UNIVERSITY OF CAMBRIDGE

Magnetism in materials
Lecture 5

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Course B: Materials for Devices

▶ Professor M does Science

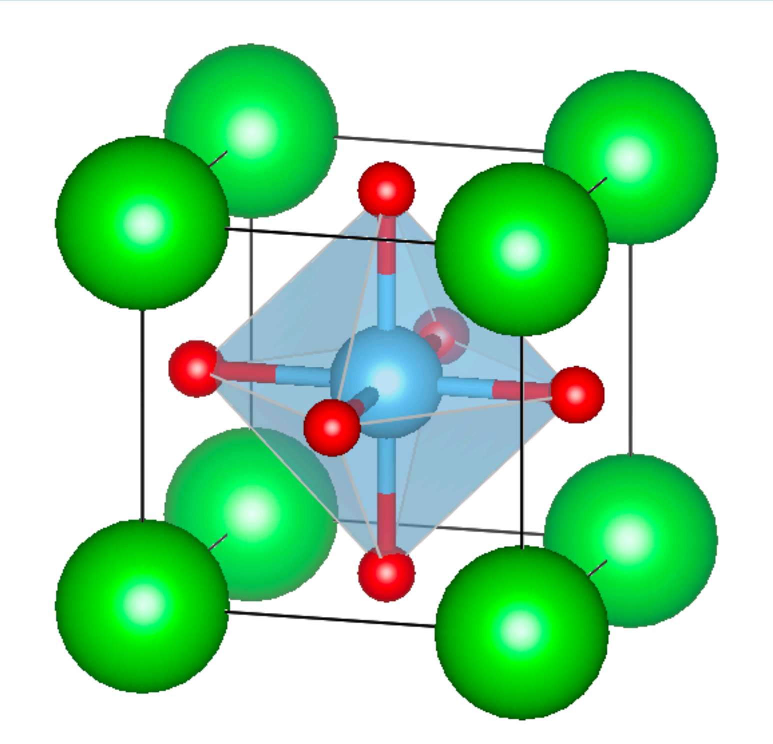
🌐 http://www.tcm.phy.cam.ac.uk/~bm418/

Course B: Materials for Devices

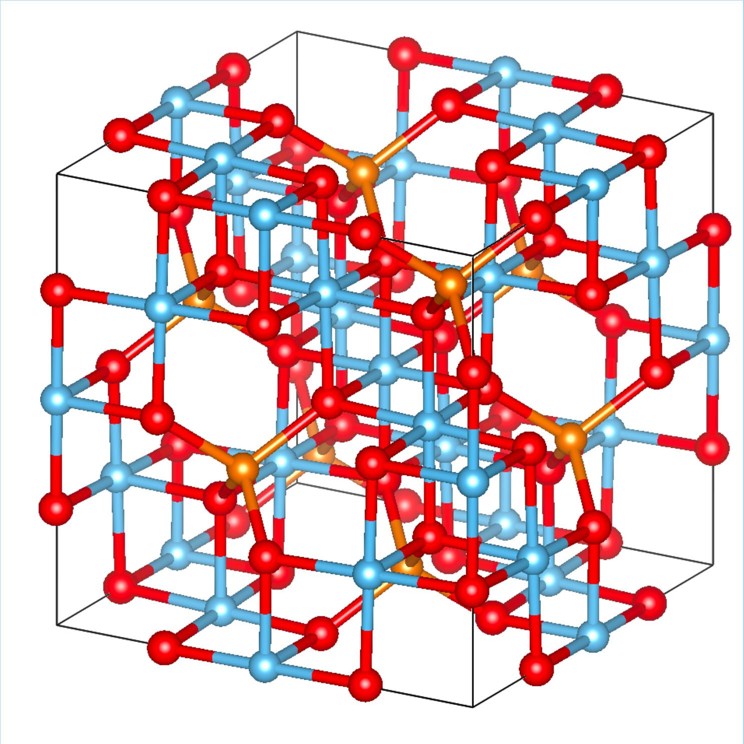
order

disorder

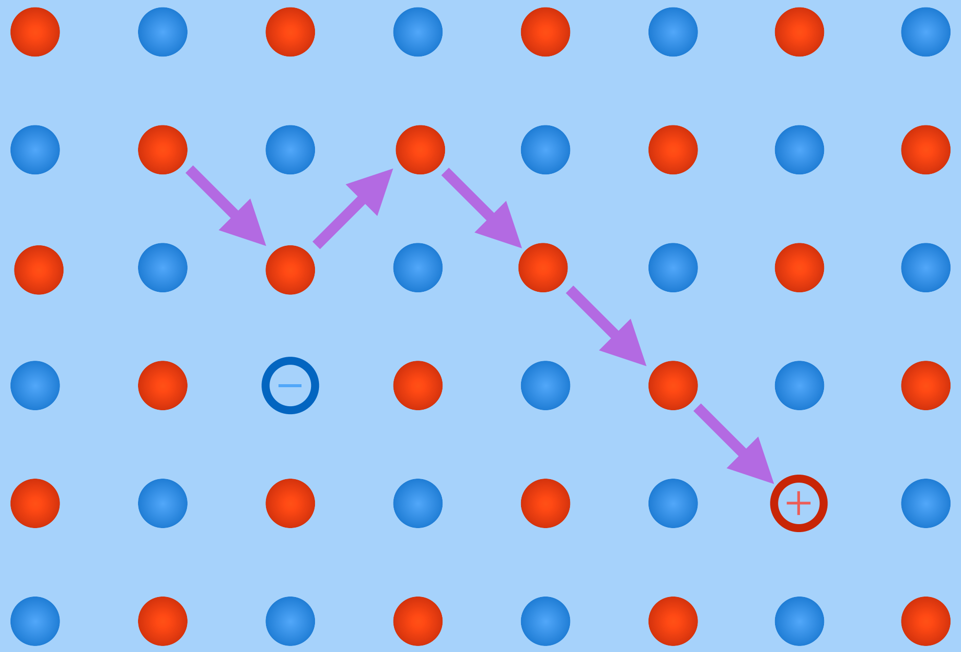
electric polarisation in materials



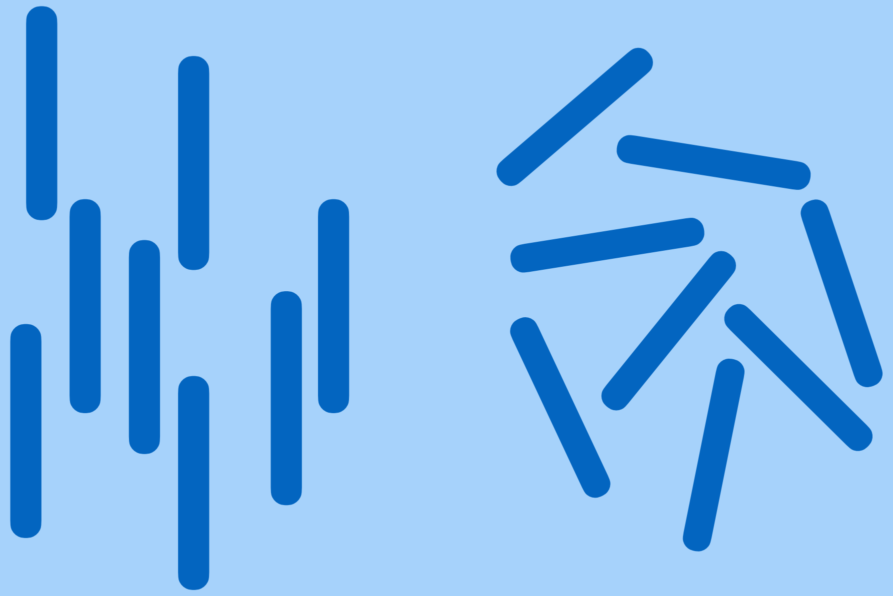
magnetism in materials



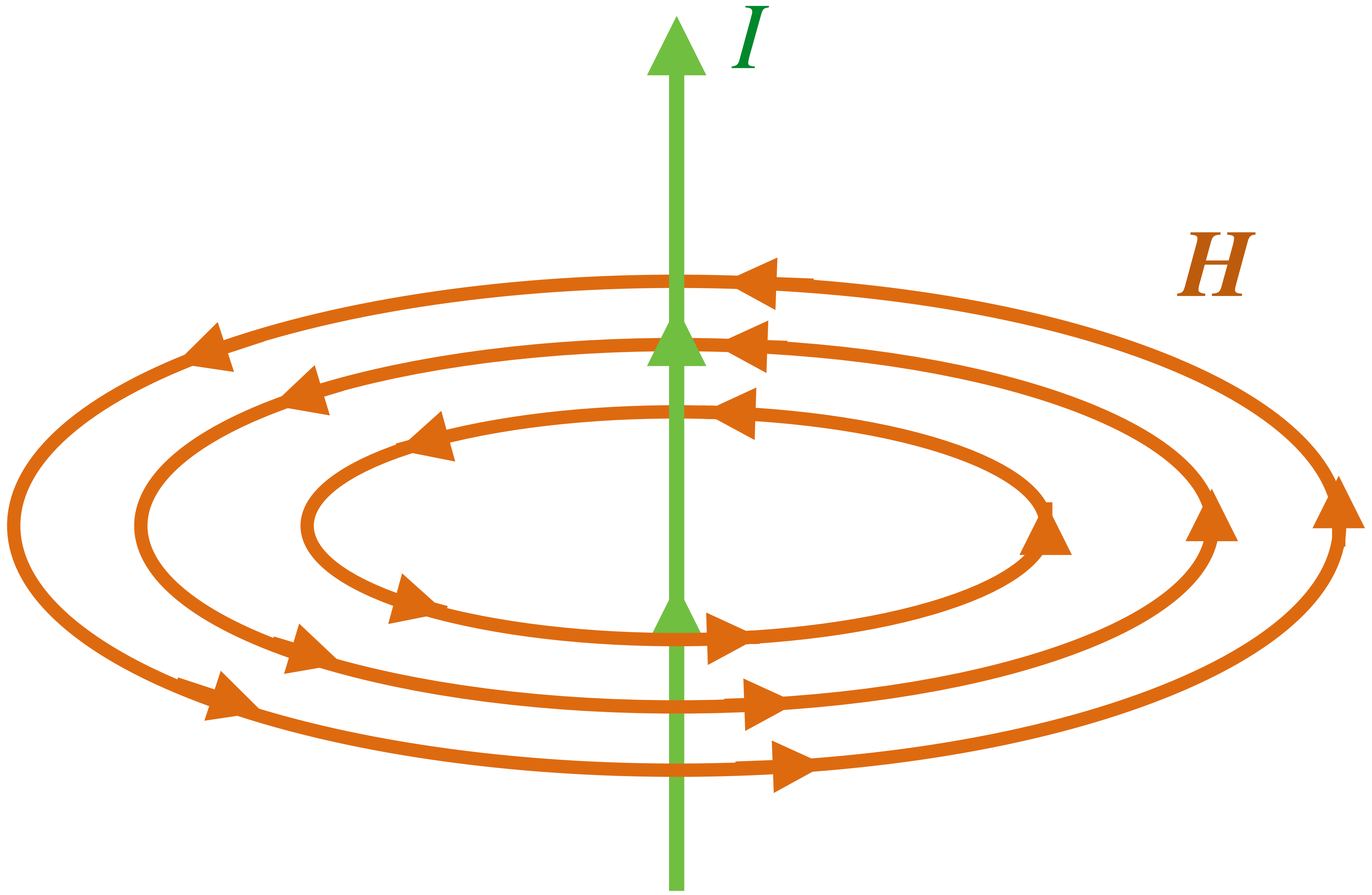
ionic conductors



liquid crystals



Magnetostatics: Ampère's law



- Ampère's law:

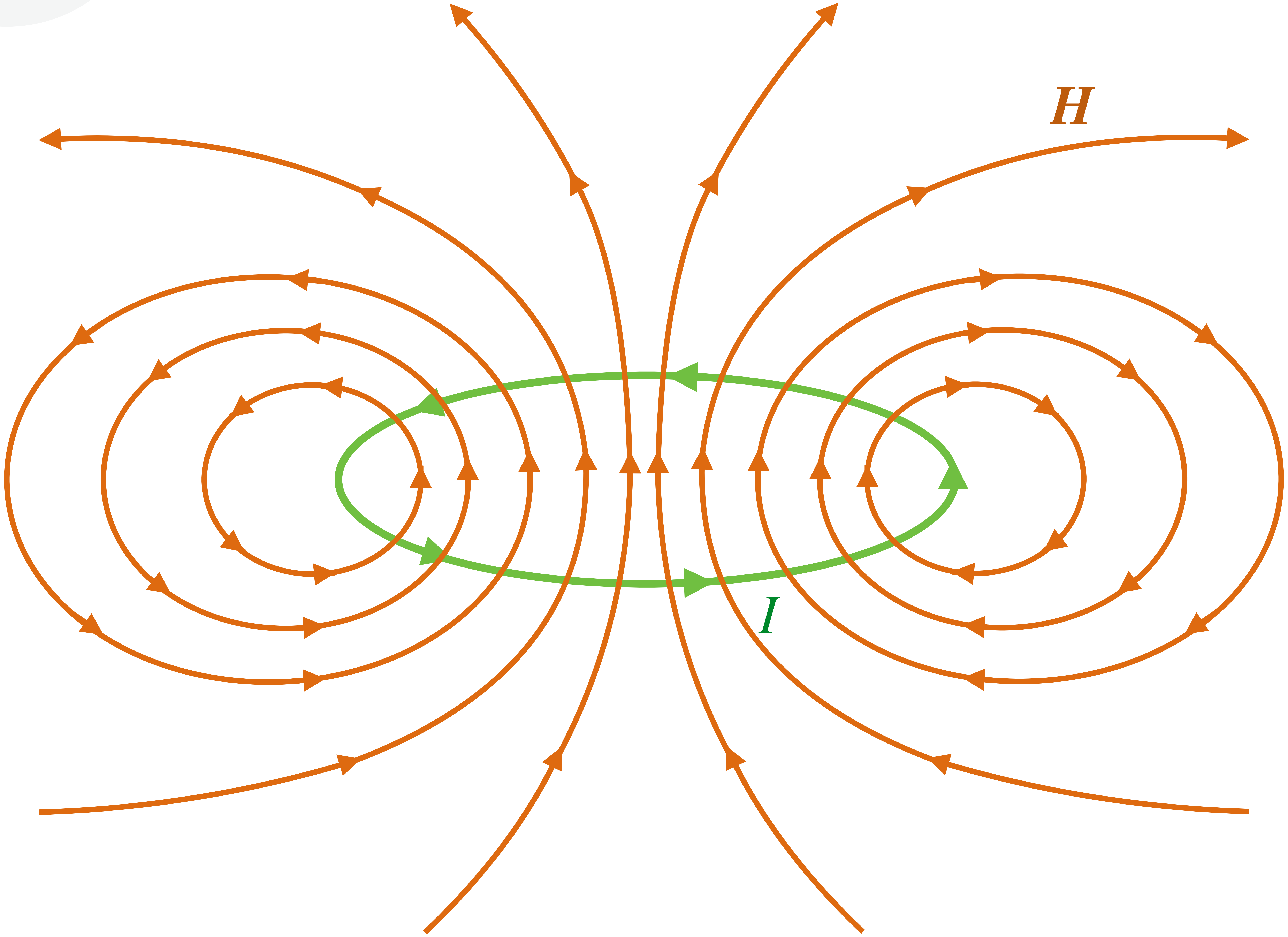
$$\boxed{\oint_P \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = I_{\text{free}}}$$

$\boldsymbol{H}$: magnetic field $[\mathrm{Am}^{-1}]$

$\mathrm{d}\boldsymbol{l}$: infinitesimal length element of path P $[\mathrm{m}]$

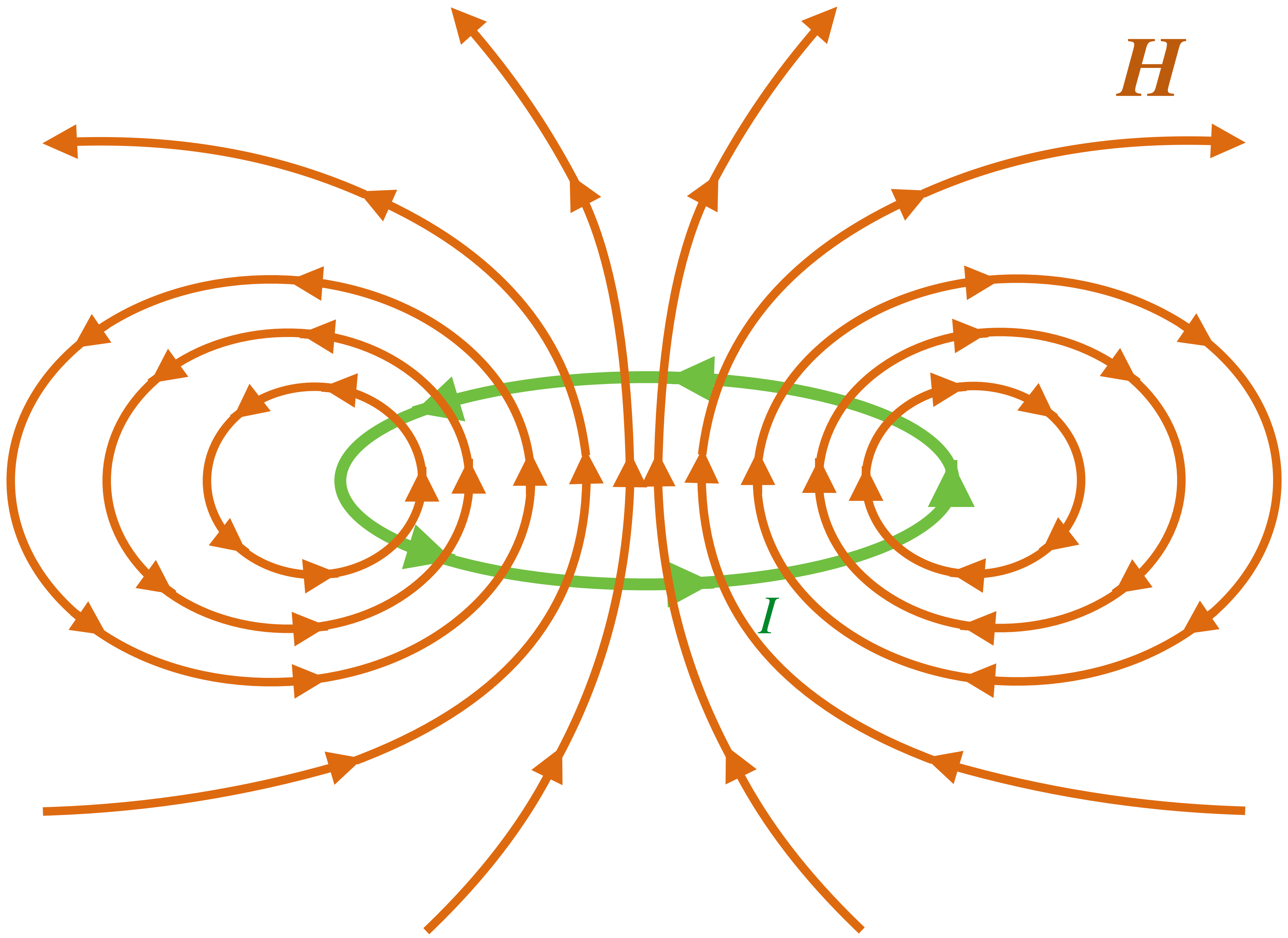
I_{free} : free current $[\mathrm{A}]$

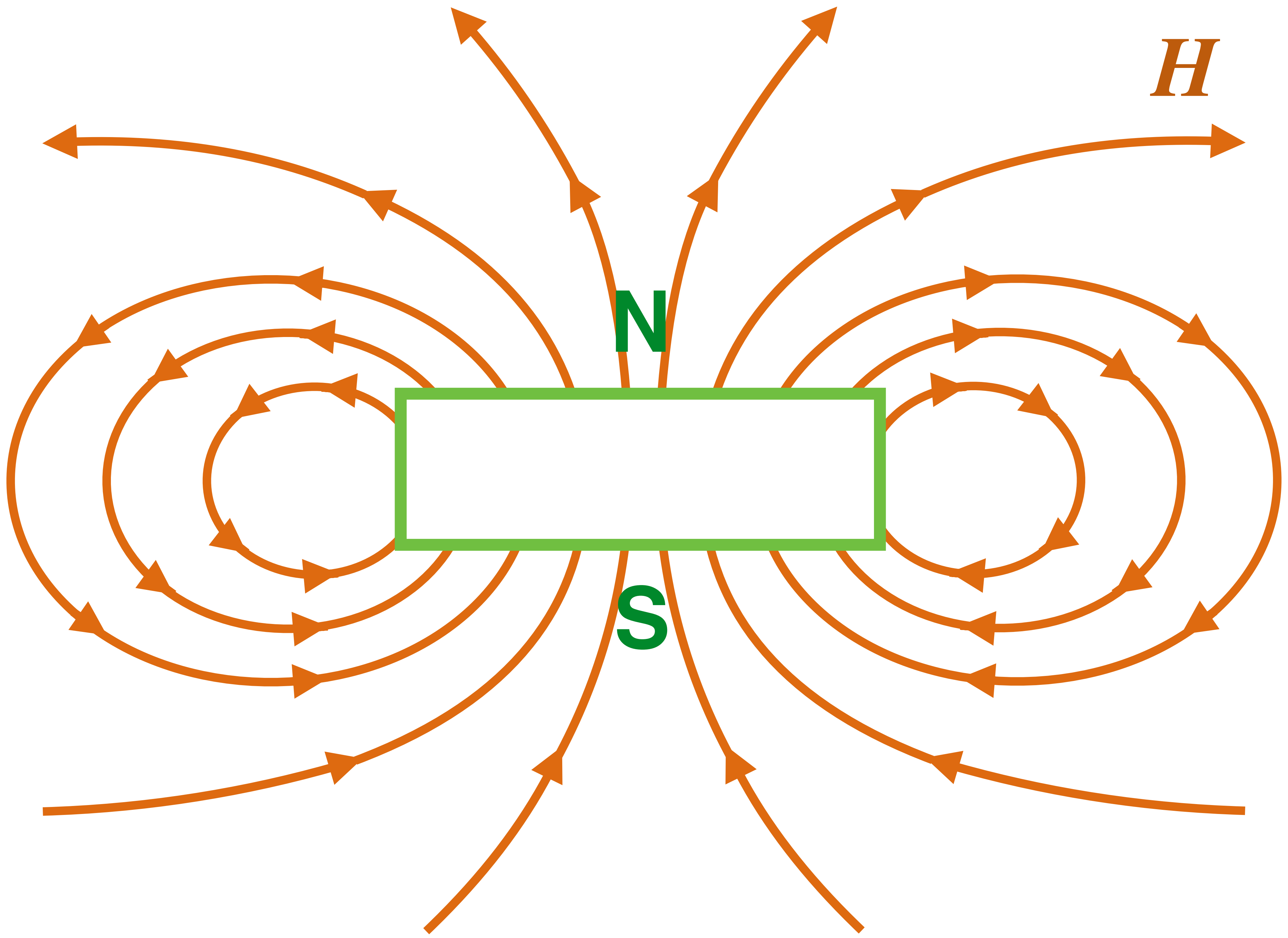
Magnetostatics: Ampère's law



$$\oint_P \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = I_{\mathrm{free}}$$

