# Electron-Electron Revisited

$$n(\underline{r},t) = \left\langle \psi \left| e^{i\int_{-\infty}^{t}(\hat{H}+\hat{H}_e)dt'} n(\underline{r}) e^{-i\int_{-\infty}^{t}(\hat{H}+\hat{H}_e)dt'} \right| \psi \right\rangle$$

$$= \left\langle \psi \left| e^{i\int_{-\infty}^{t}\hat{H}_e dt'} \underbrace{e^{i\int_{-\infty}^{t} H dt'} n(\underline{r}) e^{-i\int_{-\infty}^{t} H dt'}}_{n(\underline{r},t)} e^{-i\int_{-\infty}^{t}\hat{H}_e dt'} \right| \psi \right\rangle$$

Interaction representation

Explore linear response by expanding the exponential to first order

$$= \left\langle \psi \left| \left(1 + i\int_{-\infty}^{t}\hat{H}_e dt'\right) n(\underline{r},t) \left(1 - i\int_{-\infty}^{t}\hat{H}_e dt'\right) \right| \psi \right\rangle = n_0 + n_{ind}$$

$$n_{ind} = i\int_{-\infty}^{t} dt' \left\langle \psi \left| \left[ H_e(t'), n(r,t) \right] \right| \psi \right\rangle$$

But $\hat{H}_e = \int dr' \, U(r',t')\hat{n}(r't')$ so

$$= i\int_{-\infty}^{t} dt' \int dr' \underbrace{\left\langle \psi \left| \left[ \hat{n}(r't'), \hat{n}(rt) \right] \right| \psi \right\rangle}_{\Pi(r-r', t-t')} U(r't')$$

$$n_{ind}(q\omega) = \Pi(q\omega)U(q\omega)$$

$$\frac{1}{\varepsilon} = 1 + \frac{n_{ind}}{n_{ext}}$$

$$= 1 + \underbrace{\frac{4\pi e^2}{q^2}\Pi(q,\omega)}_{\text{Screened polarizability}} \longrightarrow \frac{1}{\varepsilon} = 1 + U\Pi$$

$$\varepsilon = 1 - U\Pi$$

The Lindhard function

for up, down spin

Fermi-Dirac function

$$\Pi(q,\omega) = -2\sum_{\underline{k}} \frac{f(\underline{k}+\underline{q}) - f(\underline{k})}{\varepsilon_{\underline{k}+\underline{q}} - \varepsilon_{\underline{k}} - \omega}$$

Spectrum $\frac{k^2}{2}$, $\mu$ cancels

$$\Pi(q,0) = -2\sum_{k} \frac{f(k+q) - f(k)}{\epsilon_{k+q} - \epsilon_{k} - i\eta}$$

Static screening limit

$$\approx -2\sum_{k} \frac{q\cdot\partial_k f(\epsilon_k)}{q\cdot\partial_k \epsilon_k}$$

$$\approx -2\int \frac{d^3k}{(2\pi)^3}\, \partial_{\epsilon_k} f(\epsilon_k)$$

$$= -2\int d\epsilon\, \nu(\epsilon)\, \partial_\epsilon f(\epsilon)$$

$$= 2\nu(\mu)$$

Density of states at Fermi surface

$$\Pi(q,\omega) = -2\sum_{k} \frac{f(k+q) - f(k)}{\epsilon_{k+q} - \epsilon_k - \omega}$$

