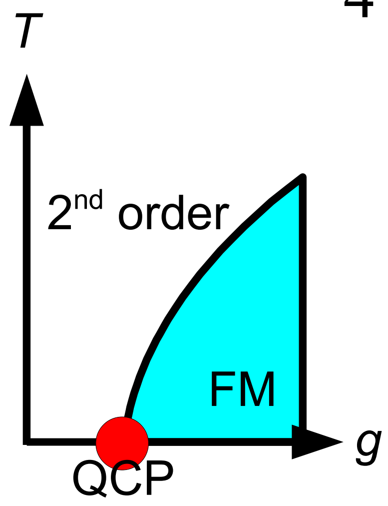
$$Z=\int D\psi \exp\left(-\int \sum_{\sigma} \bar{\psi}_{\sigma}(-i\omega+\epsilon-\mu)\psi_{\sigma}-g\int \bar{\psi}_{\uparrow}\bar{\psi}_{\downarrow}\psi_{\downarrow}\psi_{\uparrow}\right)$$

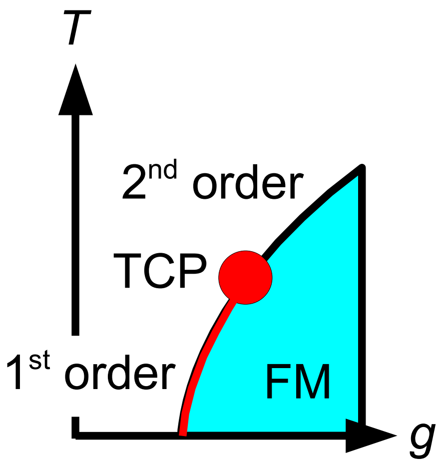
- Decouple using only the average magnetisation $m=\bar{\psi}_{\downarrow}\psi_{\uparrow}-\bar{\psi}_{\uparrow}\psi_{\downarrow}$

  gives $F\propto(1-g\nu)m^2$ i.e. the Stoner criterion

- Belitz-Kirkpatrick-Vojta (soft particle-hole & magnetisation) [Belitz *et al.* Z. Phys. B 1997]

$$F=\frac{1}{2}\left(|\omega|/\Gamma_q+r+q^2\right)m^2+\frac{u}{4}m^4\ln(m^2+T^2)+\cdots-hm$$





Quantum Monte Carlo verification

- First order transition into uniform phase with TCP

- QMC also sees first order transition

Cold atomic gases — spin

- Two fermionic atom species have a *pseudo-spin*:

^{6}Li	m_F=1/2	maps to	spin 1/2
^{6}Li	m_F=-1/2	maps to	spin -1/2

- The up-and down spin particles *cannot* interchange — population imbalance is fixed. Possible spin states are:

$\|\uparrow\uparrow\rangle$	S=1, S_z=1	State not possible as S_z has changed
$\|\downarrow\downarrow\rangle$	S=1, S_z=-1	State not possible as S_z has changed
$(\|\uparrow\downarrow\rangle+\|\downarrow\uparrow\rangle)/\sqrt{2}$	S=1, S_z=0	Magnetic moment in plane
$(\|\uparrow\downarrow\rangle-\|\downarrow\uparrow\rangle)/\sqrt{2}$	S=0, S_z=0	Non-magnetic state

- Ferromagnetism, if favourable, must form in-plane

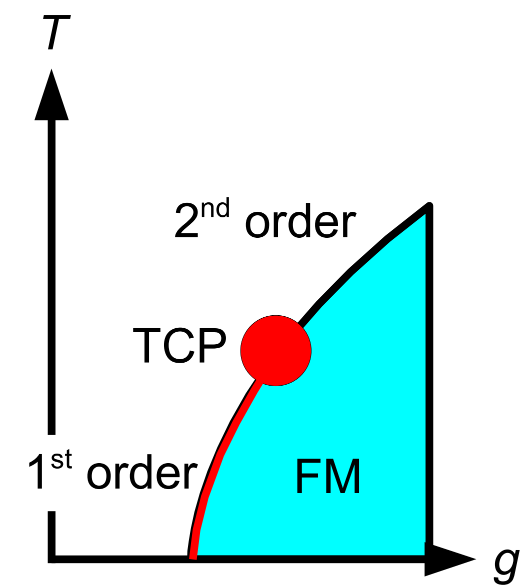
Particle-hole perspective

- To second order in g the free energy is

$$F=\sum_{\sigma,\boldsymbol{k}}\epsilon_{\boldsymbol{k}}^{\sigma}n(\epsilon_{\boldsymbol{k}}^{\sigma})+gN^{\uparrow}N^{\downarrow}$$
$$-\frac{2g^2}{V^3}\sum_{\boldsymbol{p}}\int\int\frac{\rho^{\uparrow}(\boldsymbol{p},\epsilon_{\uparrow})\rho^{\downarrow}(-\boldsymbol{p},\epsilon_{\downarrow})}{\epsilon_{\uparrow}+\epsilon_{\downarrow}}d\epsilon_{\uparrow}d\epsilon_{\downarrow}$$
$$+\frac{2g^2}{V^3}\sum_{\boldsymbol{k}_{1,2,3,4}}\frac{n(\epsilon_{\boldsymbol{k}_1}^{\uparrow})n(\epsilon_{\boldsymbol{k}_2}^{\downarrow})}{\epsilon_{\boldsymbol{k}_1}^{\uparrow}+\epsilon_{\boldsymbol{k}_2}^{\downarrow}-\epsilon_{\boldsymbol{k}_3}^{\uparrow}-\epsilon_{\boldsymbol{k}_4}^{\downarrow}}\delta(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}_3-\boldsymbol{k}_3)$$

with $\epsilon_{\boldsymbol{k}}^{\sigma}=\epsilon_{\boldsymbol{k}}+\sigma gm$ and a particle-hole density of states

$$\rho^{\sigma}(\boldsymbol{p},\epsilon)=\sum_{\boldsymbol{k}}n(\epsilon_{\boldsymbol{k}+\boldsymbol{p}/2}^{\sigma})\left[1-n(\epsilon_{\boldsymbol{k}-\boldsymbol{p}/2}^{\sigma})\right]\delta\left(\epsilon-\epsilon_{\boldsymbol{k}+\boldsymbol{p}/2}^{\sigma}+\epsilon_{\boldsymbol{k}-\boldsymbol{p}/2}^{\sigma}\right)$$



- Enhanced particle-hole phase space at zero magnetisation drives transition first order
- Recover $m^4\ln m^2$ at T=0
- Links quantum fluctuation to second order perturbation approach[1]

[1]Abrikosov 1958 & Duine & MacDonald 2005

Quantum Monte Carlo: Textured phase

- Textured phase preempted transition with $q=0.2k_F$

T=0

Modified collective modes

- Collective mode dispersion

- Collective mode damping