Magnetostatics: Ampère’s law

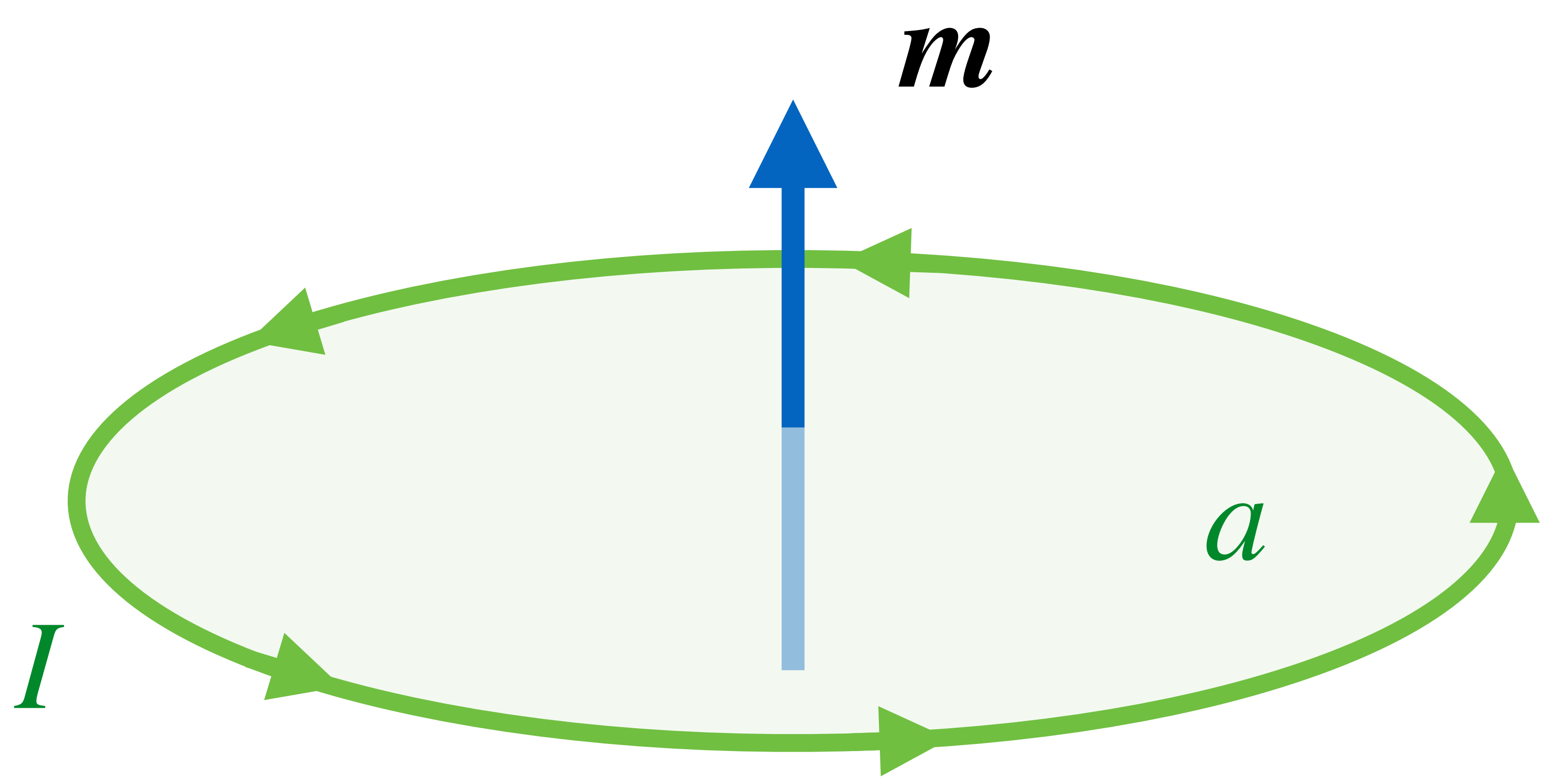




- Ampère hypothesised that all magnetic effects are due to current loops

Magnetostatics: magnetic moment

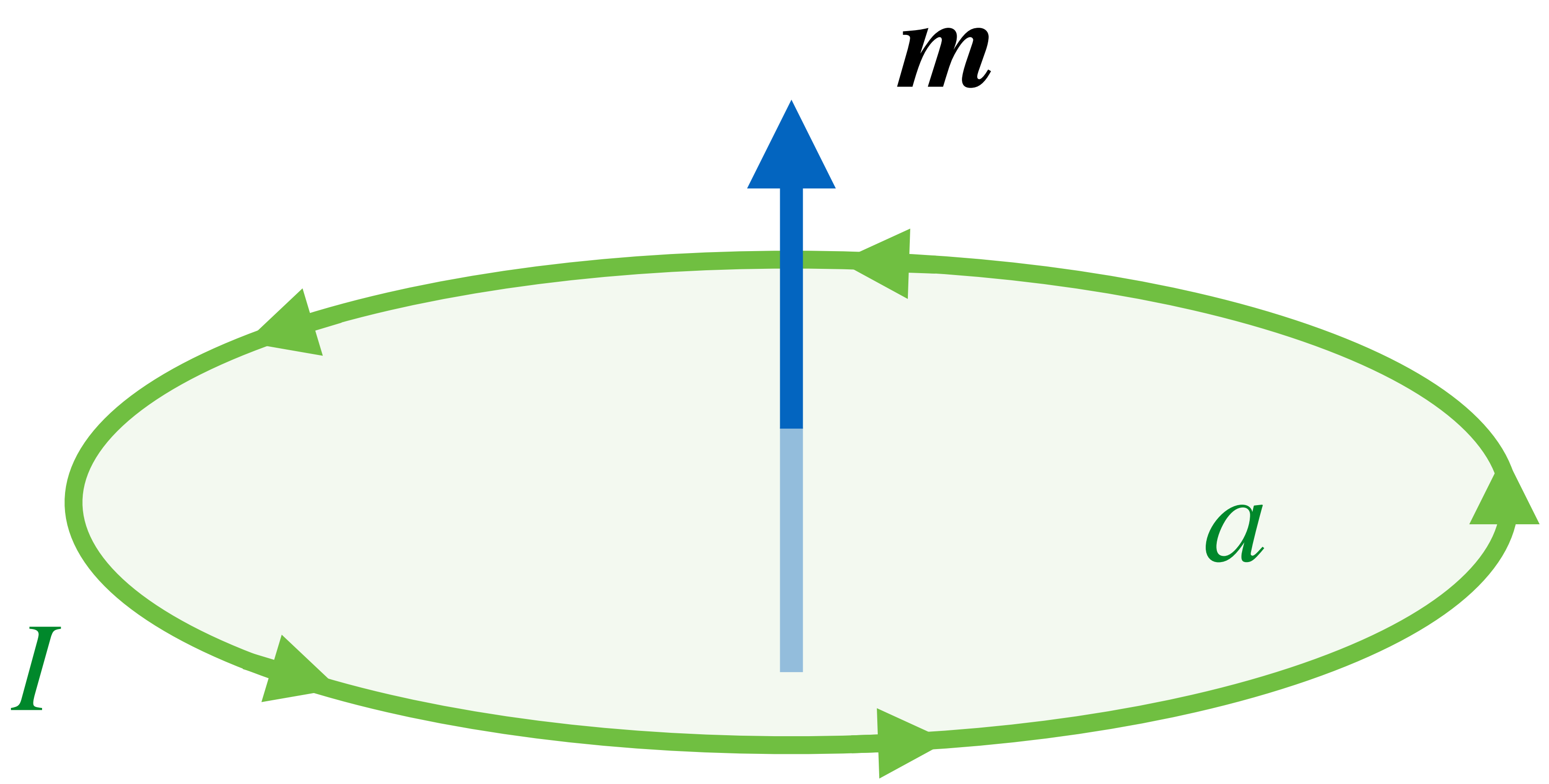
- Magnetic moment: characterises the strength and direction of *something* that generates a magnetic field



$$\boxed{m = Ia} \qquad [\mathrm{A\,m^2}]$$

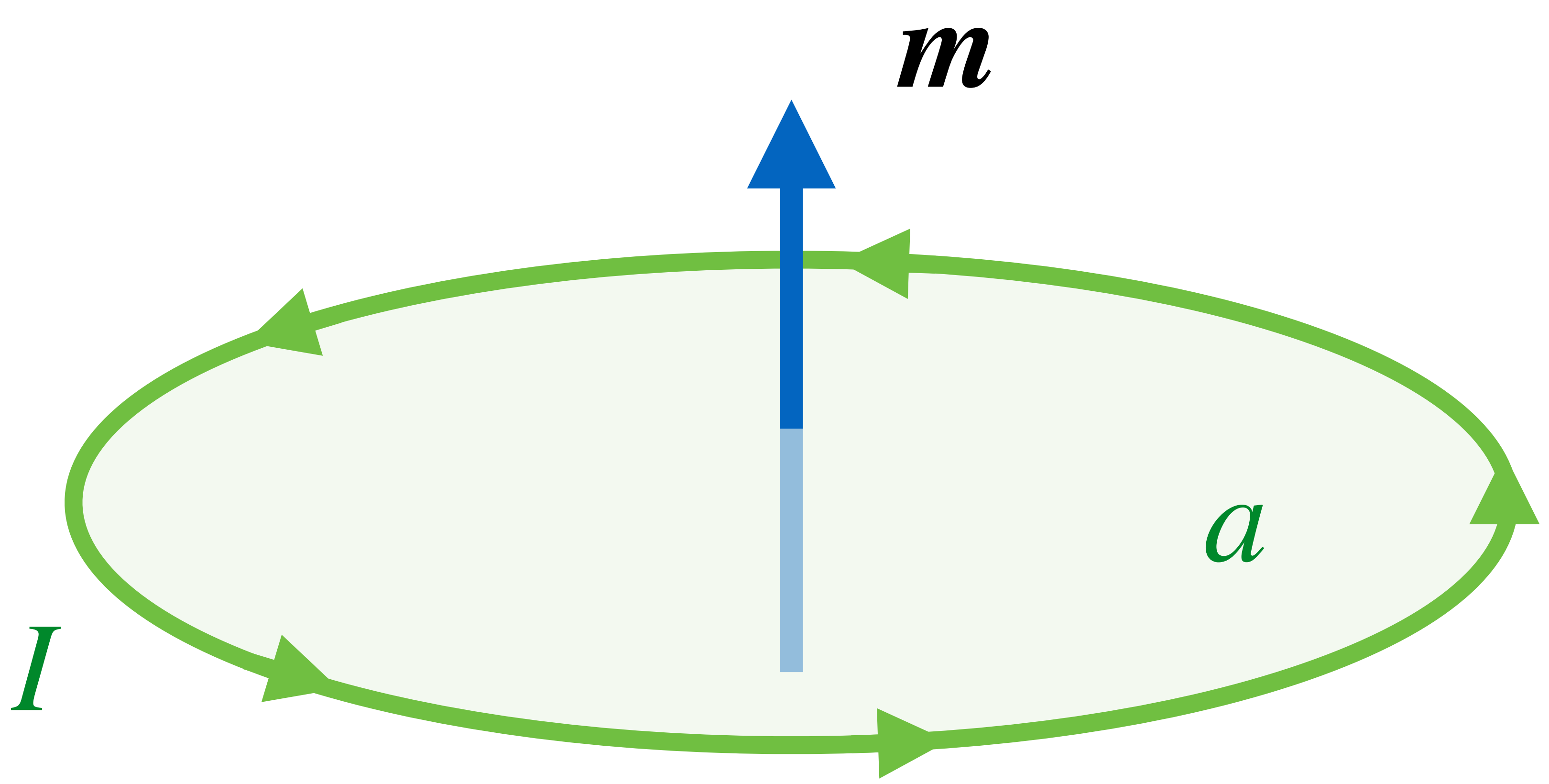
Aside: Atomic origin of magnetism

- Electrons carry two types of angular momentum:
  - Orbital angular momentum (analogous to classical angular momentum)
  - Spin angular momentum (no classical analogue)



Aside: Atomic origin of magnetism

- Atoms are made of many interacting electrons organised in atomic shells due to the Pauli exclusion principle:
  - Filled shells carry no net angular momentum: no magnetic moment
  - Partially filled shells carry angular momentum: magnetic moment



Aside: Atomic origin of magnetism

- **Angular momentum in quantum mechanics**

  https://www.youtube.com/playlist?list=PL8W2boV7eVfmm5SZRjbhOKNziRXy6yIvI

- **Hydrogen atom**

  https://www.youtube.com/playlist?list=PL8W2boV7eVfnJbLf-p3-_7d51tskA0-Sa

Magnetostatics: magnetisation

- Magnetisation: magnetic moment $\boldsymbol{m}$ per unit volume

$$\boxed{\boldsymbol{M} = \frac{\boldsymbol{m}}{V}}$$

$\boldsymbol{M}$: magnetisation $[\mathrm{Am^{-1}}]$

$\boldsymbol{m}$: magnetic moment $[\mathrm{A\,m^{2}}]$

V : volume $[\mathrm{m^{3}}]$

Magnetostatics: susceptibility

- Magnetic susceptibility: quantifies the magnetisation of a material placed in a magnetic field $\boldsymbol{H}$

$$\boxed{\boldsymbol{M} = \chi \boldsymbol{H}}$$

$\boldsymbol{M}$: magnetisation $[\mathrm{Am}^{-1}]$

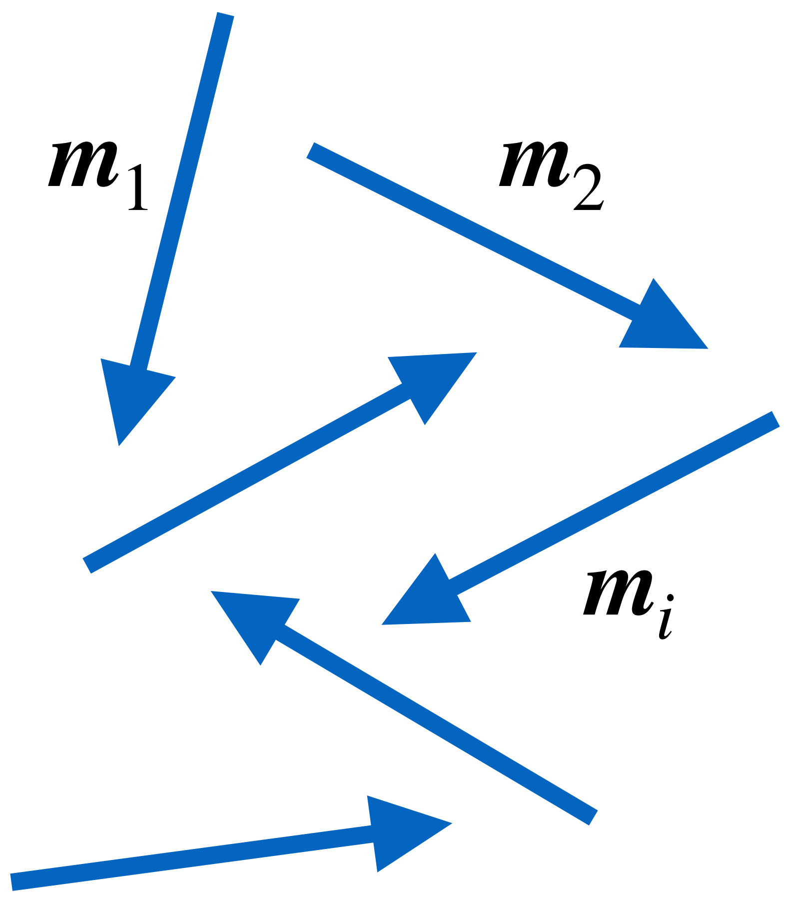
$\boldsymbol{H}$: magnetic field $[\mathrm{Am}^{-1}]$

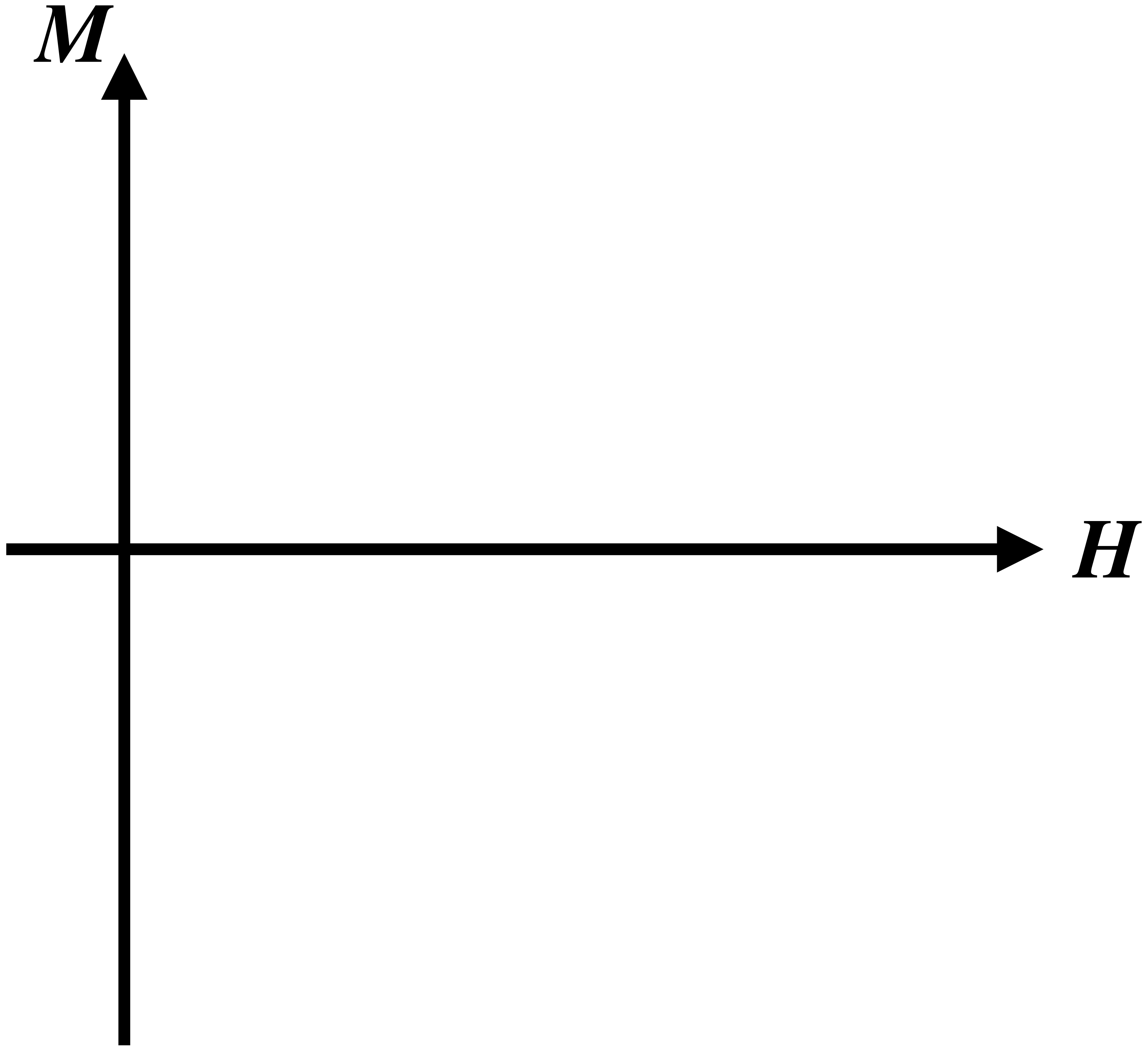
χ : susceptibility $[\mathrm{dimensionless}]$

Classifying magnetic materials

magnetisation $\boldsymbol{M}$

susceptibility χ $[\boldsymbol{M} = \chi \boldsymbol{H}]$





Diamagnetism

- Diamagnetism: change in orbital motion of electrons due to applied magnetic field
  - Lenz's law:

$$\boxed{\mathscr{E} = -\frac{d\Phi_B}{dt}}$$

$\mathscr{E}$: electromotive force $[\mathrm{Wb\,s^{-1}}]$

Φ_B : magnetic flux $[\mathrm{Wb}]$

t : time $[\mathrm{s}]$

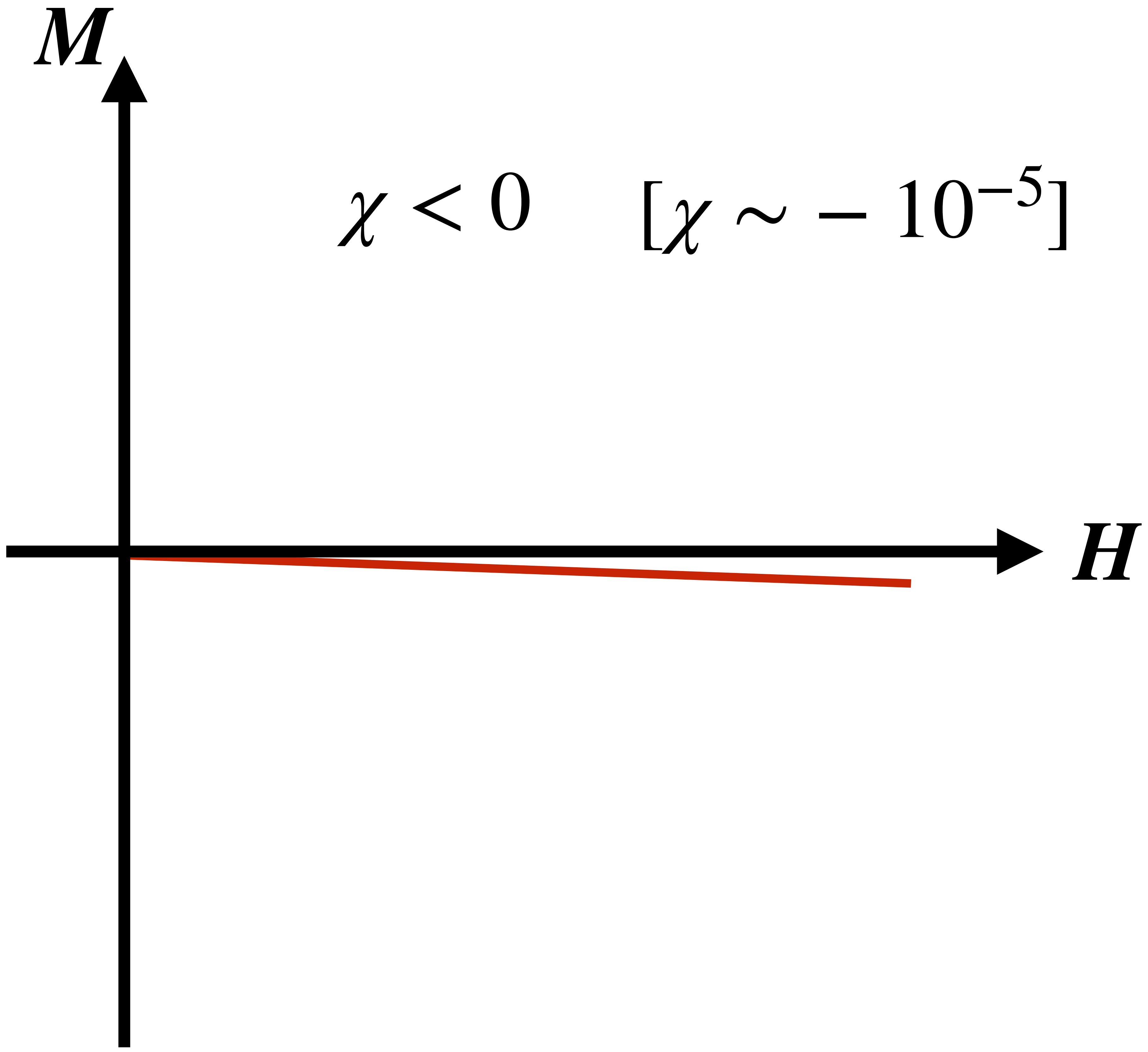
- All atoms are diamagnetic
- Diamagnetism is very weak, so masked by other effects in most materials