High frequency response

$$\simeq -2\sum_{k} \frac{q\cdot\partial_k f(\epsilon_k)}{q\cdot\partial_k \epsilon_k - \omega}$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{2}{i\omega}\left(1 + \frac{q\cdot k}{m\omega}\right) q\cdot\partial_k f(\epsilon_k)$$

$$= \int \frac{\partial^3 k}{(2\pi)^3} \frac{2}{m\omega^2}(q\cdot k)\, q\cdot\partial_k f(\epsilon_k)$$

$$= \cdots \int \frac{d^3k}{(2\pi)^3} \frac{2q^2}{m\omega^2} f(\epsilon_k)$$

$$= \cdots \frac{1}{(2\pi)^3} \frac{4}{3}\pi k_F^3 \frac{2q^2}{m\omega^2}$$

$$= \frac{nq^2}{m\omega^2}$$

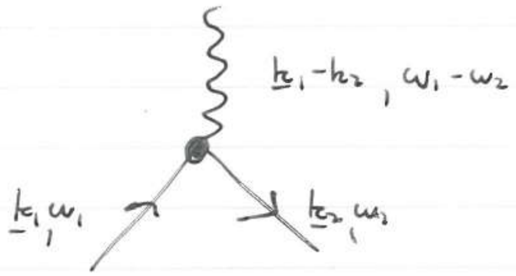
→ plasmons

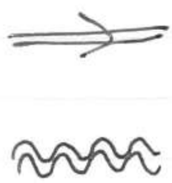
- Tool to book-keep and sum over different contributions to electron propagation:

Electrons :  Carry momentum $\underline{k}$, energy $\omega$

Photons :  Momentum and energy exchange

Vertex at interaction; strength $U$



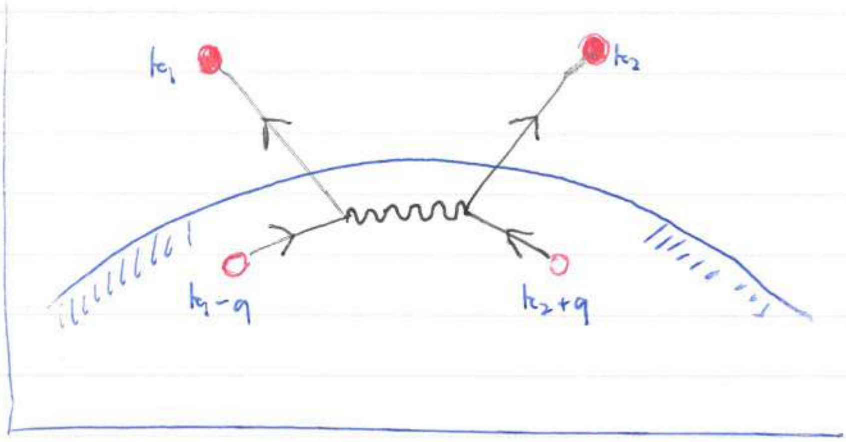



Bold diagrams already summed over

External lines $\longrightarrow$ momentum / energy in/out of system

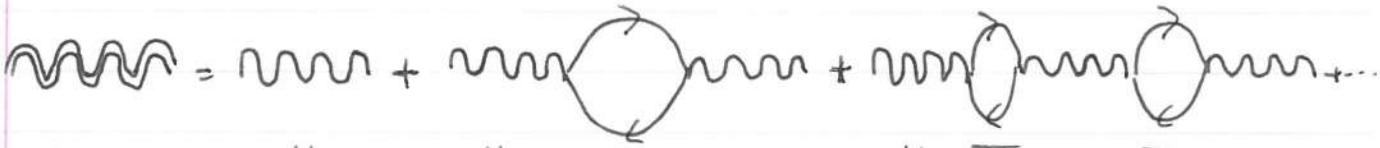
Internal lines $\longrightarrow$ momentum / energy integrated over