Diamagnetic materials

magnetisation $\boldsymbol{M}$

susceptibility χ $[\boldsymbol{M} = \chi \boldsymbol{H}]$

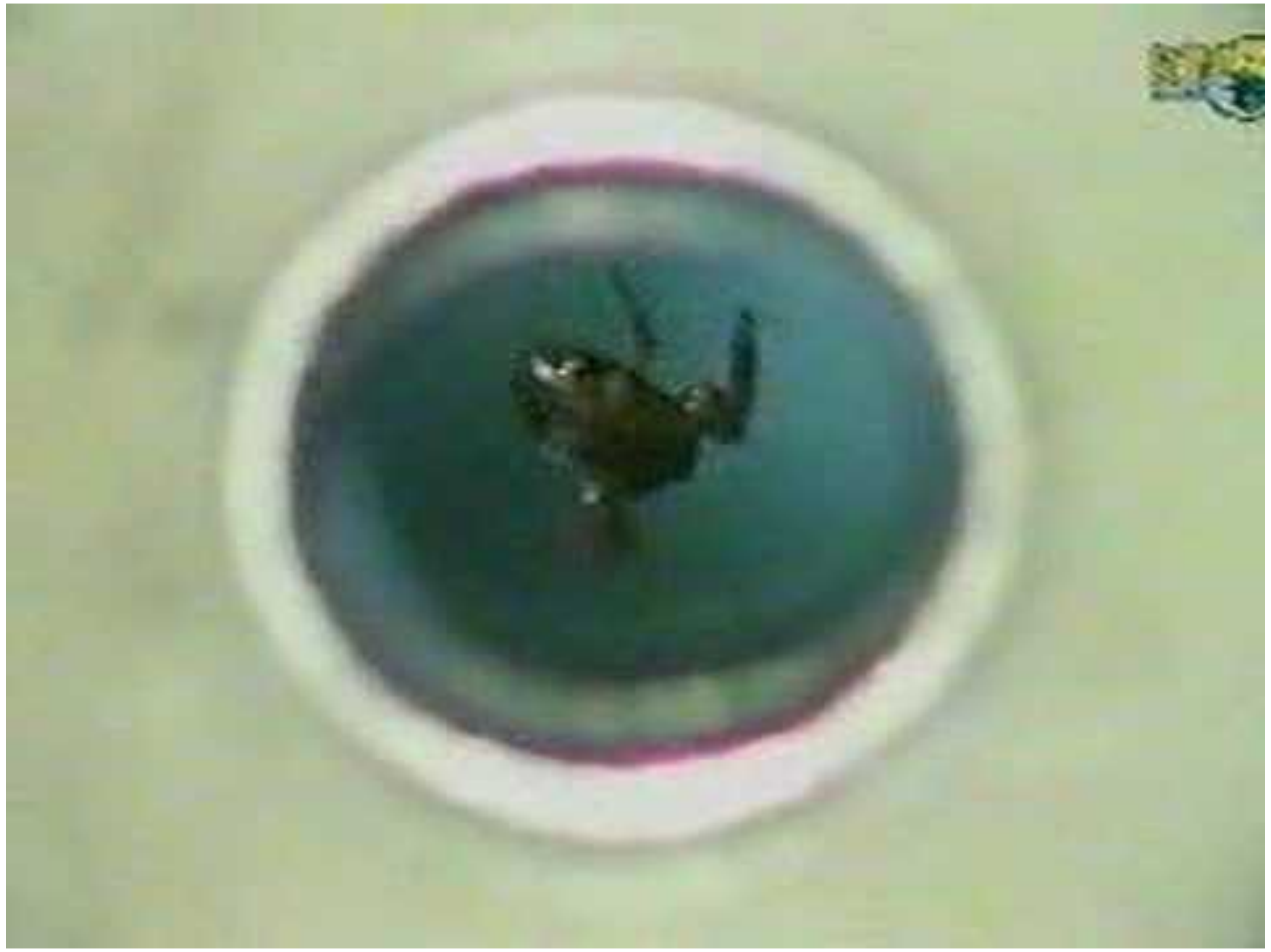
$\boldsymbol{M} = \mathbf{0}$



$\chi < 0 \quad [\chi \sim -10^{-5}]$

Diamagnetic materials

- Examples: copper, graphite, bismuth, lead, water



Andre Geim

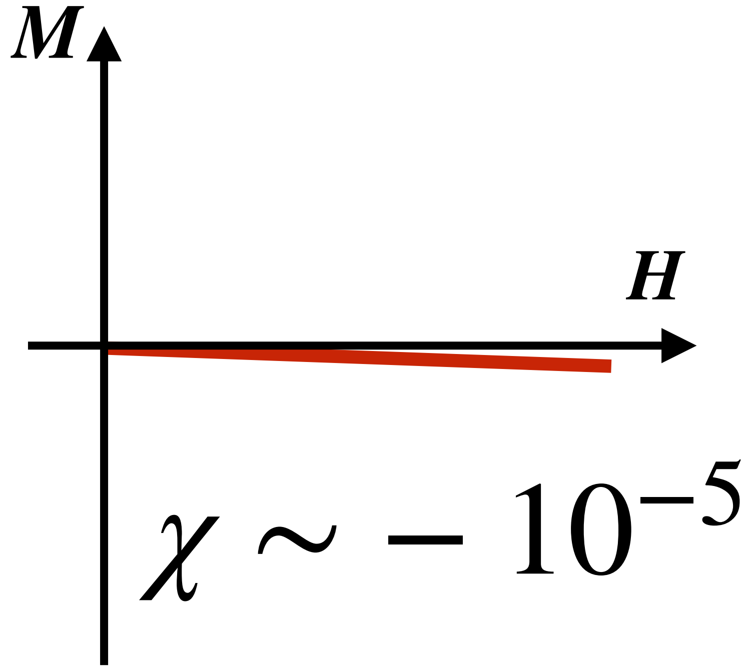
Ig Nobel Prize 2000
Nobel Prize 2010

Credit: Bengt Oberger [CC BY-SA 4.0] via Wikimedia Commons

Classification of magnetic materials

diamagnetic

$$\boldsymbol{M} = \mathbf{0}$$



$\chi \sim -10^{-5}$

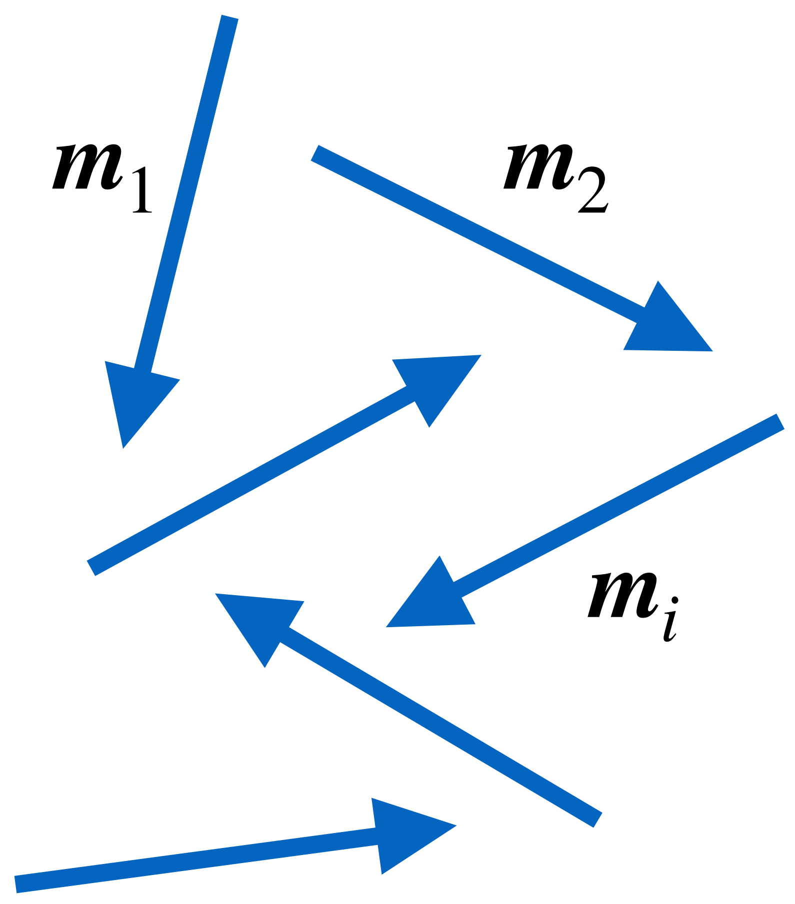
copper, water

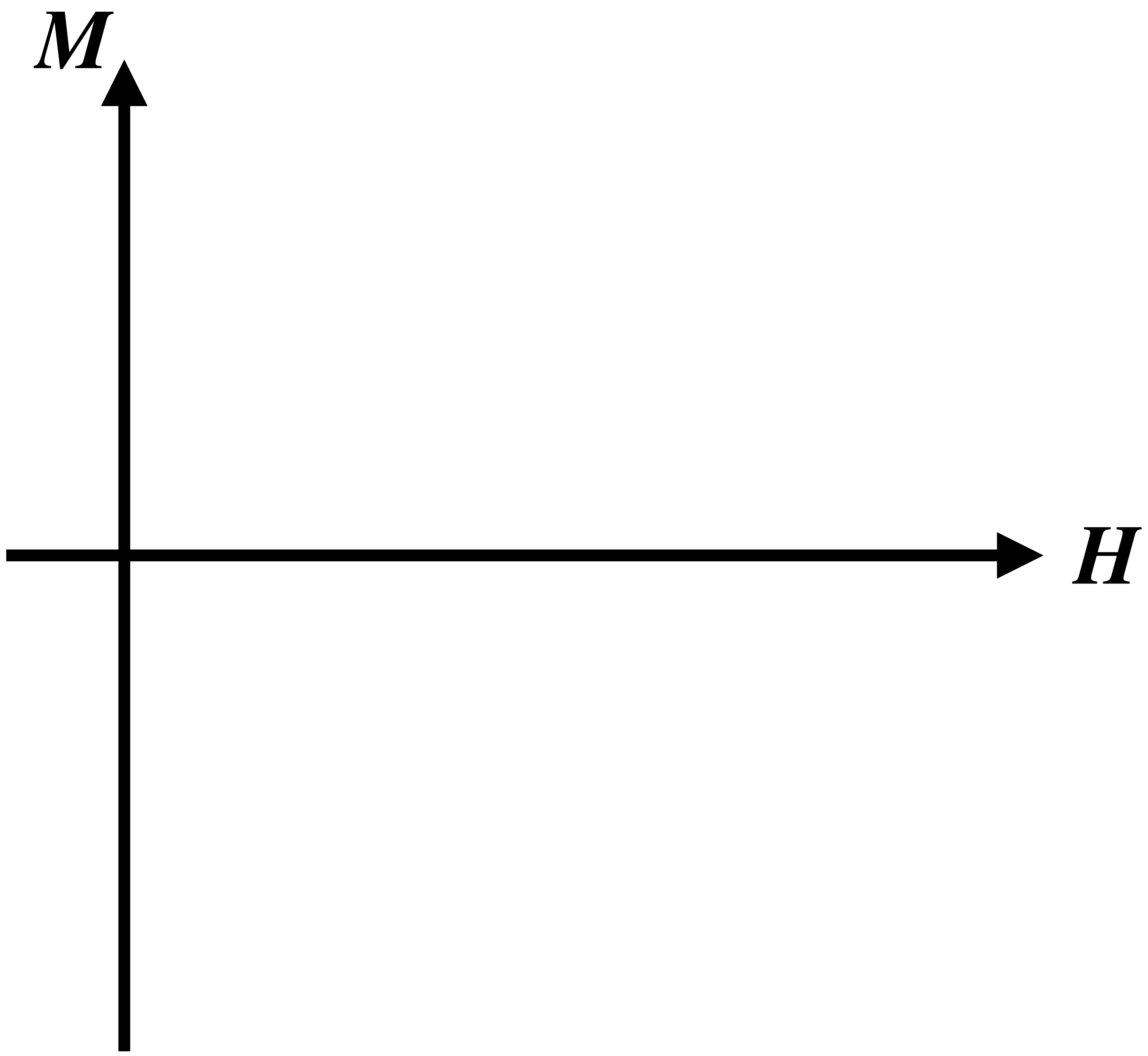
Paramagnetic materials

magnetisation $\boldsymbol{M}$

susceptibility χ $[\boldsymbol{M} = \chi \boldsymbol{H}]$

$\boldsymbol{M} = \boldsymbol{0}$



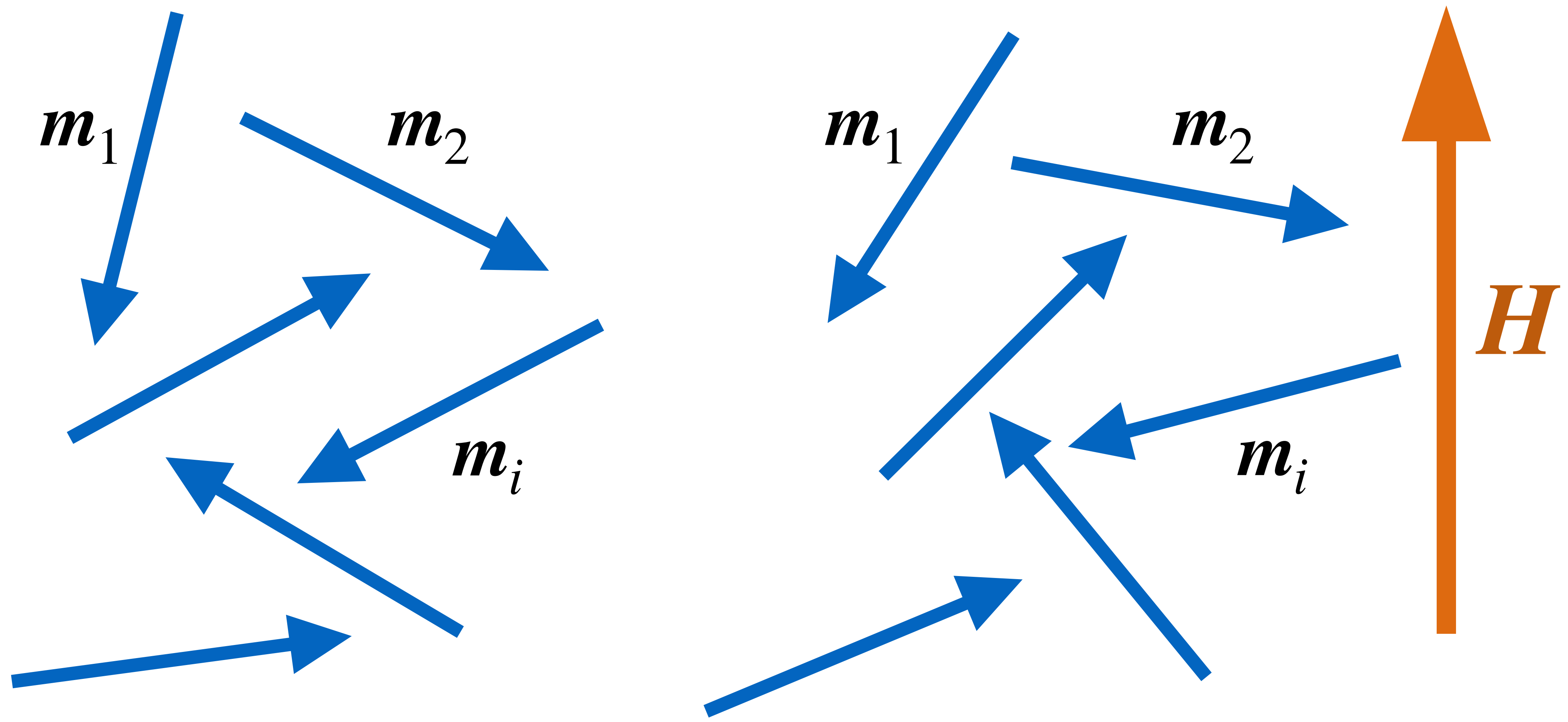


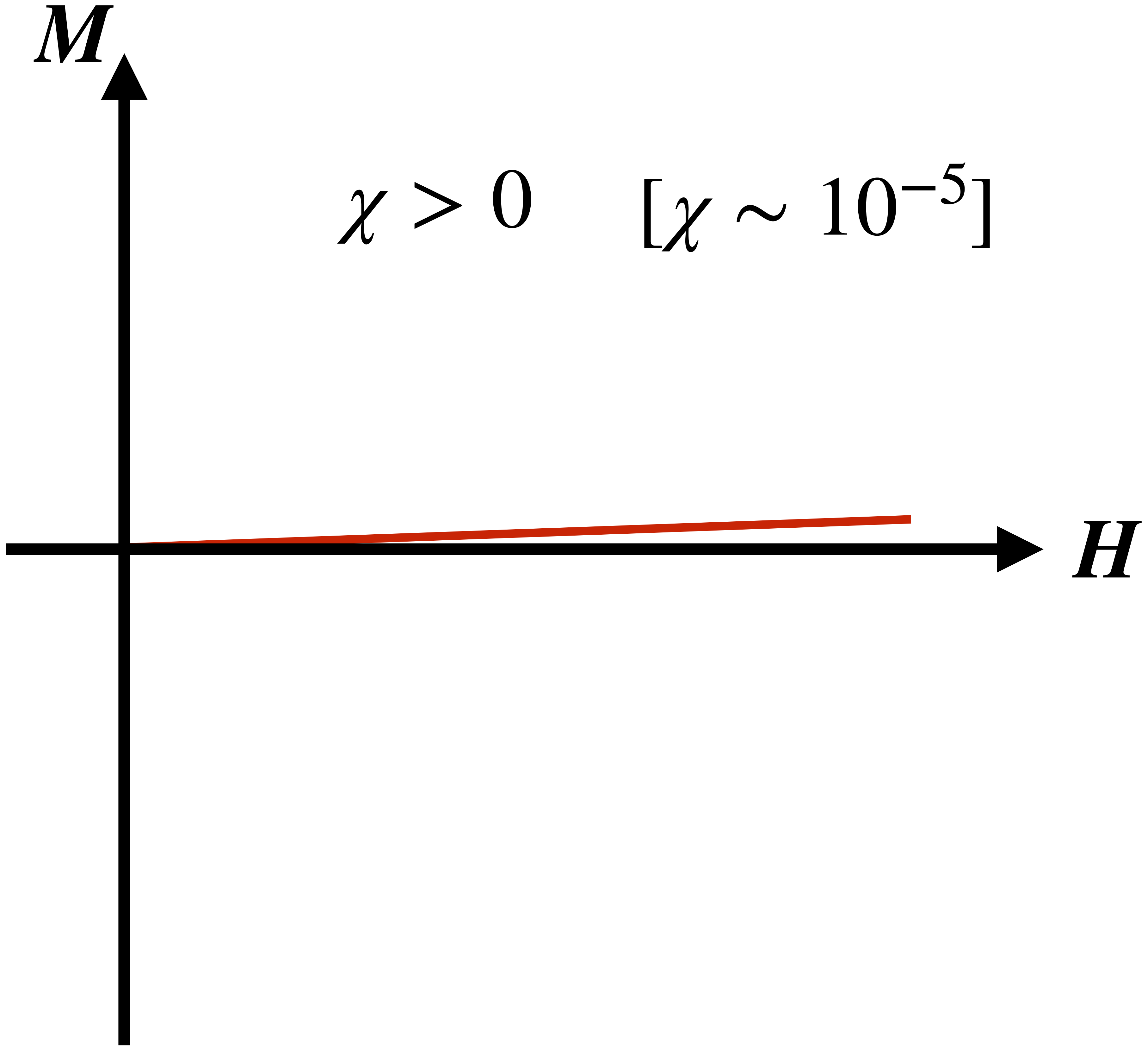
Paramagnetic materials

magnetisation $\boldsymbol{M}$

susceptibility χ $[\boldsymbol{M} = \chi \boldsymbol{H}]$

$\boldsymbol{M} = \mathbf{0}$ $\boldsymbol{M} > \mathbf{0}$





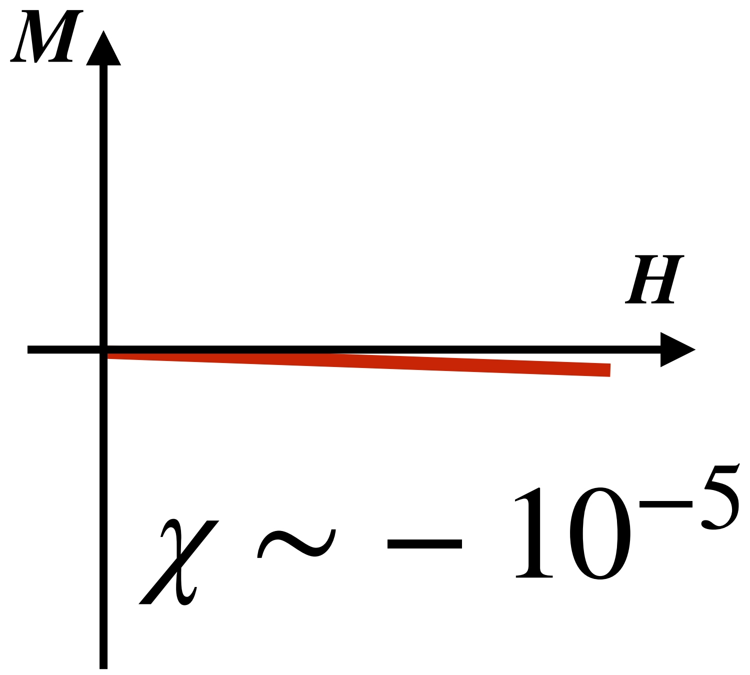
$\chi > 0$ $[\chi \sim 10^{-5}]$

- Examples: aluminium, magnesium

Classification of magnetic materials

diamagnetic

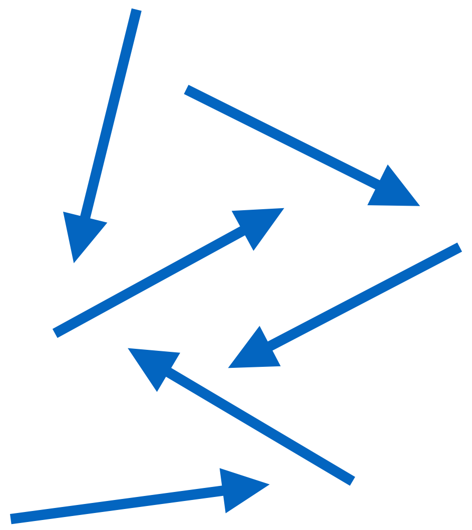
$$\boldsymbol{M} = \mathbf{0}$$

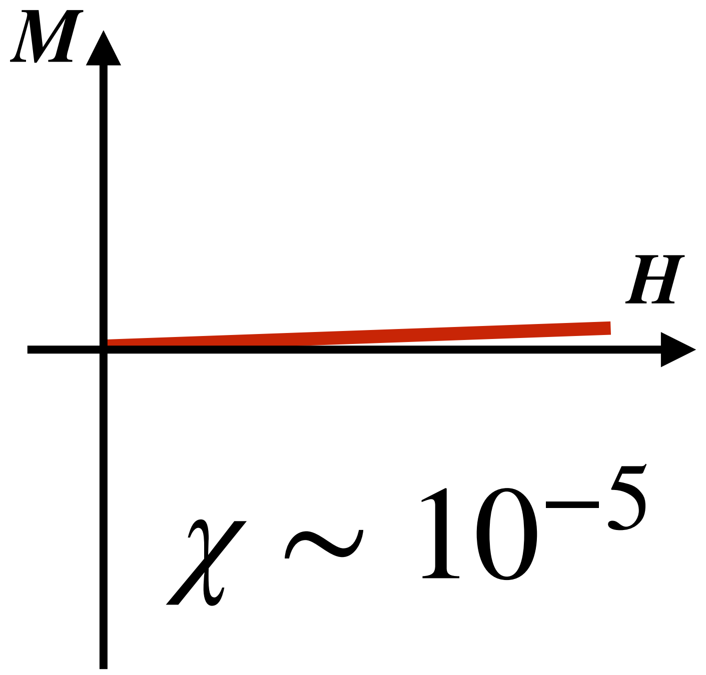


$\chi \sim -10^{-5}$

copper, water

paramagnetic





$\chi \sim 10^{-5}$

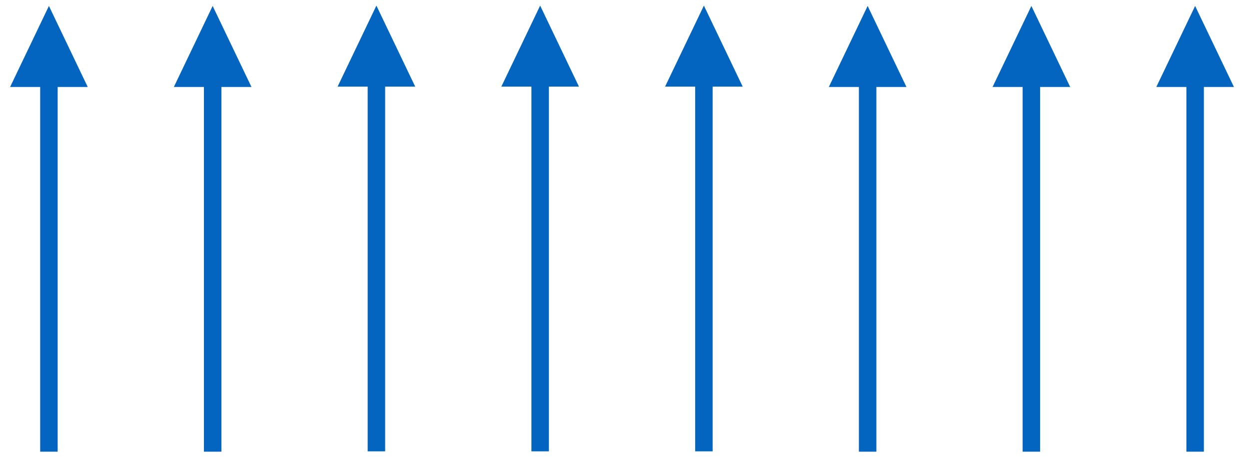
aluminium,
magnesium

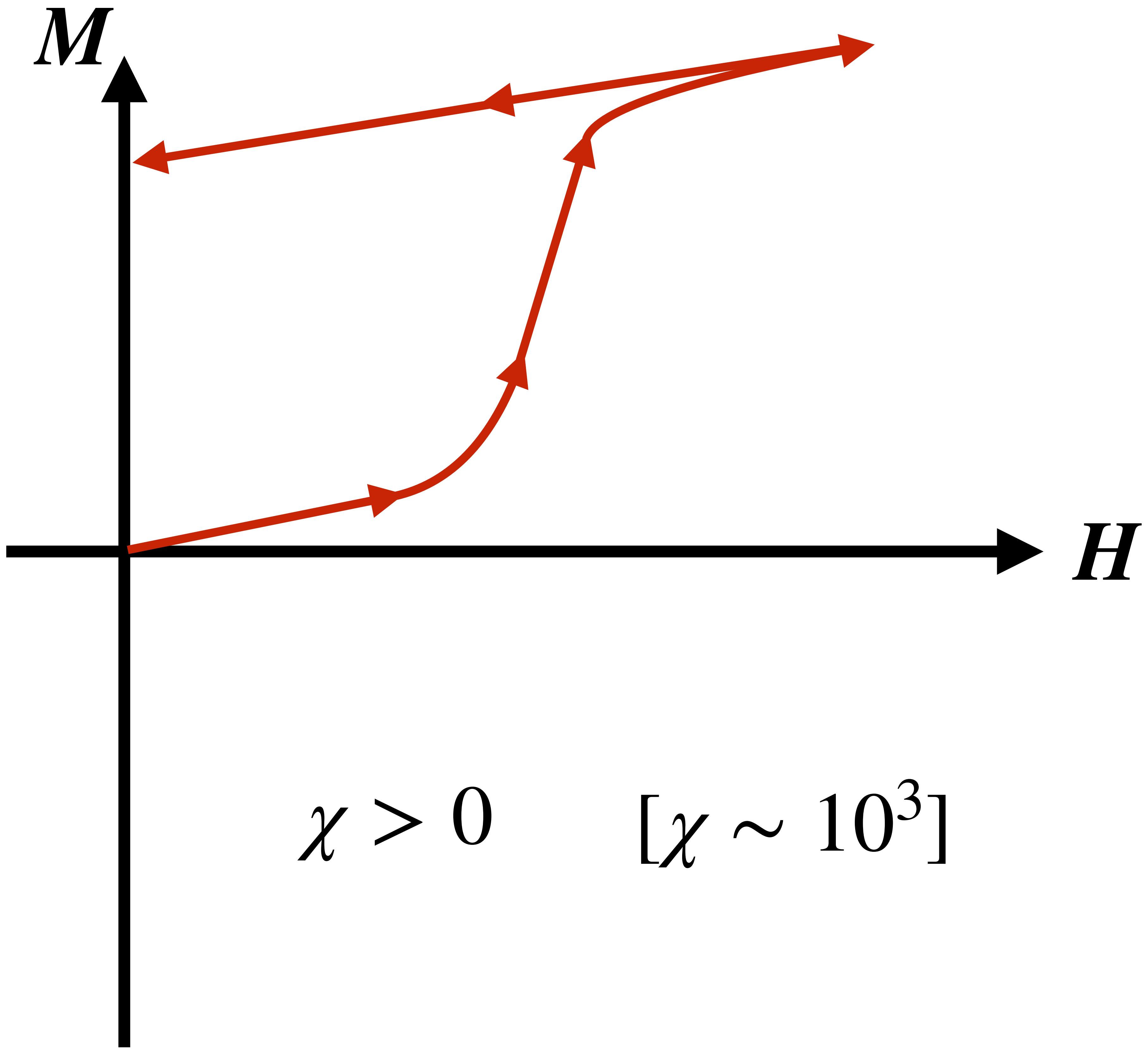
Ferromagnetic materials

magnetisation $\boldsymbol{M}$

susceptibility χ $[\boldsymbol{M} = \chi \boldsymbol{H}]$

$\boldsymbol{M} > \boldsymbol{0}$





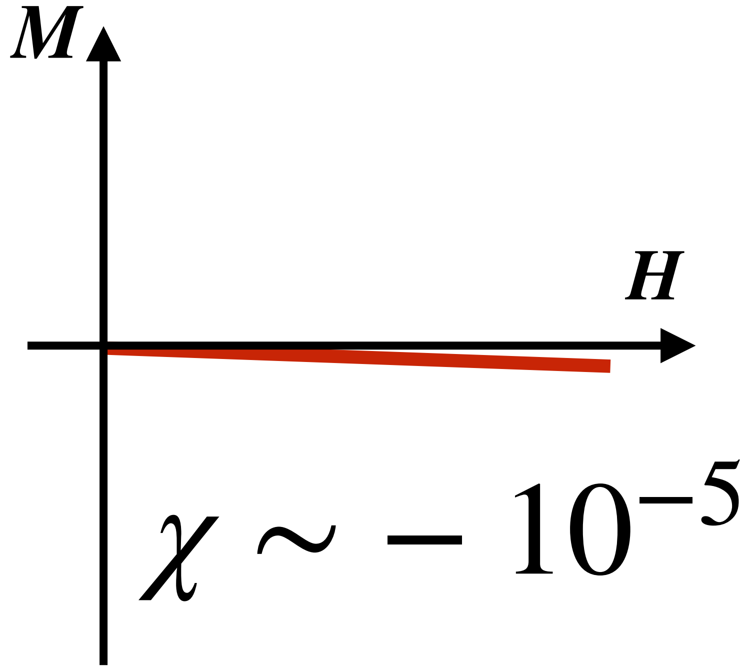
$\chi > 0$ $[\chi \sim 10^3]$

- Examples: iron, cobalt, nickel

Classification of magnetic materials

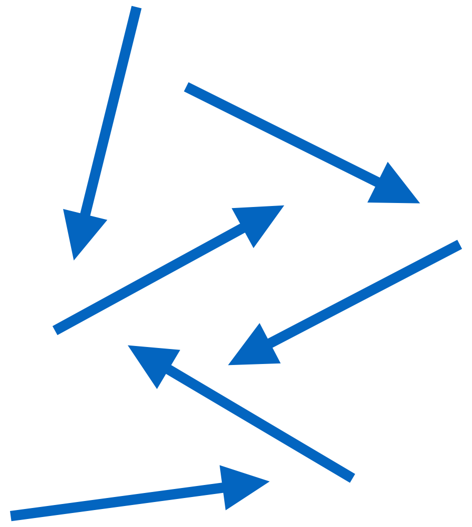
diamagnetic

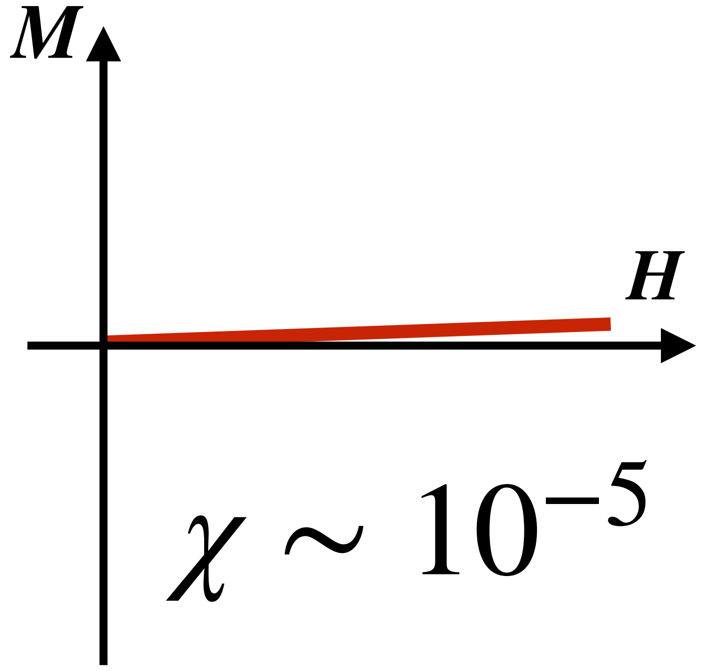
$$M = 0$$



copper, water

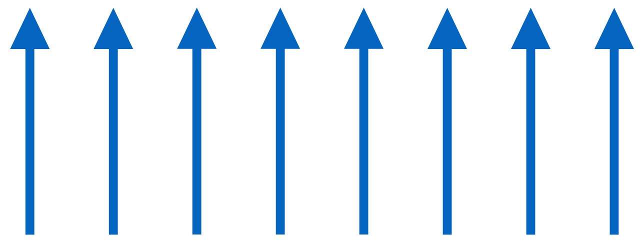
paramagnetic

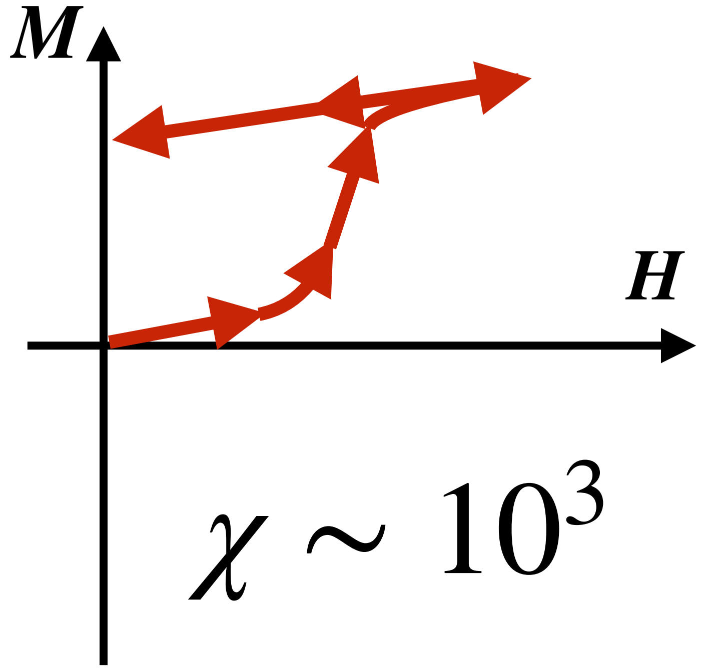




aluminium, magnesium

ferromagnetic



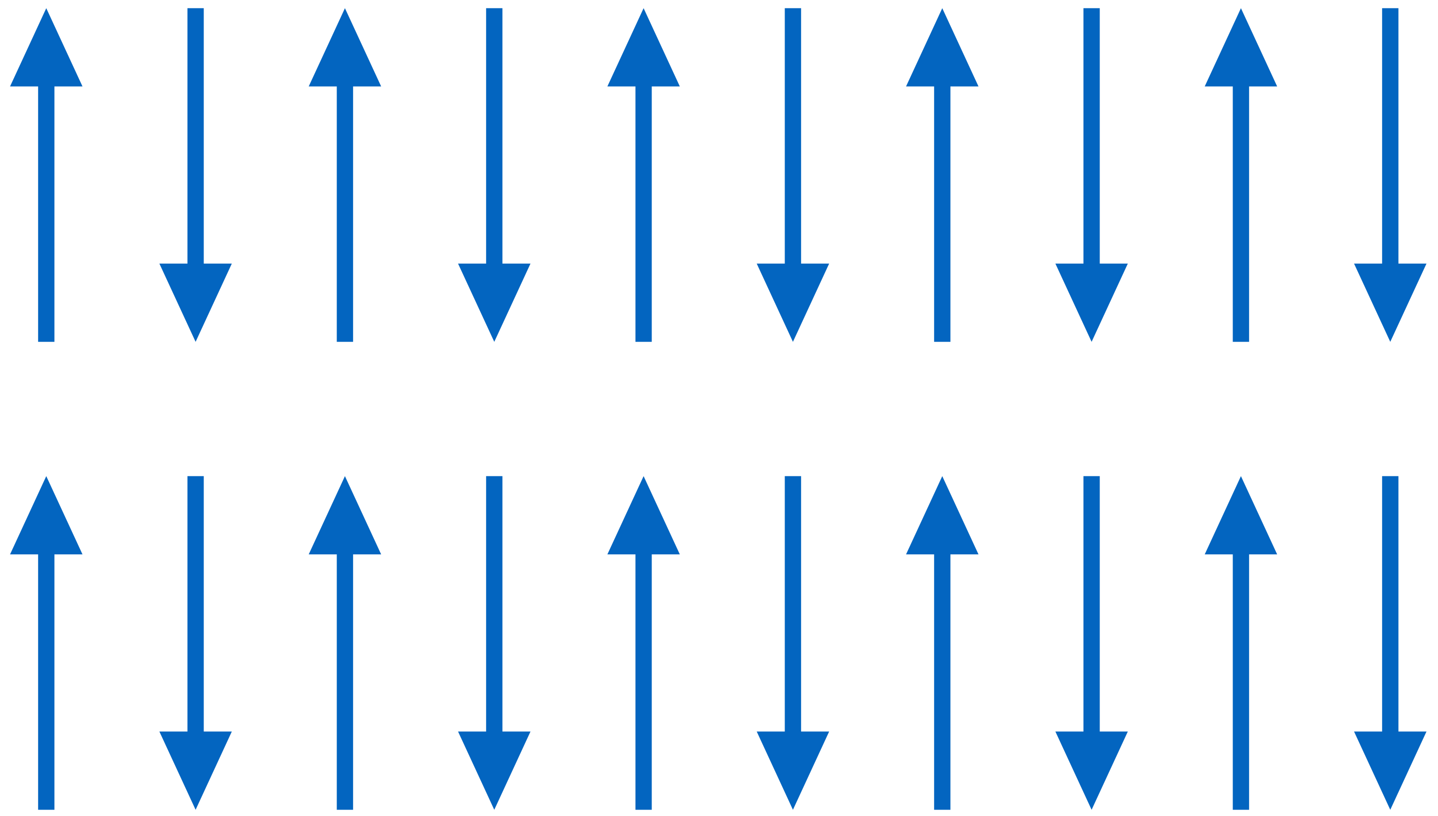


iron, cobalt, nickel

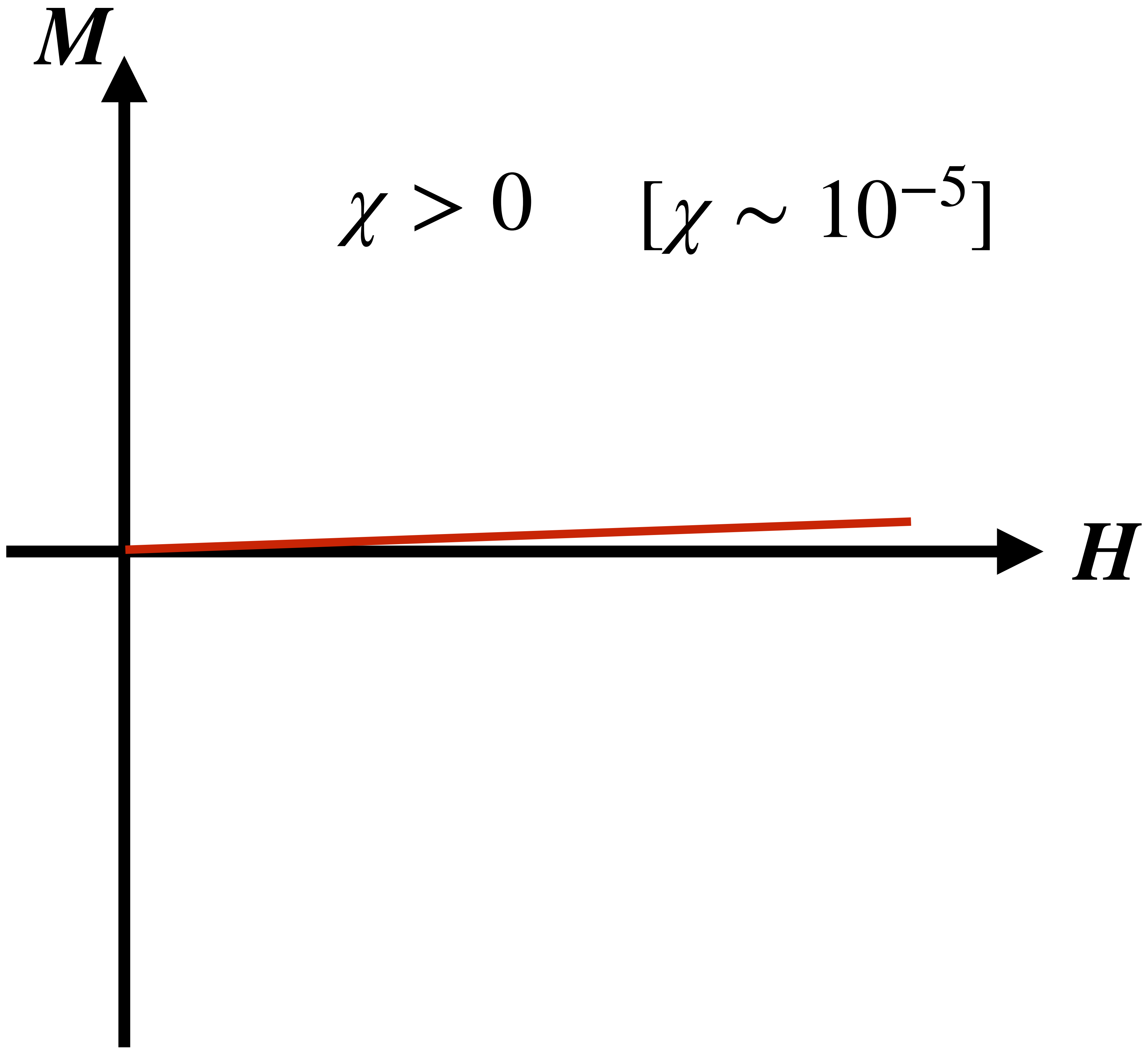
Antiferromagnetic materials

magnetisation $\boldsymbol{M}$

$\boldsymbol{M} = \mathbf{0}$



susceptibility χ $[\boldsymbol{M} = \chi \boldsymbol{H}]$