$$\sum V(q)\, a^{\dagger}_{k_1} a^{\dagger}_{k_2} a_{k_2+q} a_{k_1-q}$$

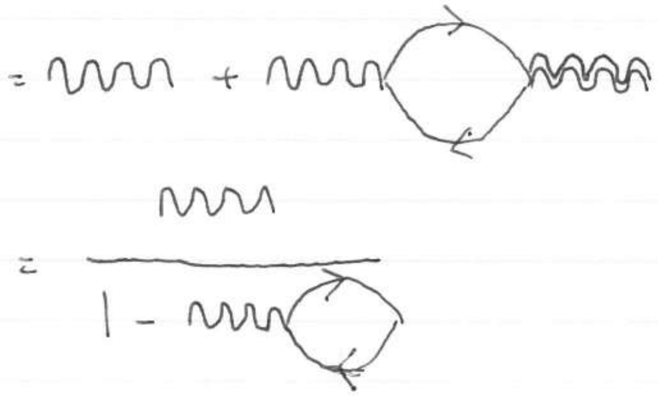



$$n_{ext}\frac{1}{\varepsilon} = n_{ext} + n_{ind}$$

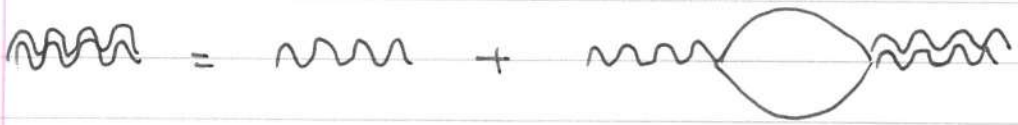
$$U_{eff} = \frac{1}{\varepsilon}U = \frac{U}{1-U\Pi} = U + U\Pi U + \dots$$



$$U_{eff} = U_0 + U_0 \times \Pi \quad U_0 + U_0 \times \Pi \times U_0 \times \Pi \times U_0$$



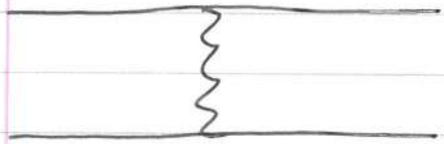
# 1 Particle propagator



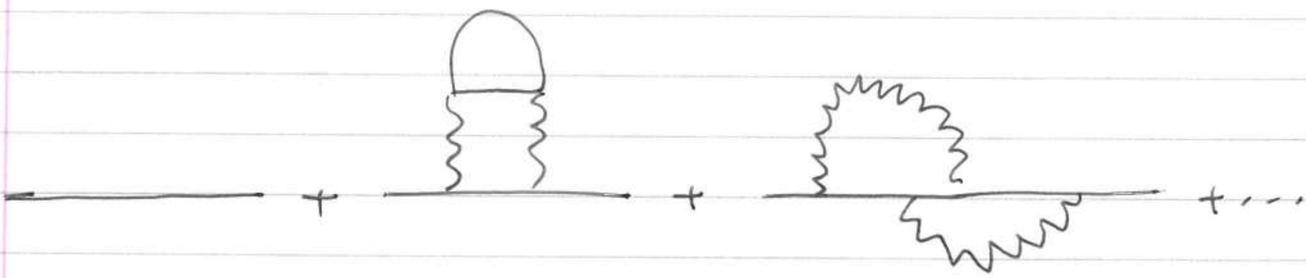
Photon

Electron

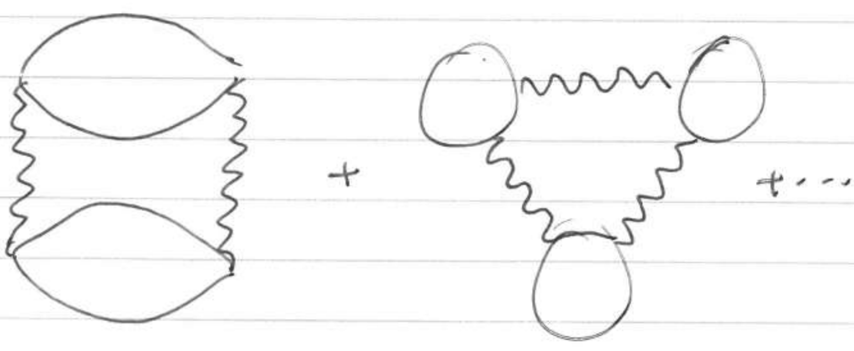
## Effective interaction / polarizability



## One particle eg $n(k)$



## Energy eg uniform electron gas



From Electron Correlations and Material Properties on google books