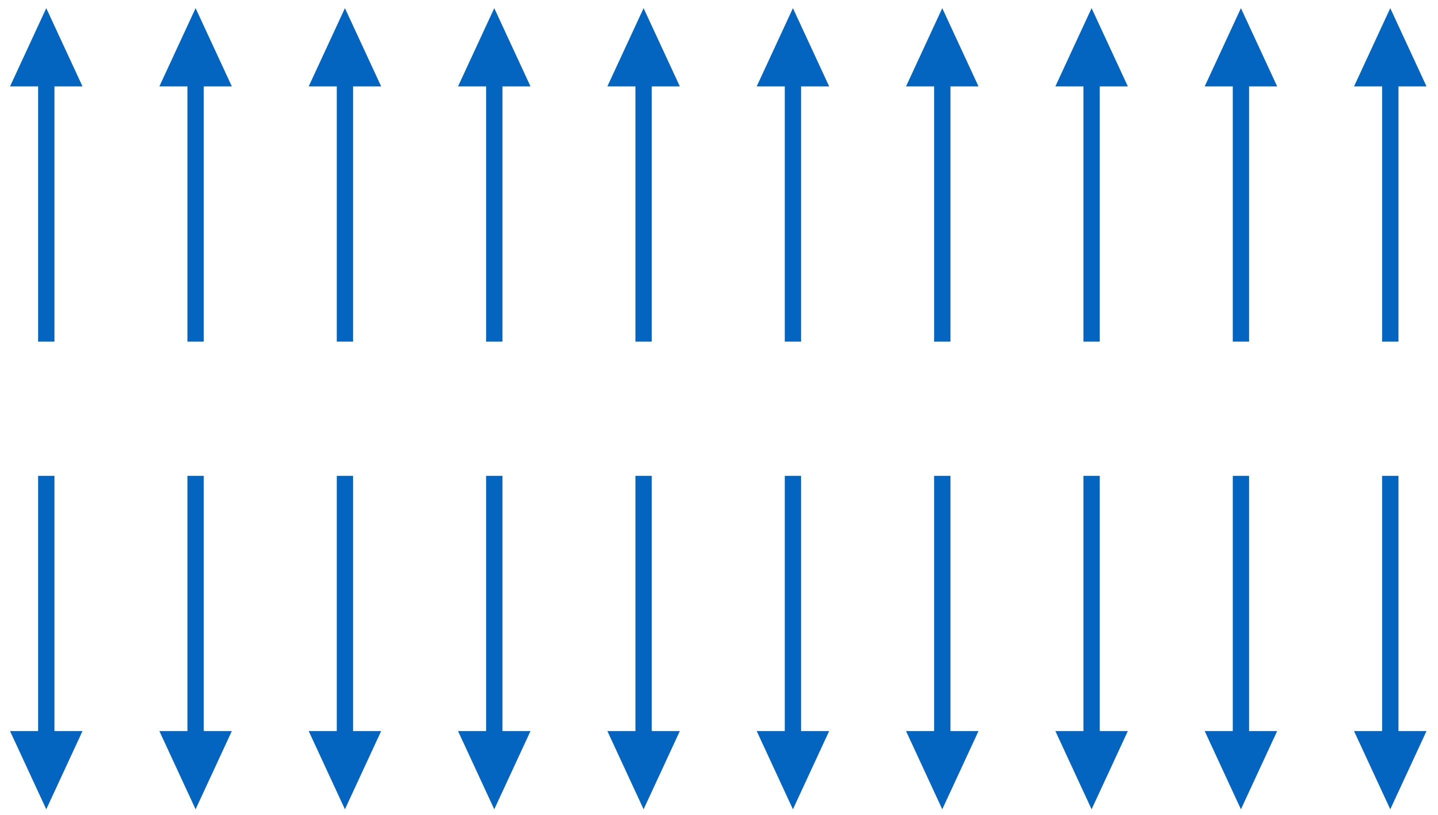
- Examples: FeMn, NiO

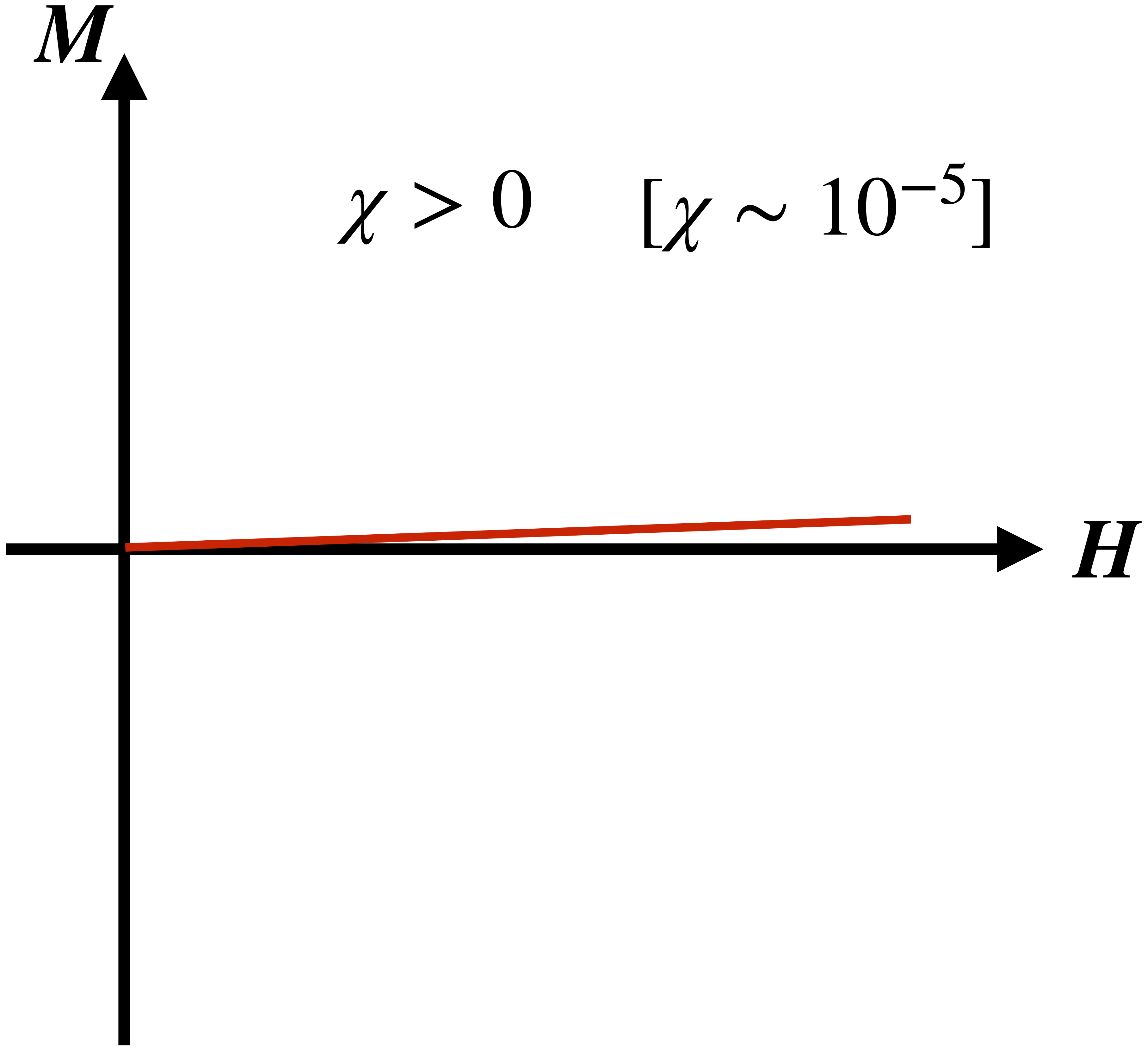
Antiferromagnetic materials

magnetisation $\boldsymbol{M}$

susceptibility χ $[\boldsymbol{M} = \chi \boldsymbol{H}]$

$\boldsymbol{M} = \mathbf{0}$



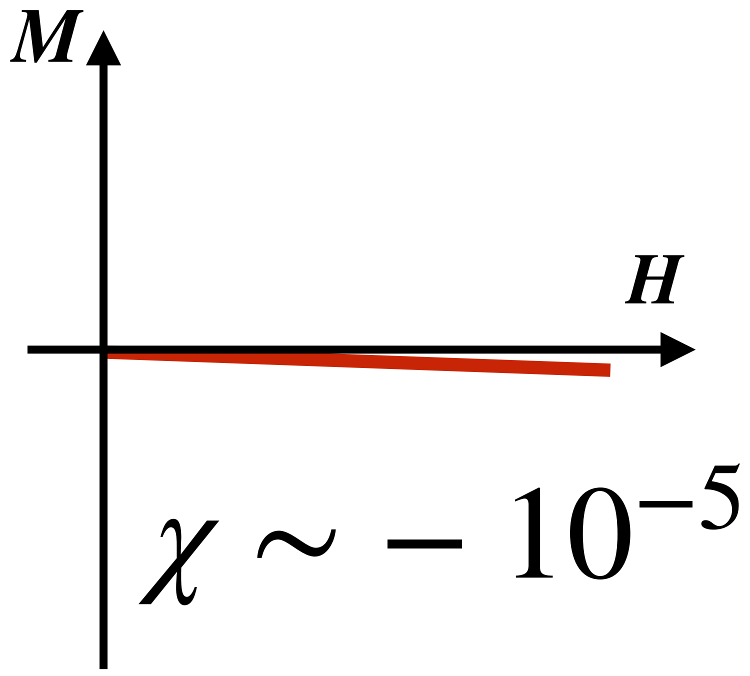


- Examples: FeMn, NiO

Classification of magnetic materials

diamagnetic

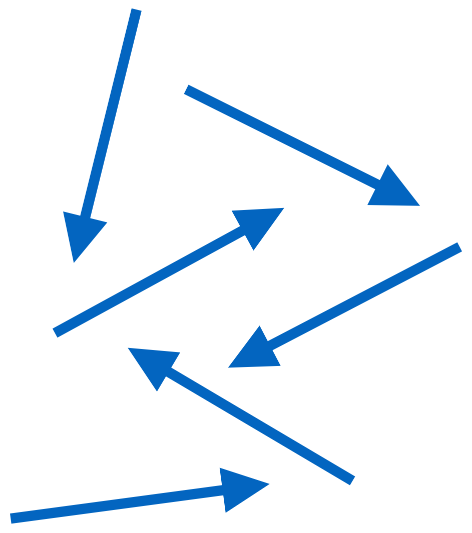
$\boldsymbol{M = 0}$

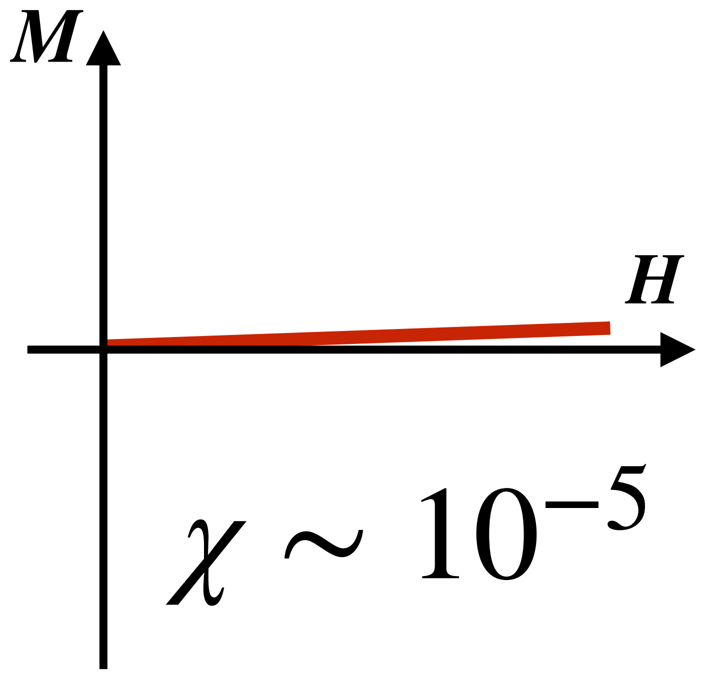


$\chi \sim -10^{-5}$

copper, water

paramagnetic

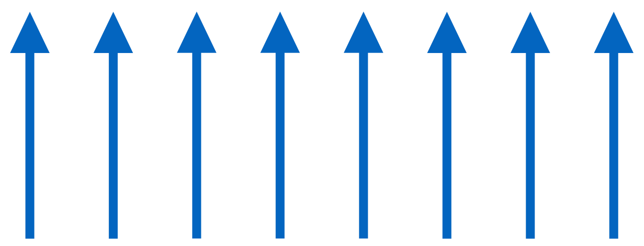


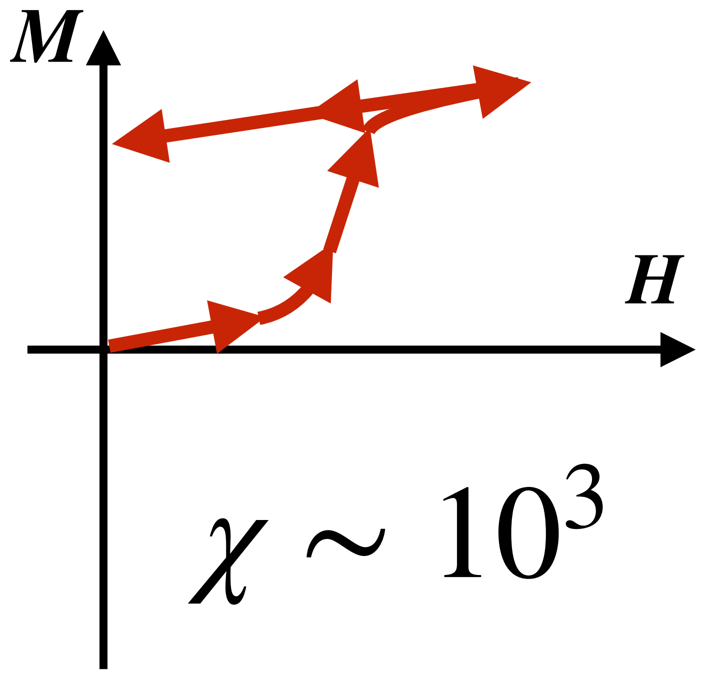


$\chi \sim 10^{-5}$

aluminium, magnesium

ferromagnetic

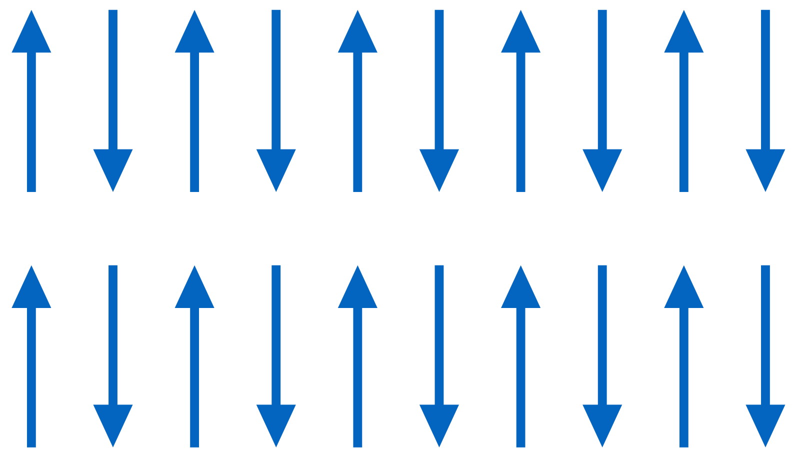


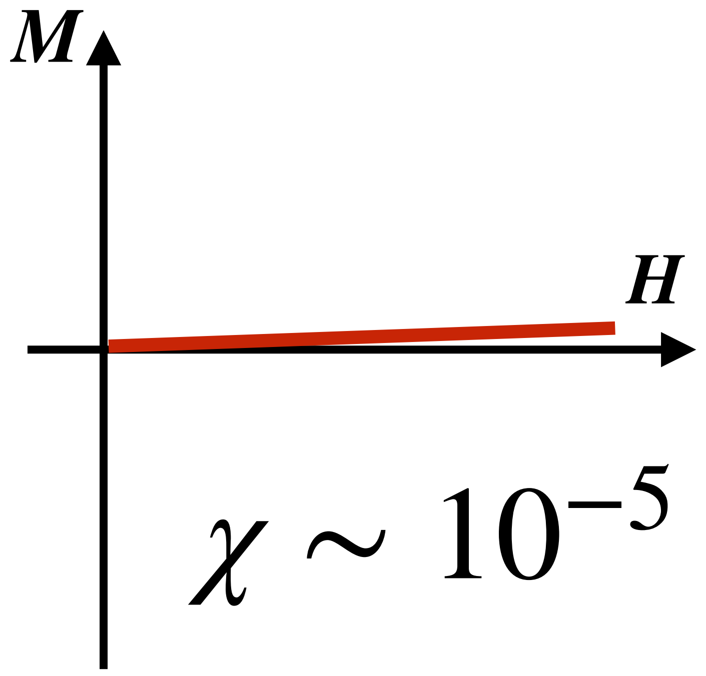


$\chi \sim 10^{3}$

iron, cobalt, nickel

antiferromagnetic





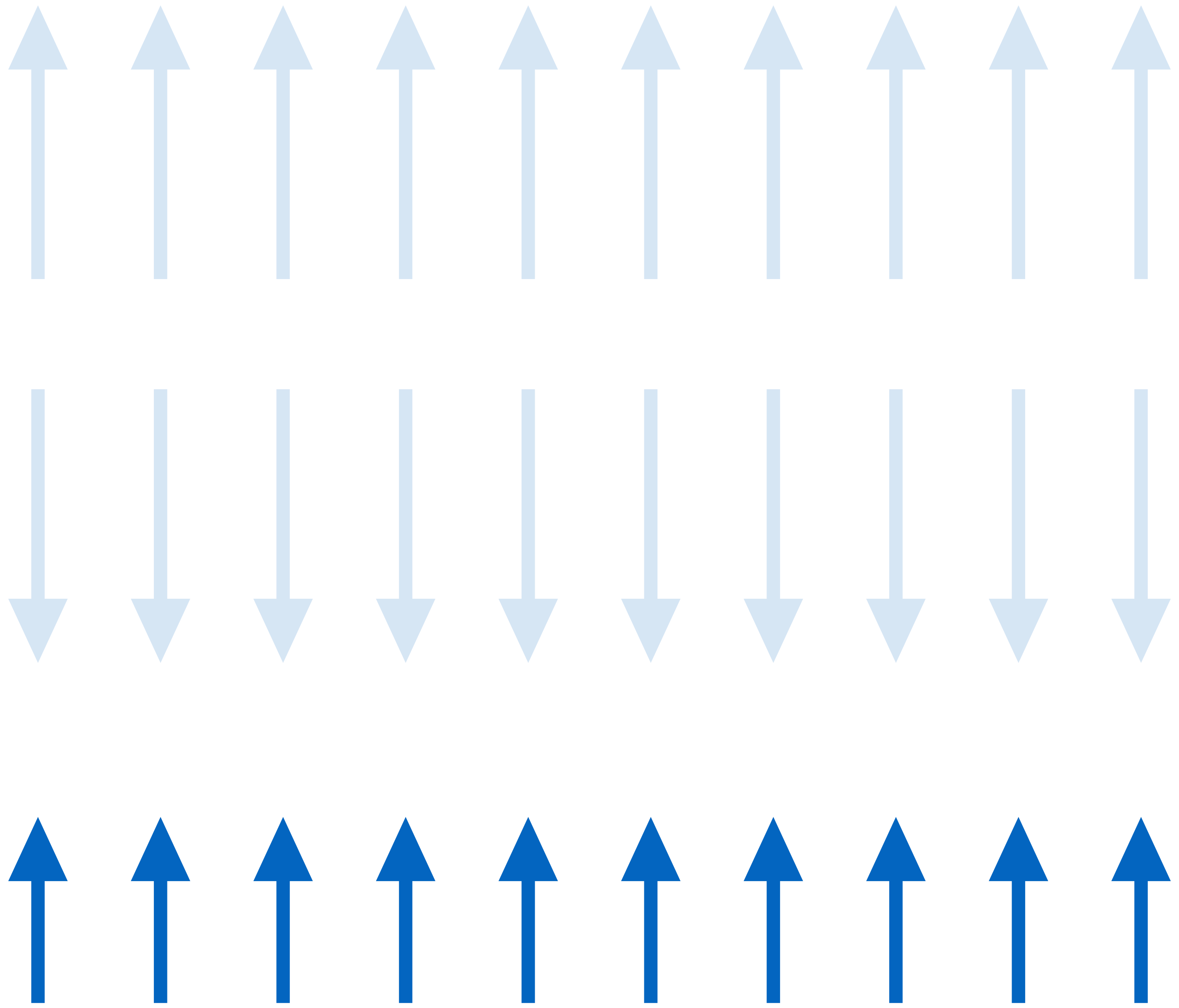
$\chi \sim 10^{-5}$

FeMn, NiO

Ferrimagnetic materials

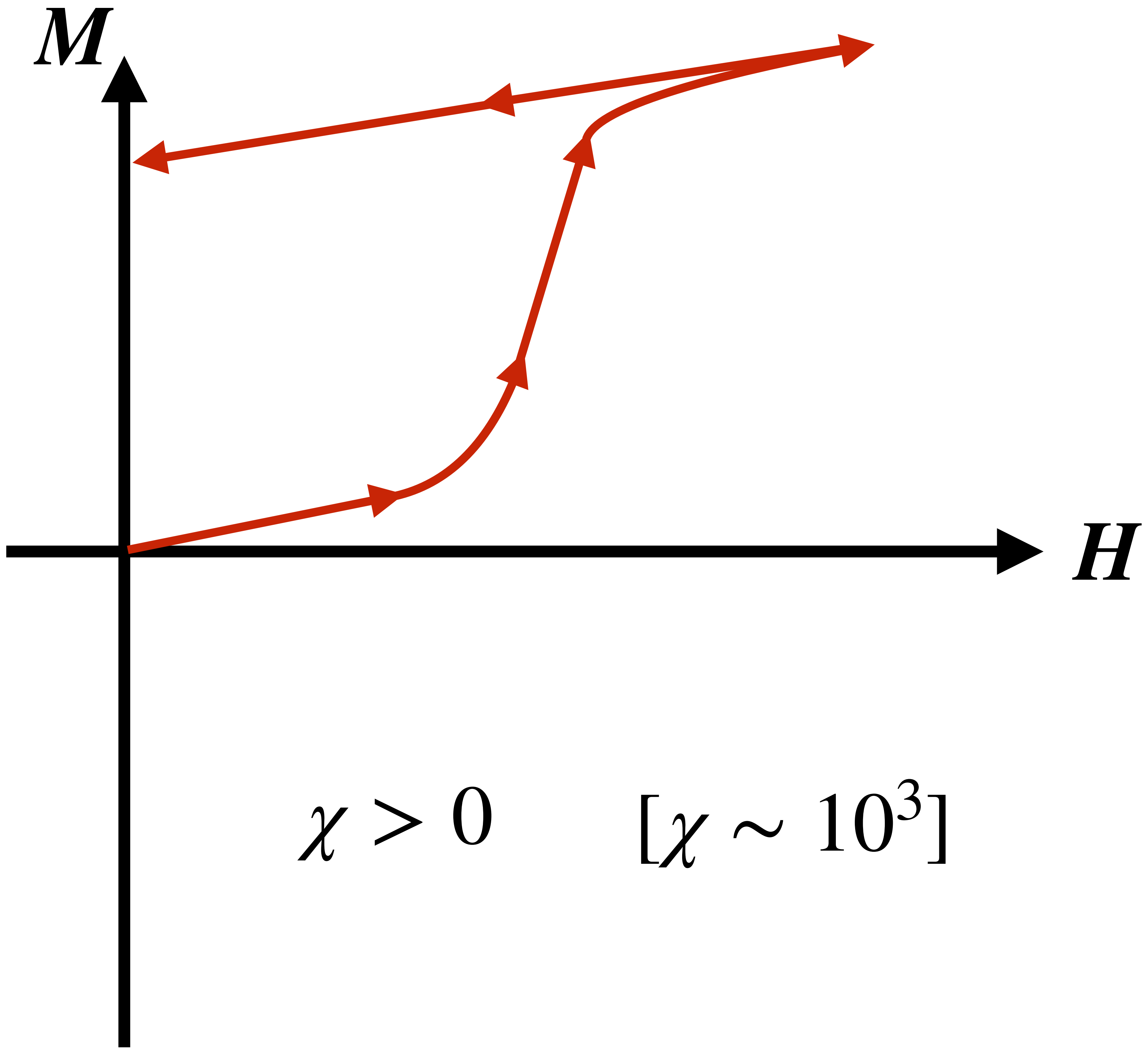
magnetisation $\boldsymbol{M}$

$$\boldsymbol{M} > \mathbf{0}$$



- Examples: Fe_3O_4, $NiFe_2O_4$, $Y_3Fe_5O_{12}$

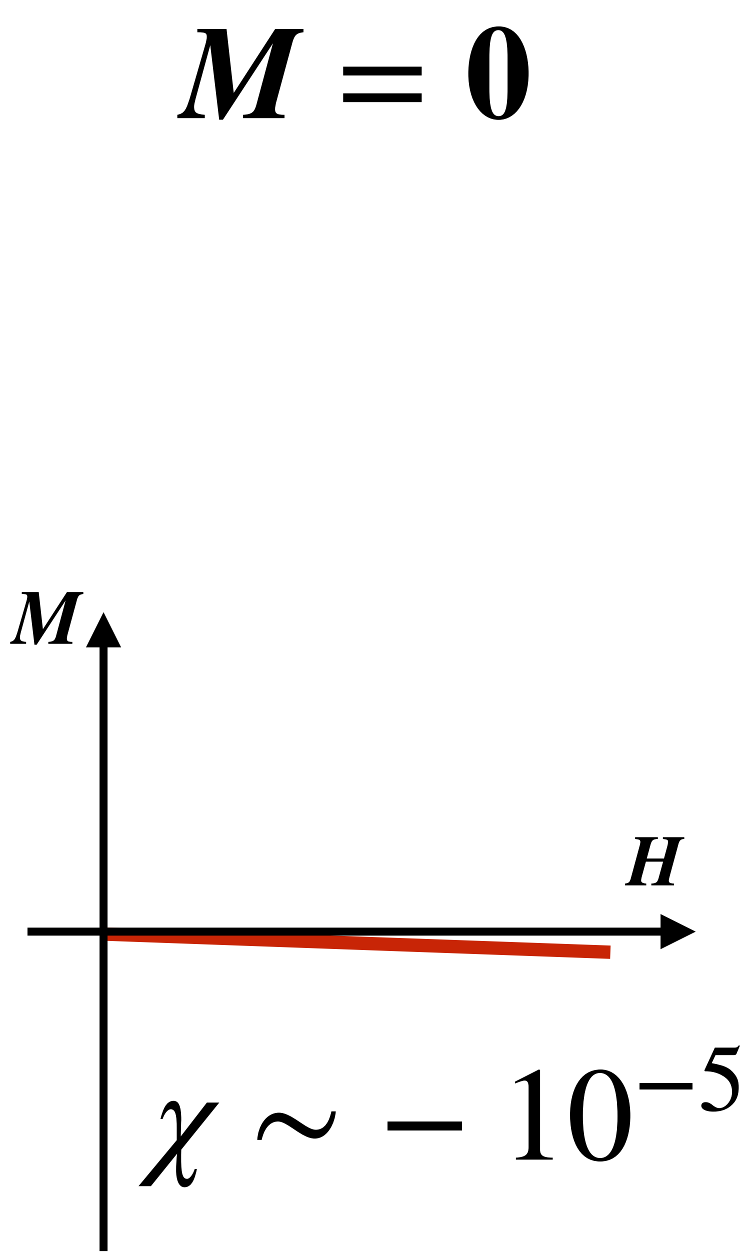
susceptibility χ $[\boldsymbol{M} = \chi \boldsymbol{H}]$



$\chi > 0$ $[\chi \sim 10^3]$

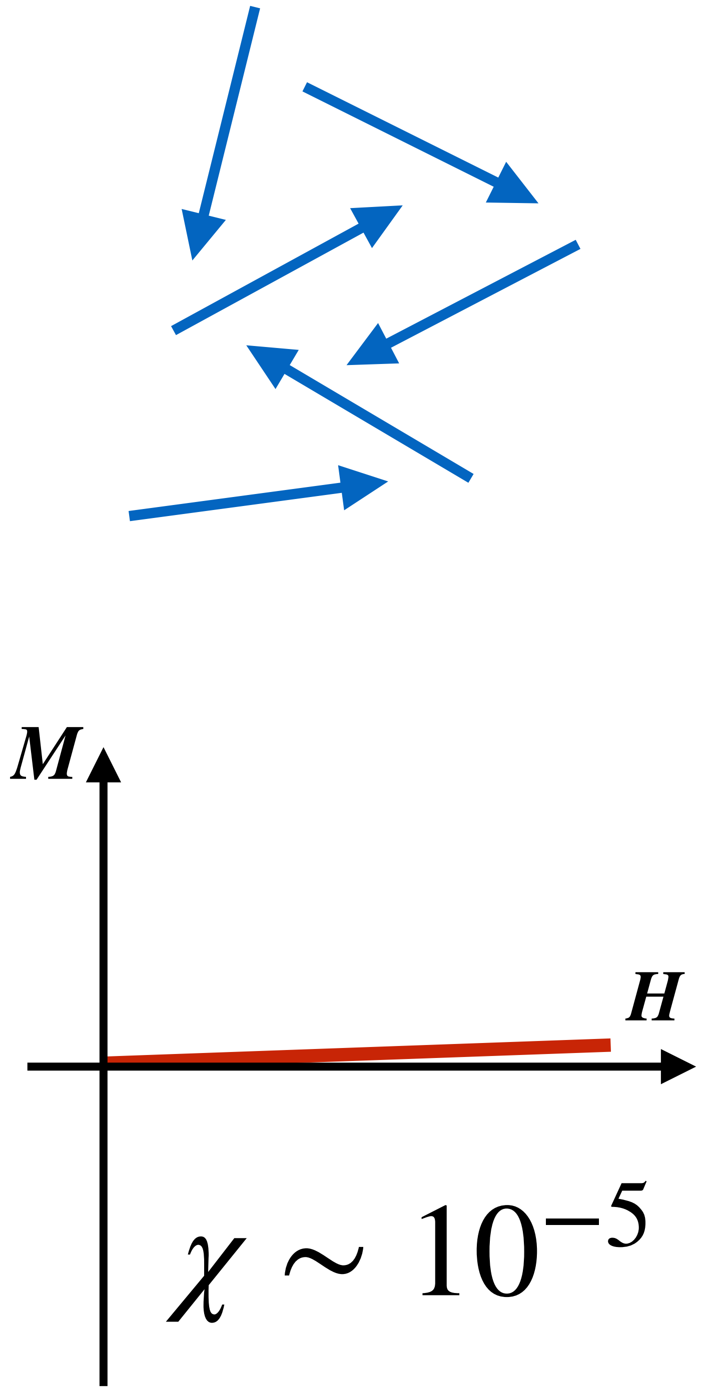
Classification of magnetic materials

diamagnetic



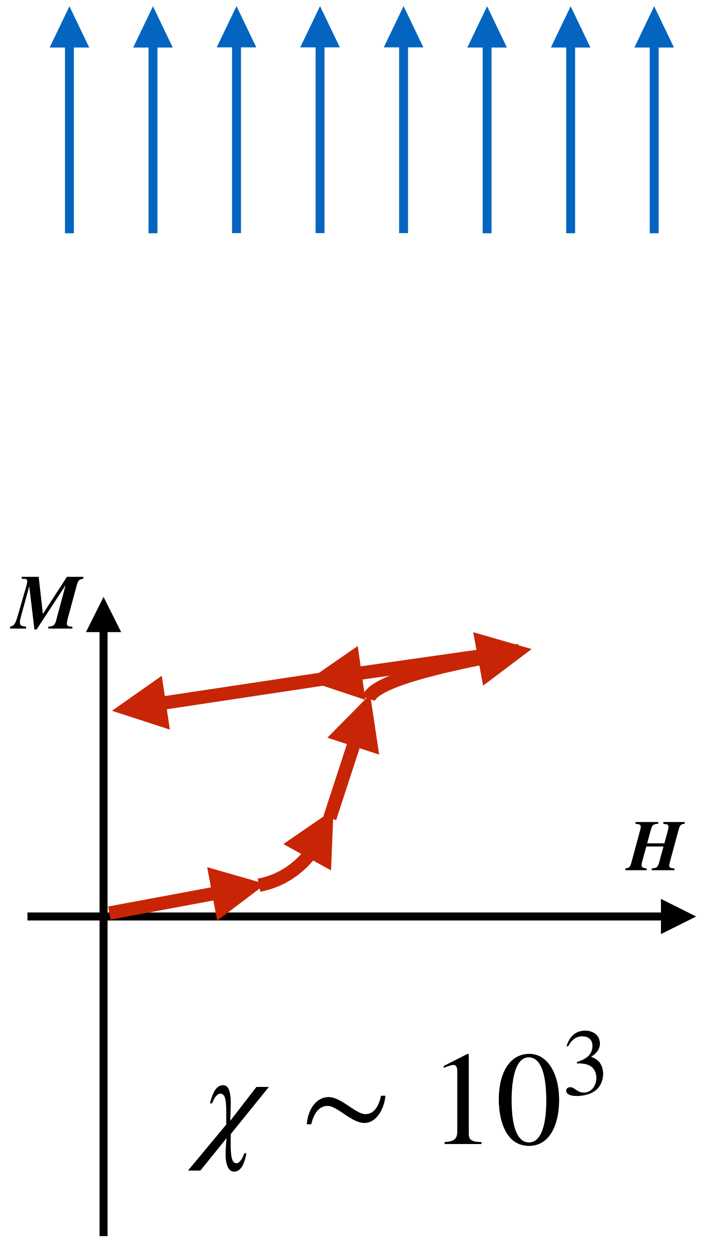
copper, water

paramagnetic



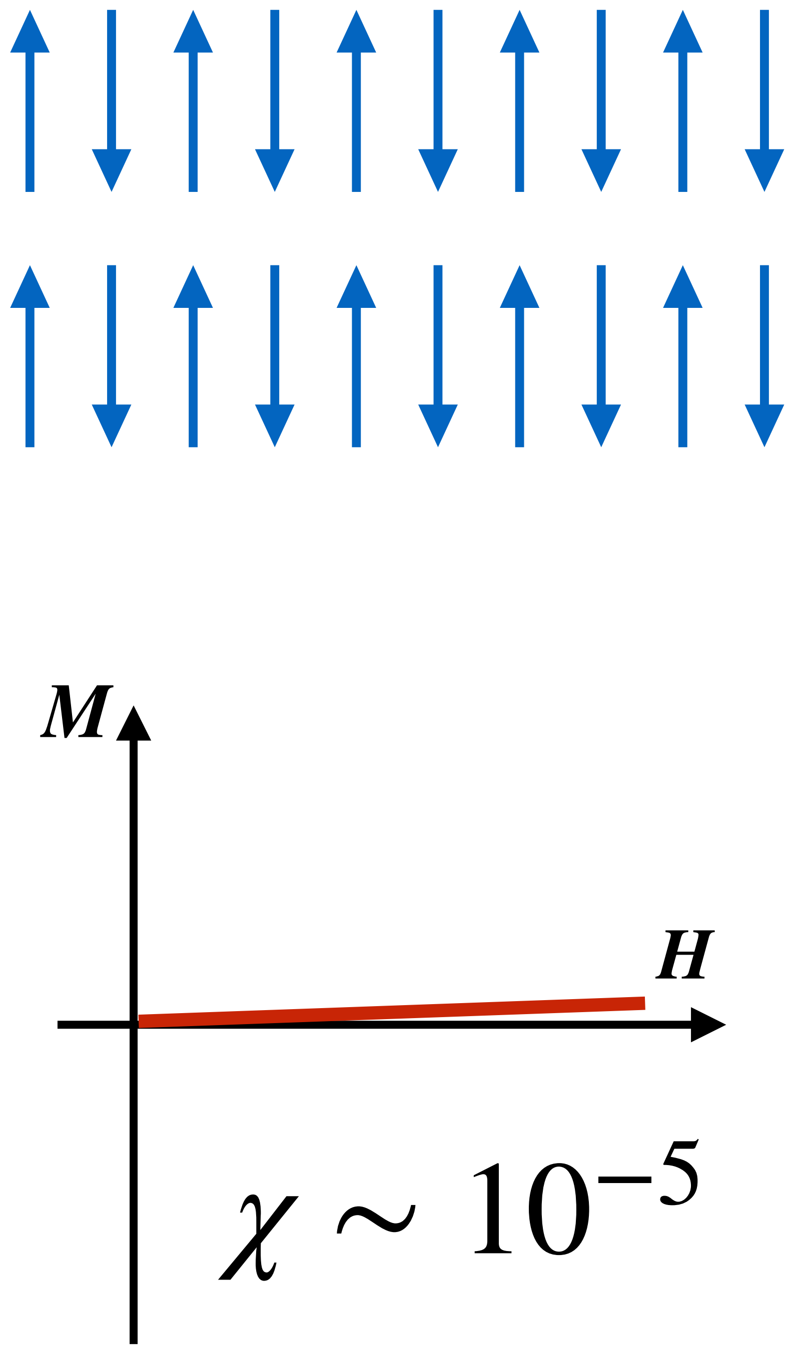
aluminium,
magnesium

ferromagnetic



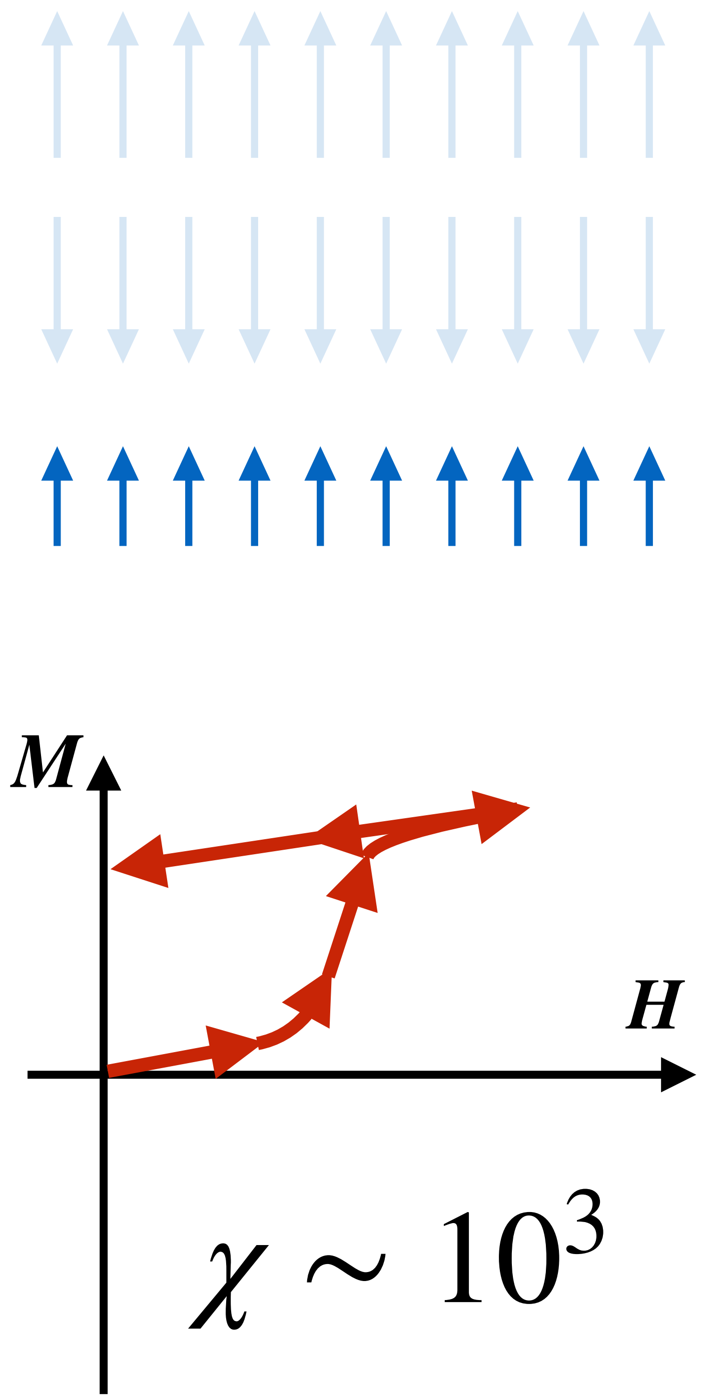
iron, cobalt,
nickel

antiferromagnetic



FeMn, NiO

ferrimagnetic

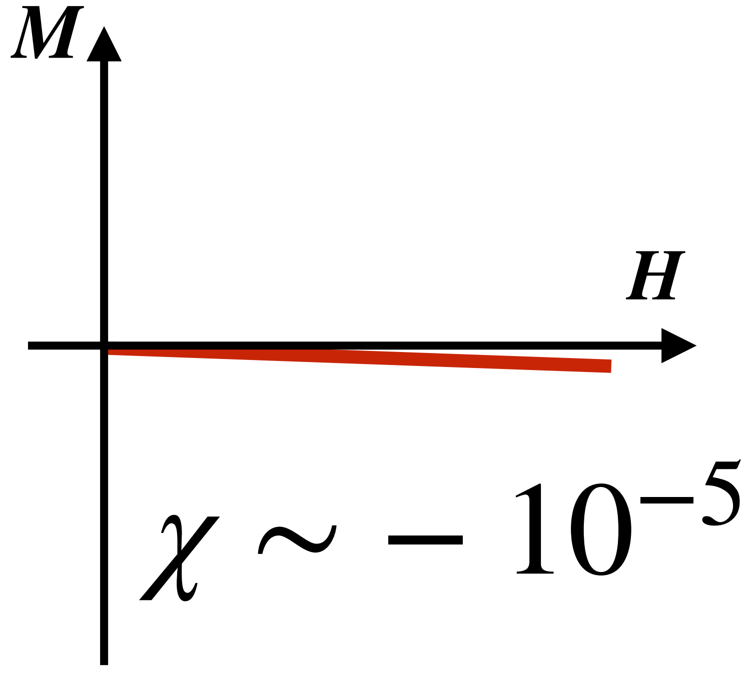


Fe_3O_4, $NiFe_2O_4$,
$Y_3Fe_5O_{12}$

Classification of magnetic materials

diamagnetic

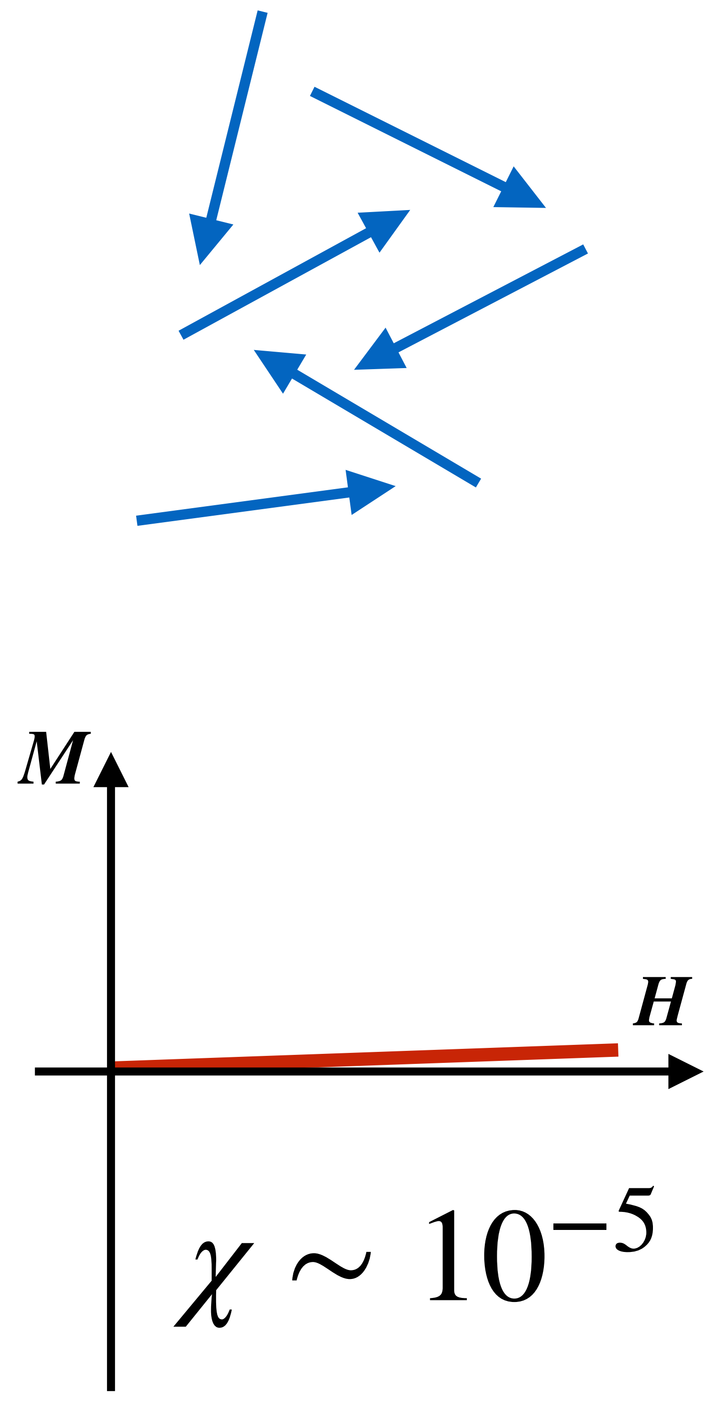
$M = 0$



$\chi \sim -10^{-5}$

copper, water

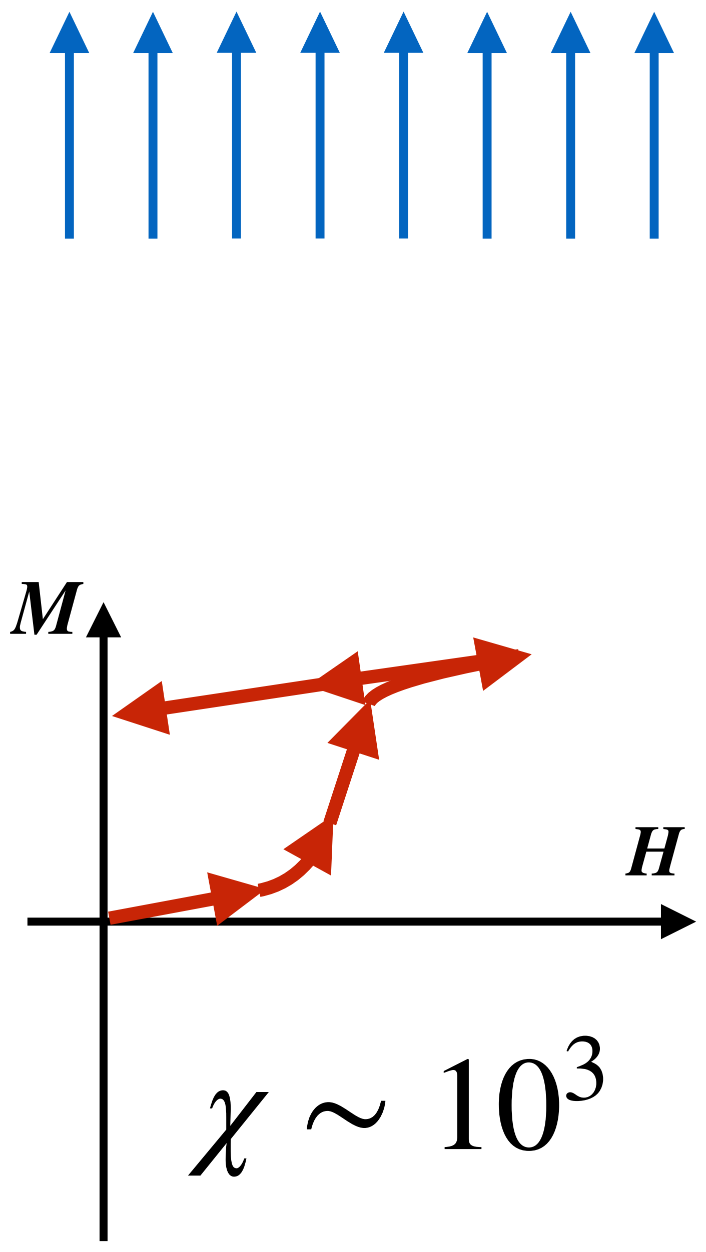
paramagnetic



$\chi \sim 10^{-5}$

aluminium,
magnesium

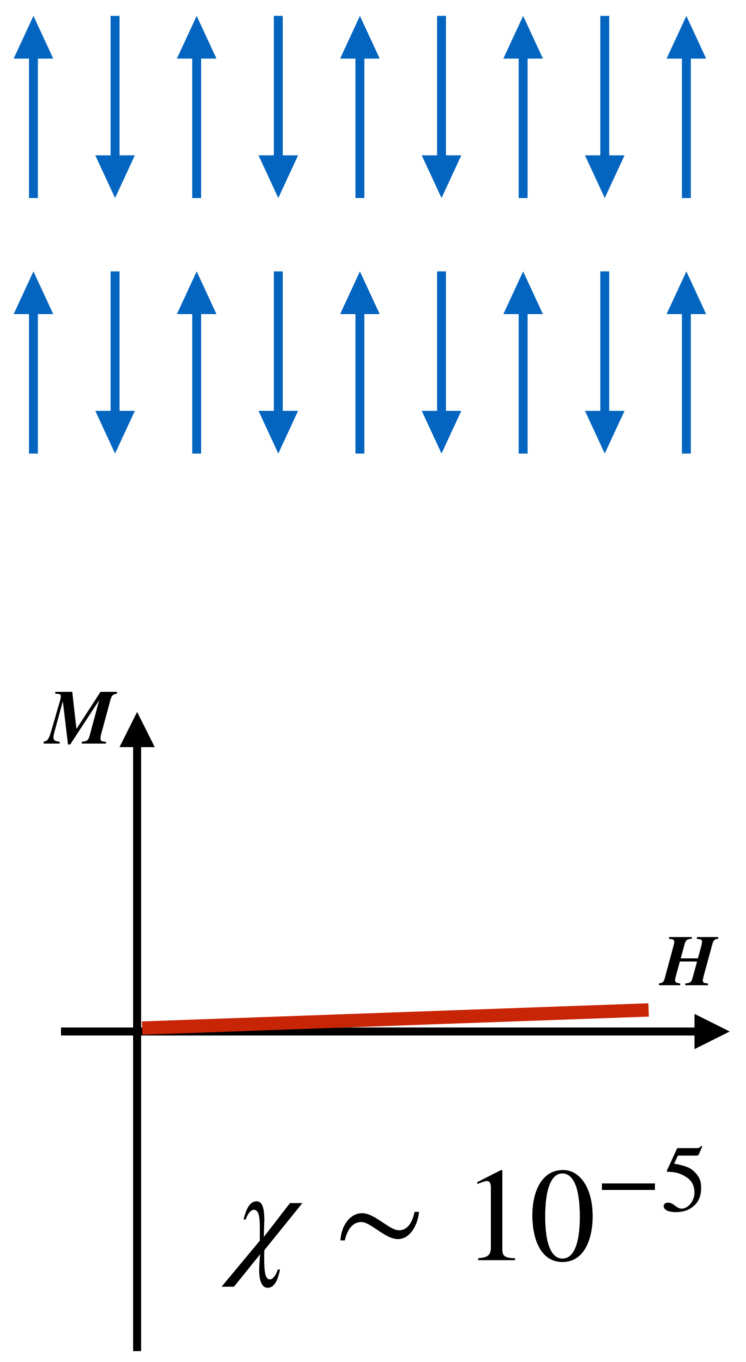
ferromagnetic



$\chi \sim 10^{3}$

iron, cobalt,
nickel

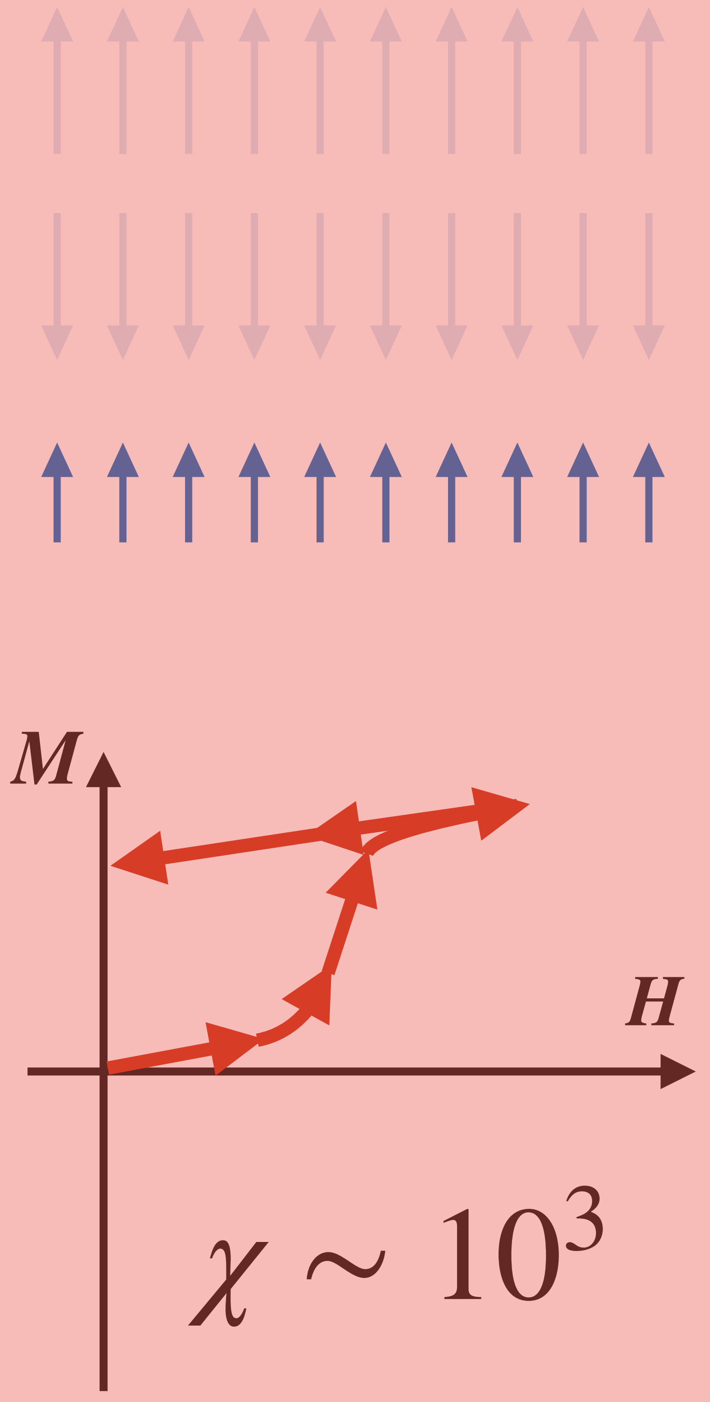
antiferromagnetic



$\chi \sim 10^{-5}$

FeMn, NiO

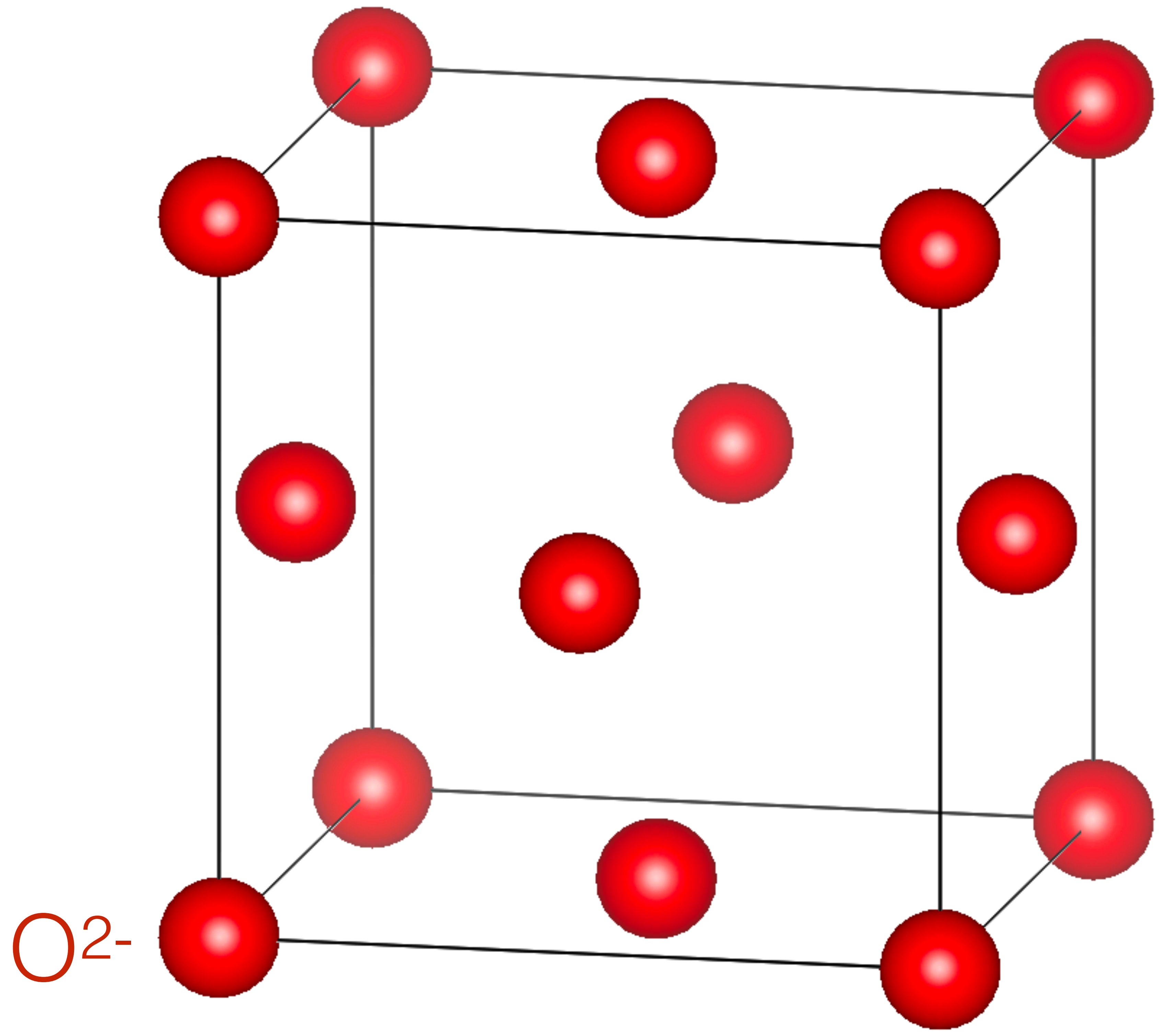
ferrimagnetic



$\chi \sim 10^{3}$

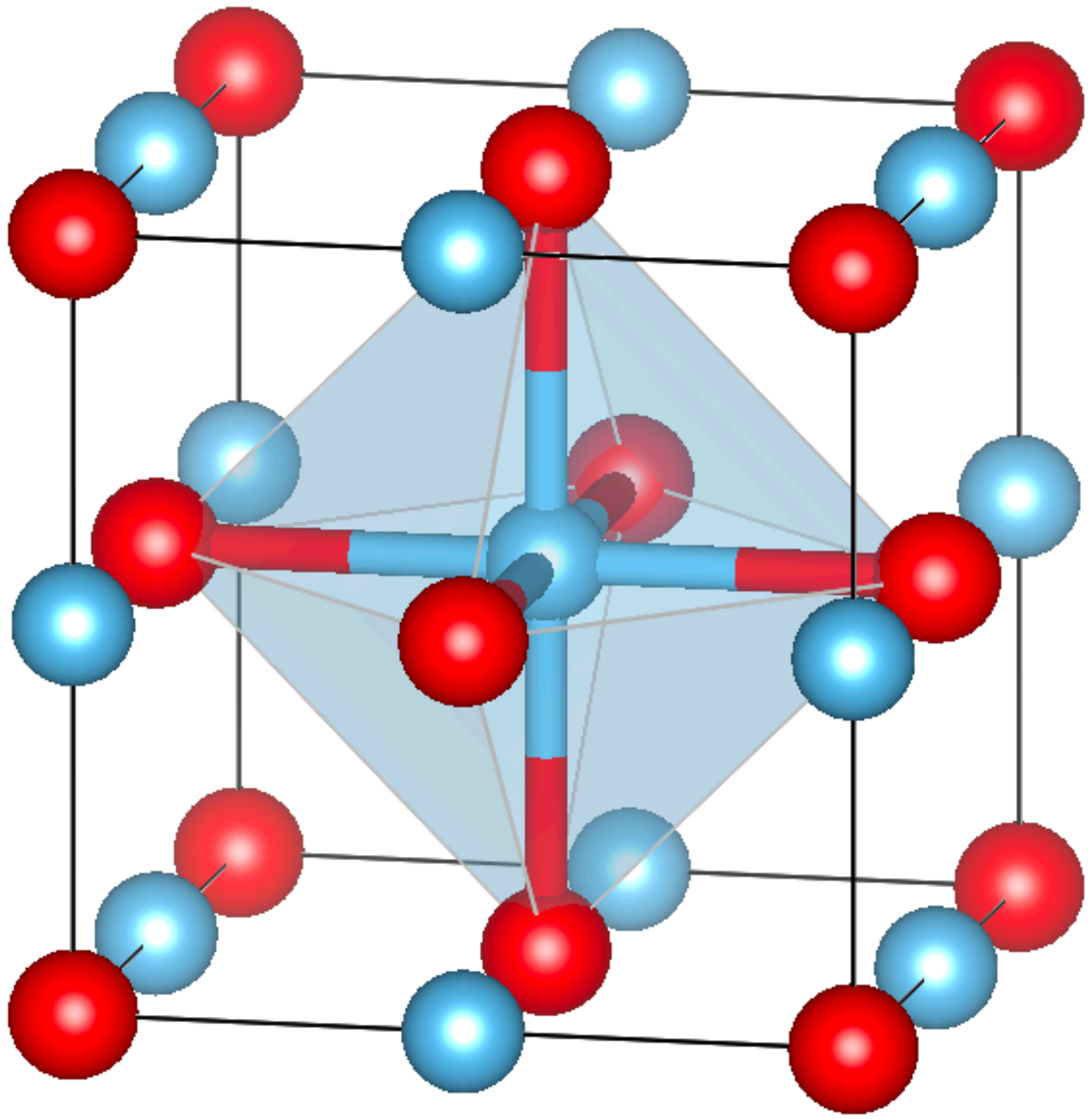
Fe_3O_4, $NiFe_2O_4$,
$Y_3Fe_5O_{12}$

Spinel structure $MgAl_2O_4$



- Oxygen sublattice
- Face-centred cubic (fcc)
- Conventional cell

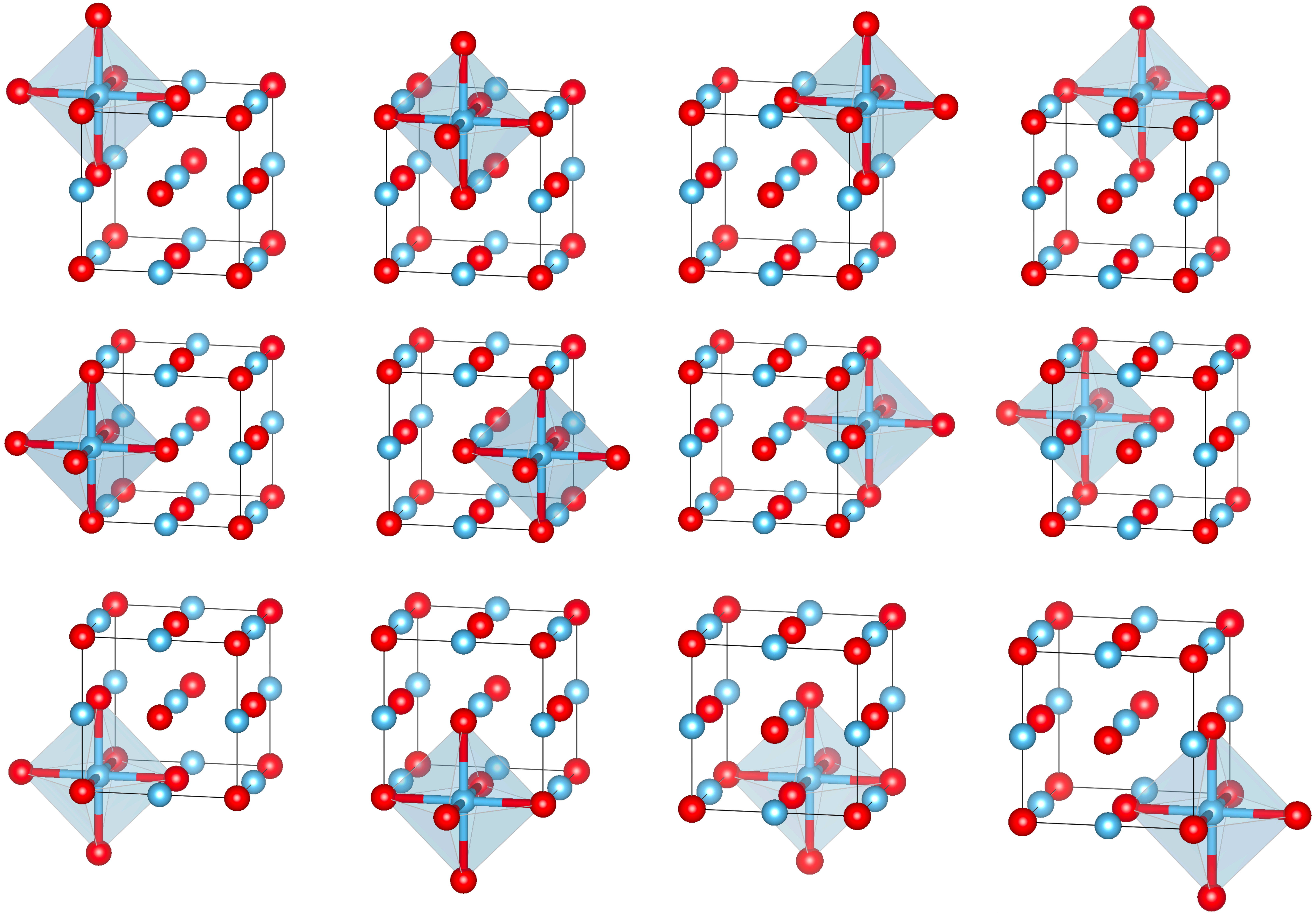
Spinel structure $MgAl_2O_4$



octahedral
site

- Octahedral interstices:
  - One at the centre

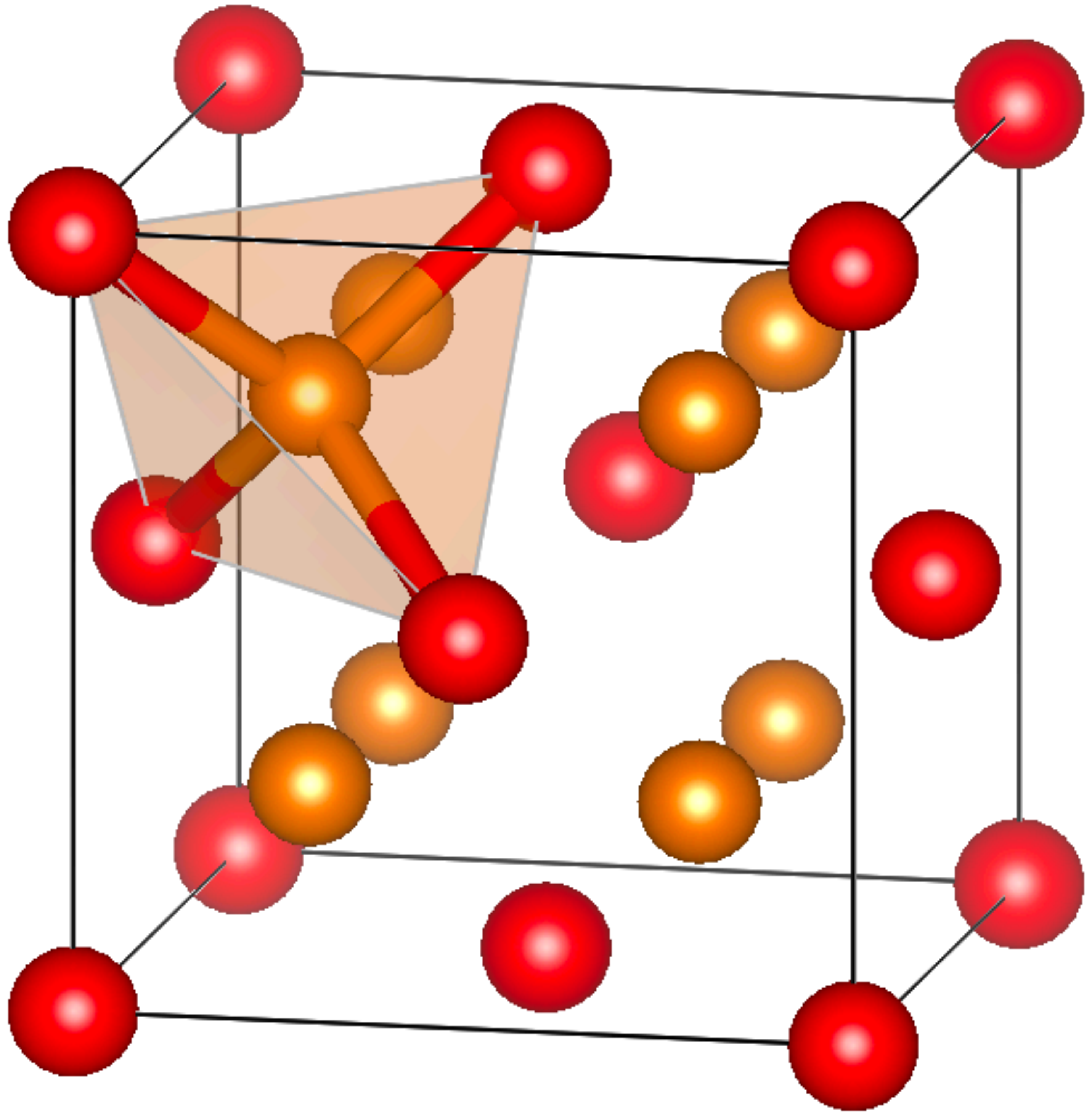
Spinel structure $MgAl_2O_4$



- Octahedral interstices:
  - One at the centre
  - 12 at the edge

$$1 + \frac{12}{4} = 4$$

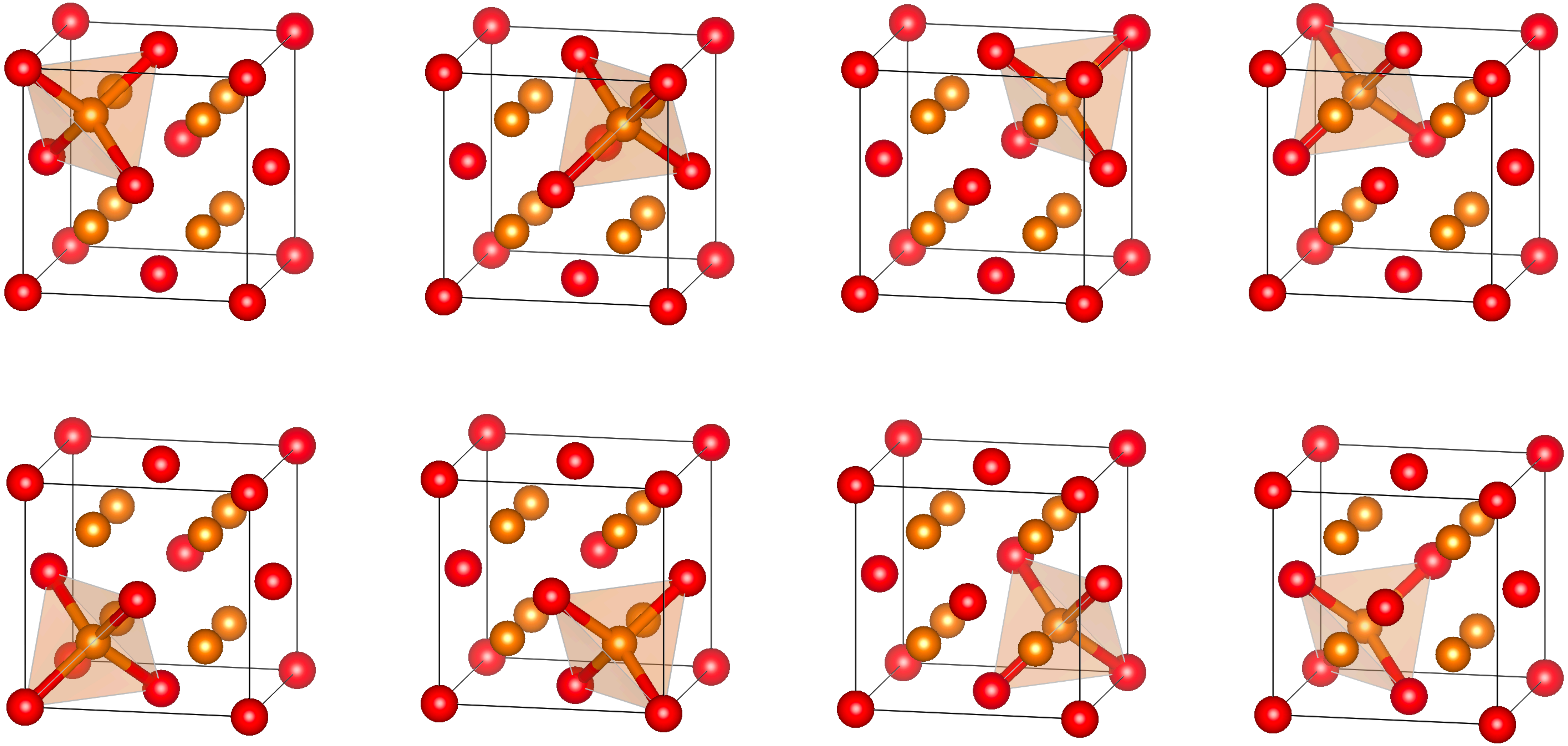
Spinel structure $MgAl_2O_4$



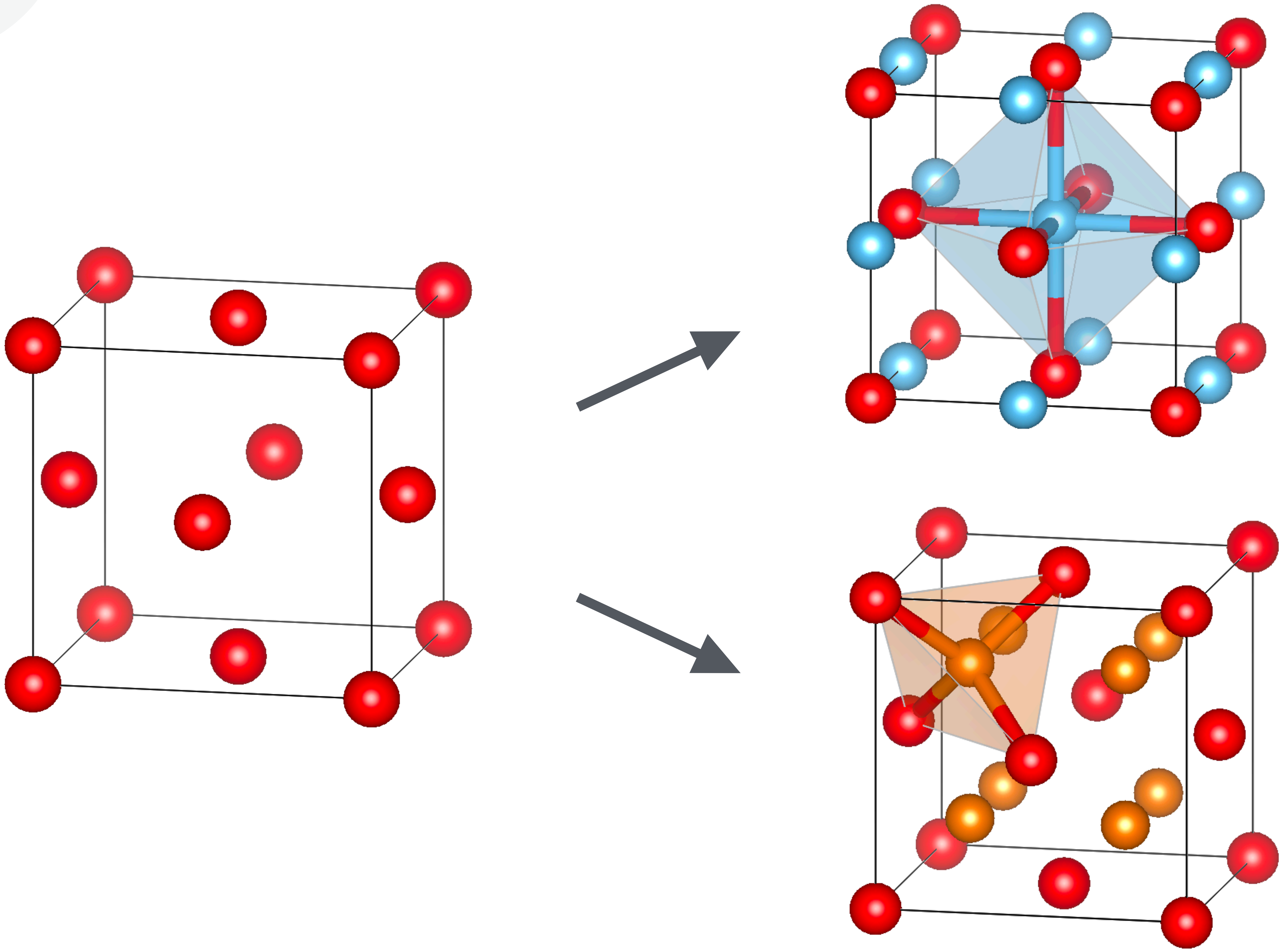
tetrahedral site

Spinel structure $MgAl_2O_4$

tetrahedral site



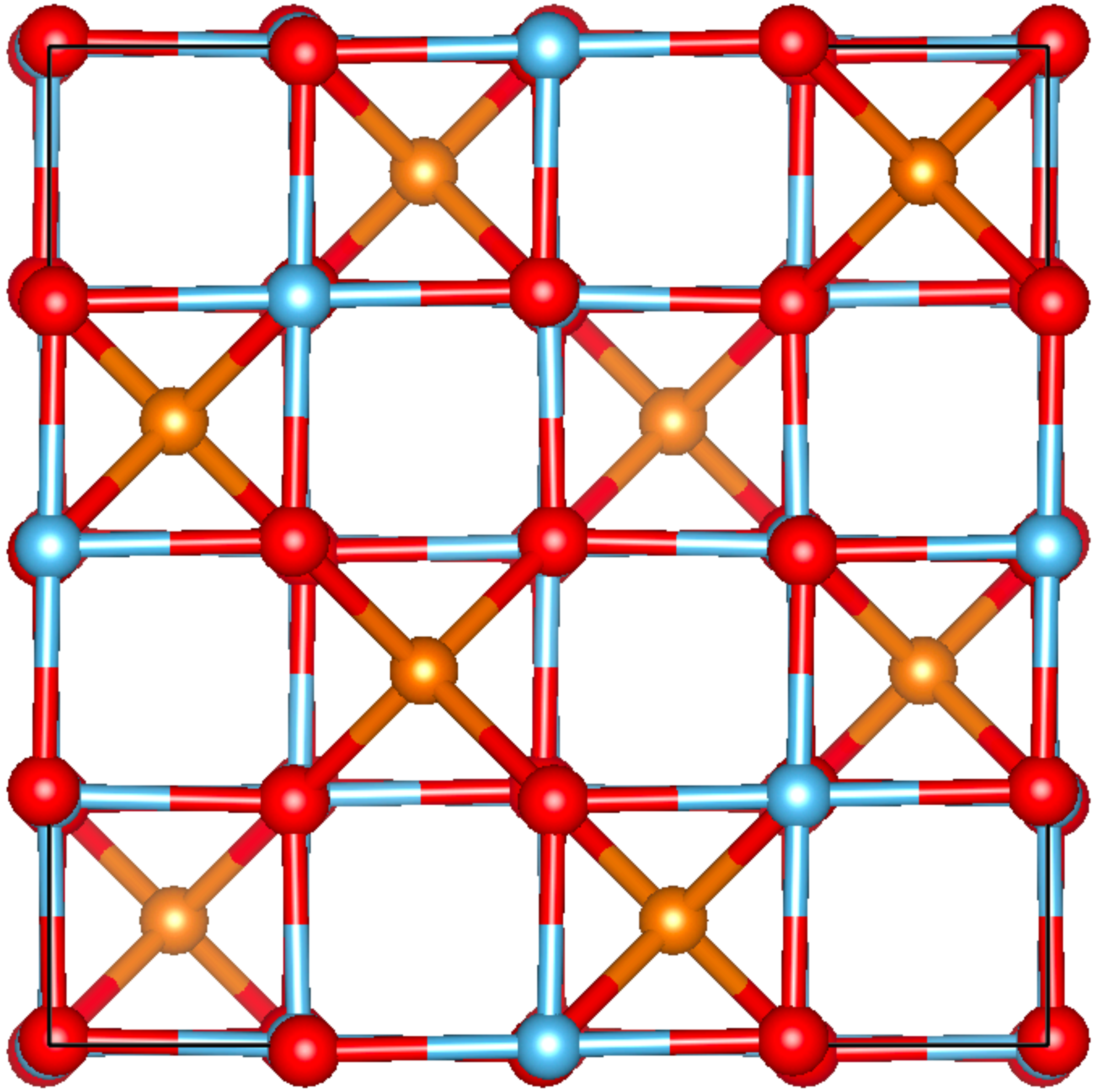
Spinel structure $MgAl_2O_4$



4 octahedral sites

8 tetrahedral sites

Spinel structure $MgAl_2O_4$



$2 \times 2 \times 2$ supercell of fcc conventional O^{2-} sublattice

$8 \times 4 = 32$ octahedral sites

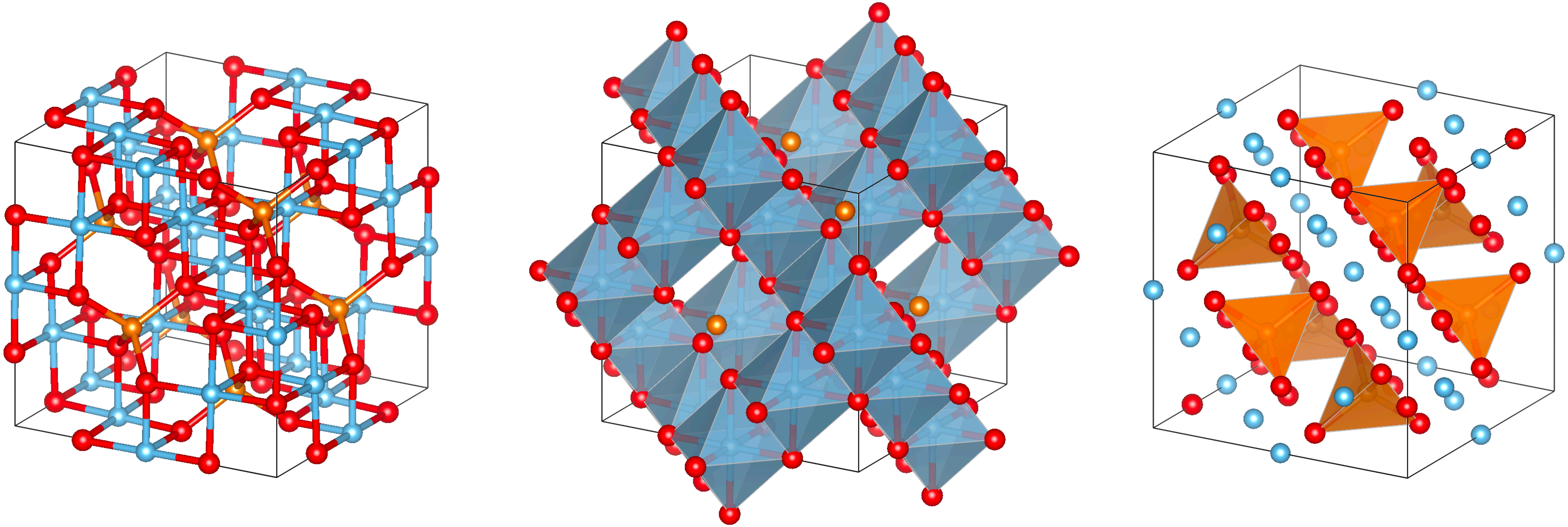
$\frac{1}{2} \times 32 = 16$ sites occupied by Al^{3+}

$8 \times 8 = 64$ tetrahedral sites

$\frac{1}{8} \times 64 = 8$ sites occupied by Mg^{2+}

- *See structure model in 3D*

Spinel structure $MgAl_2O_4$



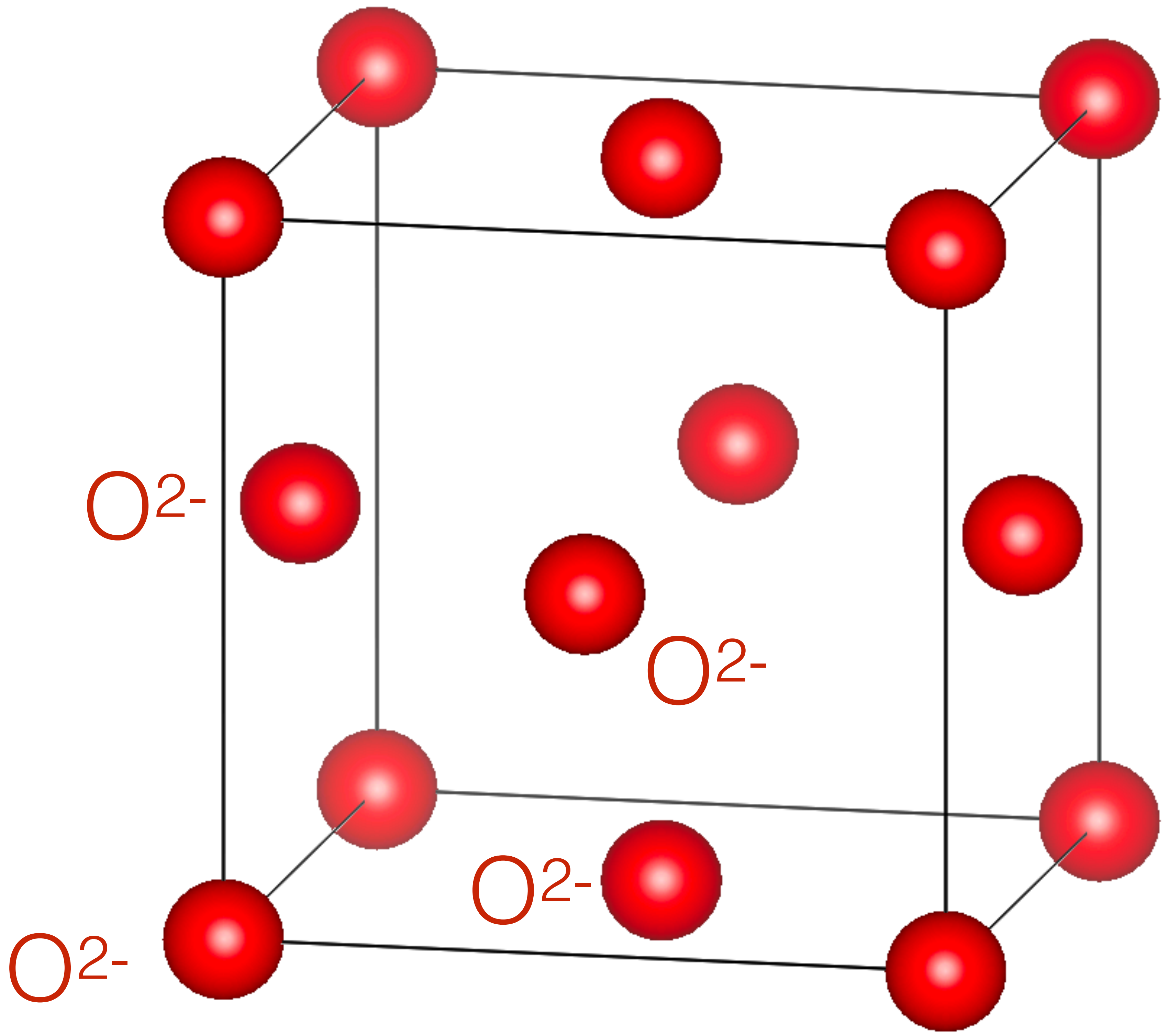
$\frac{1}{2} \times 32 = 16$ octahedral sites

are occupied by Al^{3+}

$\frac{1}{8} \times 64 = 8$ tetrahedral sites

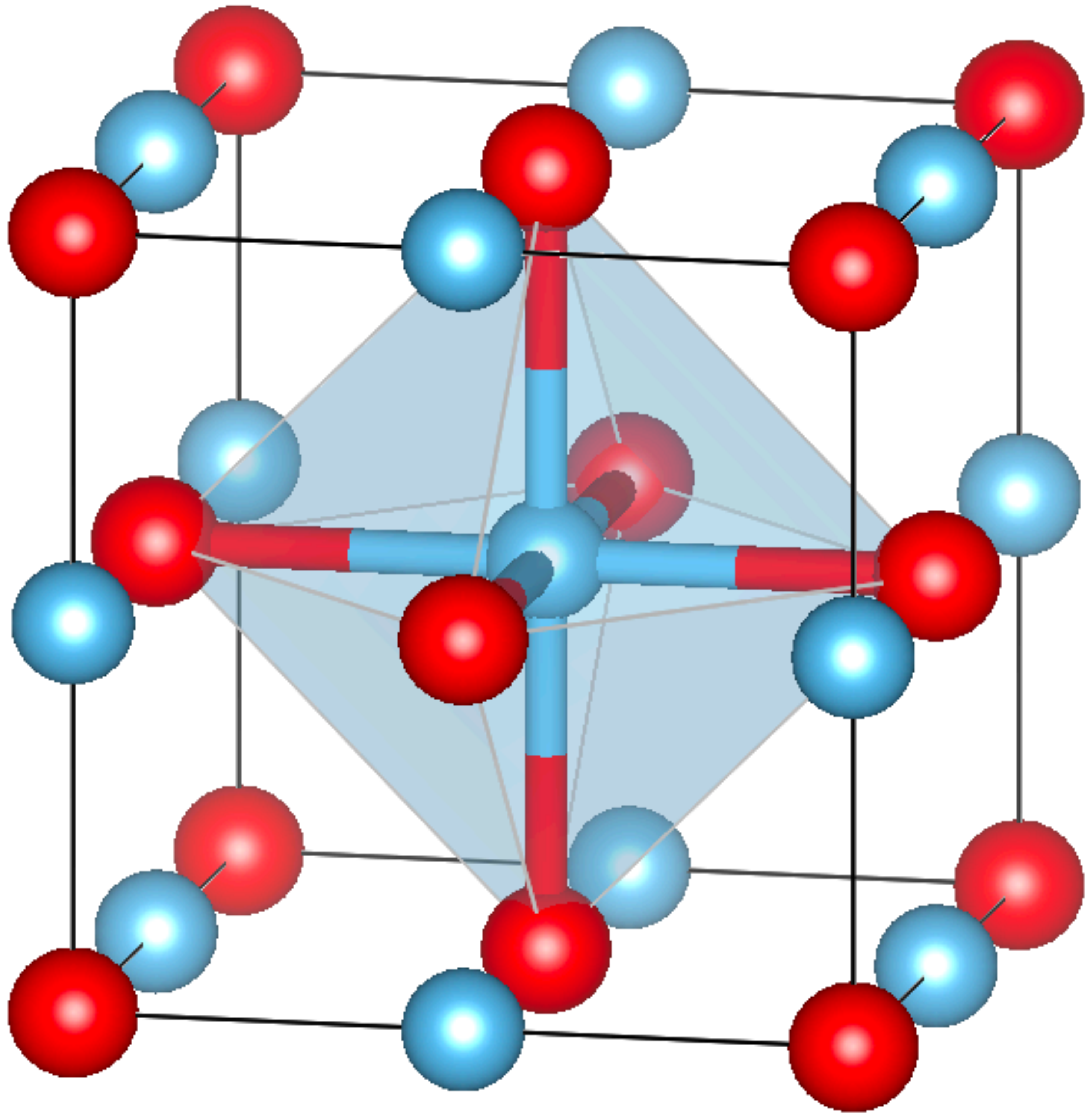
are occupied by Mg^{2+}

Spinel structure $MgAl_2O_4$



4 O^{2-}

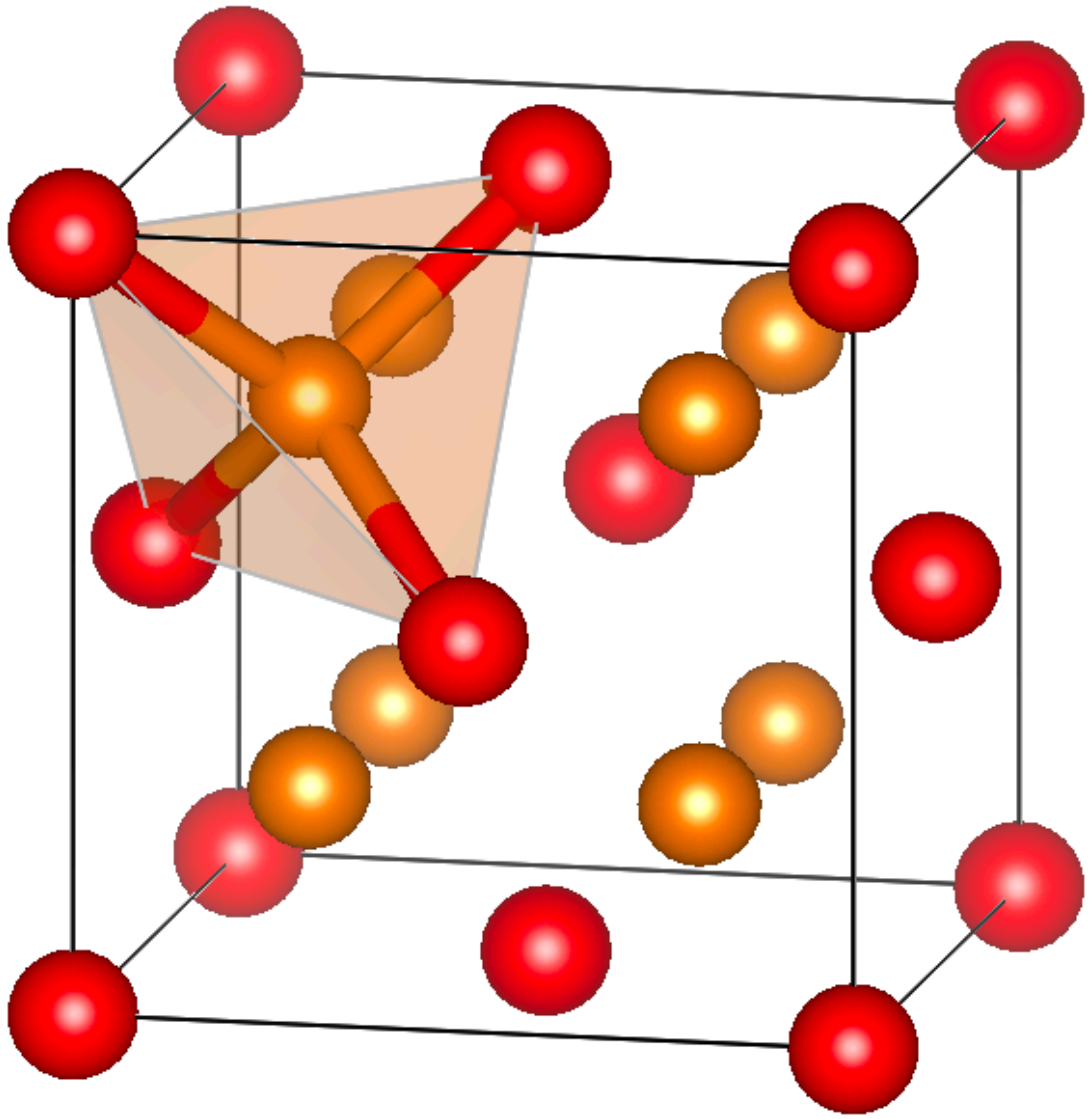
Spinel structure $MgAl_2O_4$



$4\ O^{2-}$

4 octahedral sites: $\frac{1}{2} \times 4 = 2\ Al^{3+}$

Spinel structure $MgAl_2O_4$

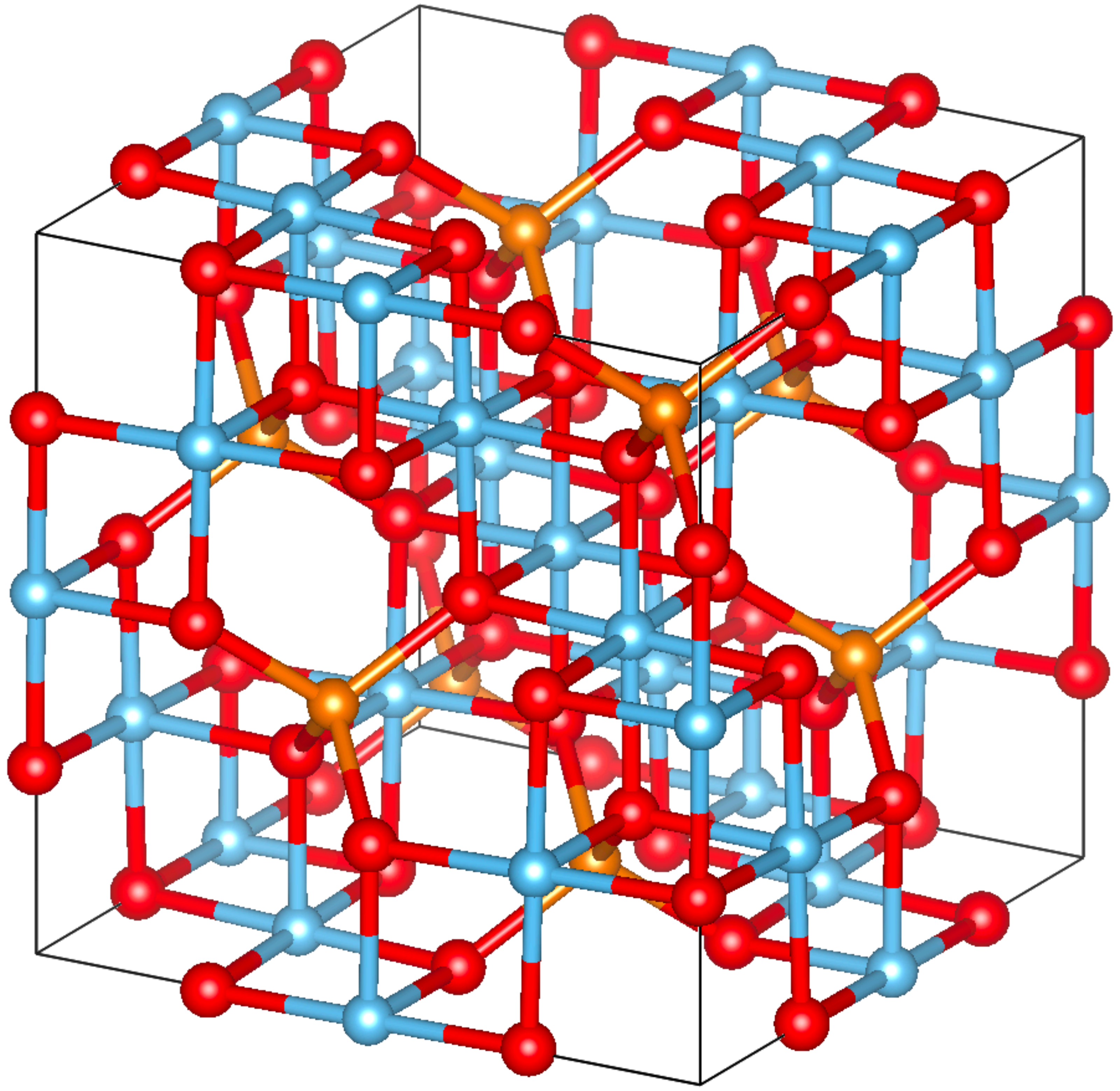


$4\ O^{2-}$

4 octahedral sites: $\frac{1}{2} \times 4 = 2\ Al^{3+}$

8 tetrahedral sites: $\frac{1}{8} \times 8 = 1\ Mg^{2+}$

Spinel structure $MgAl_2O_4$



$4\ O^{2-}$

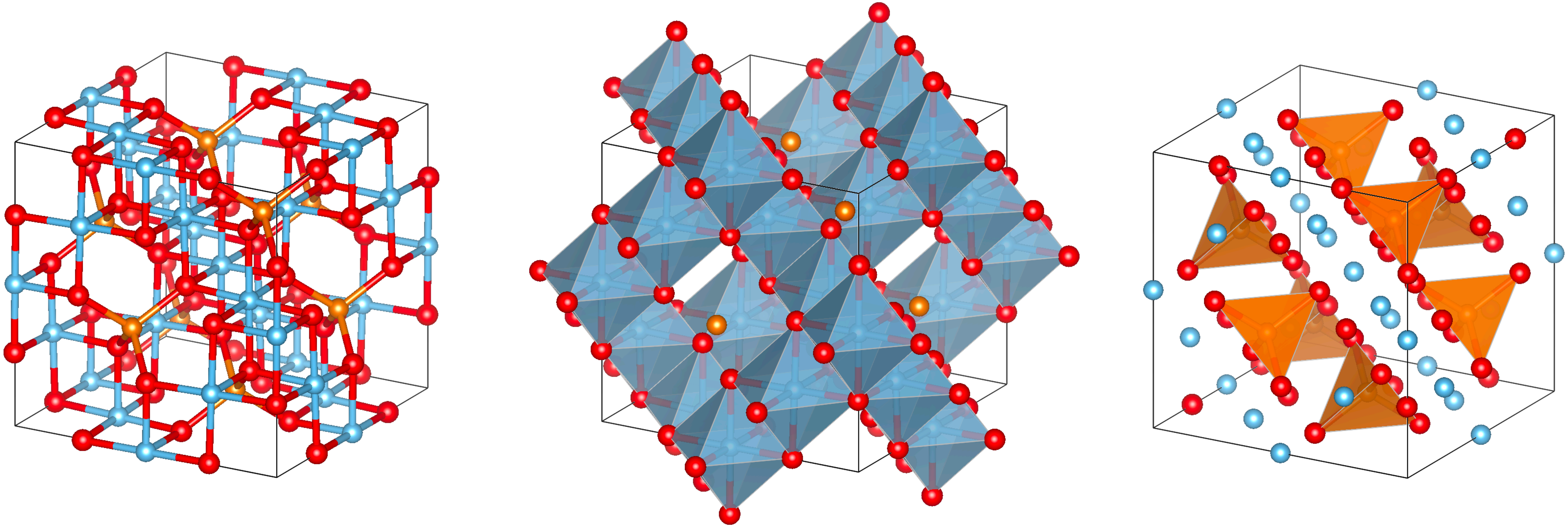
4 octahedral sites: $\frac{1}{2} \times 4 = 2\ Al^{3+}$

8 tetrahedral sites: $\frac{1}{8} \times 8 = 1\ Mg^{2+}$

- Charge balance:

$$4 \times (-2) + 2 \times (+3) + 1 \times (+2) = -8 + 6 + 2 = 0$$

Inverse spinel structure Fe_3O_4 (magnetite)



$\frac{1}{2} \times 32 = 16$ octahedral sites

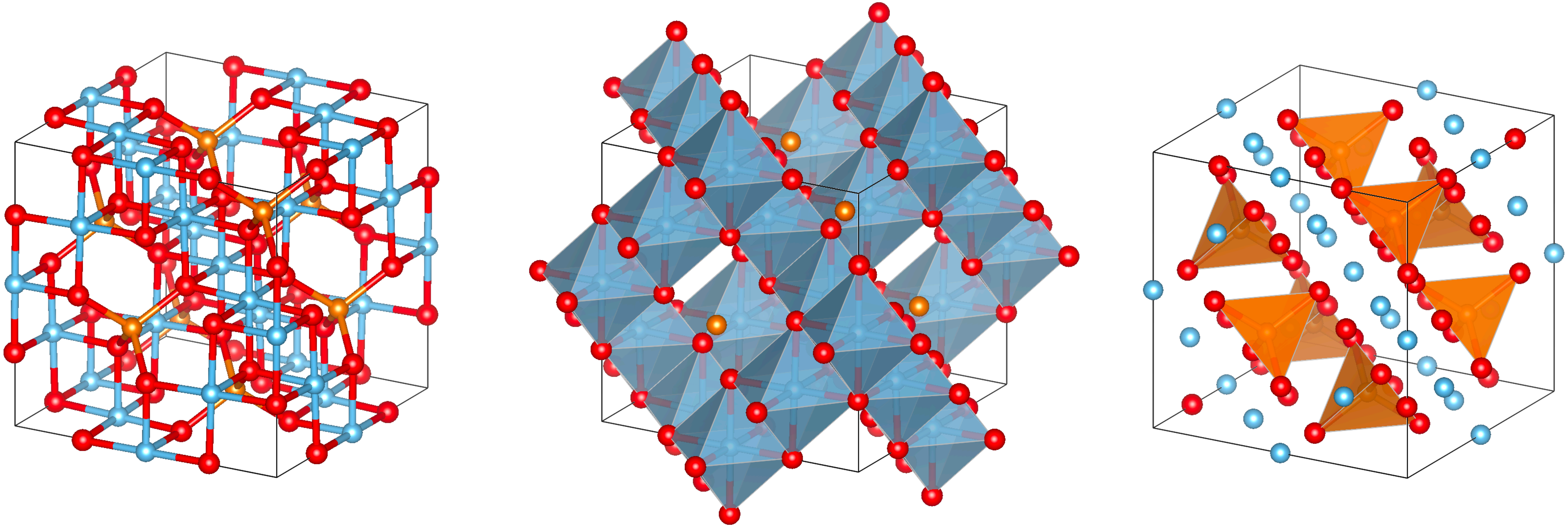
8 are occupied by Fe^{2+}

8 are occupied by Fe^{3+}

$\frac{1}{8} \times 64 = 8$ tetrahedral sites

8 are occupied by Fe^{3+}

Inverse spinel structure Fe_3O_4 (magnetite)



$$\boxed{(\mathrm{Fe}^{3+})_{\mathrm{tetra}}(\mathrm{Fe}^{2+}\mathrm{Fe}^{3+})_{\mathrm{octa}}\mathrm{O}_4^{2-}}$$

$\frac{1}{2} \times 32 = 16$ octahedral sites

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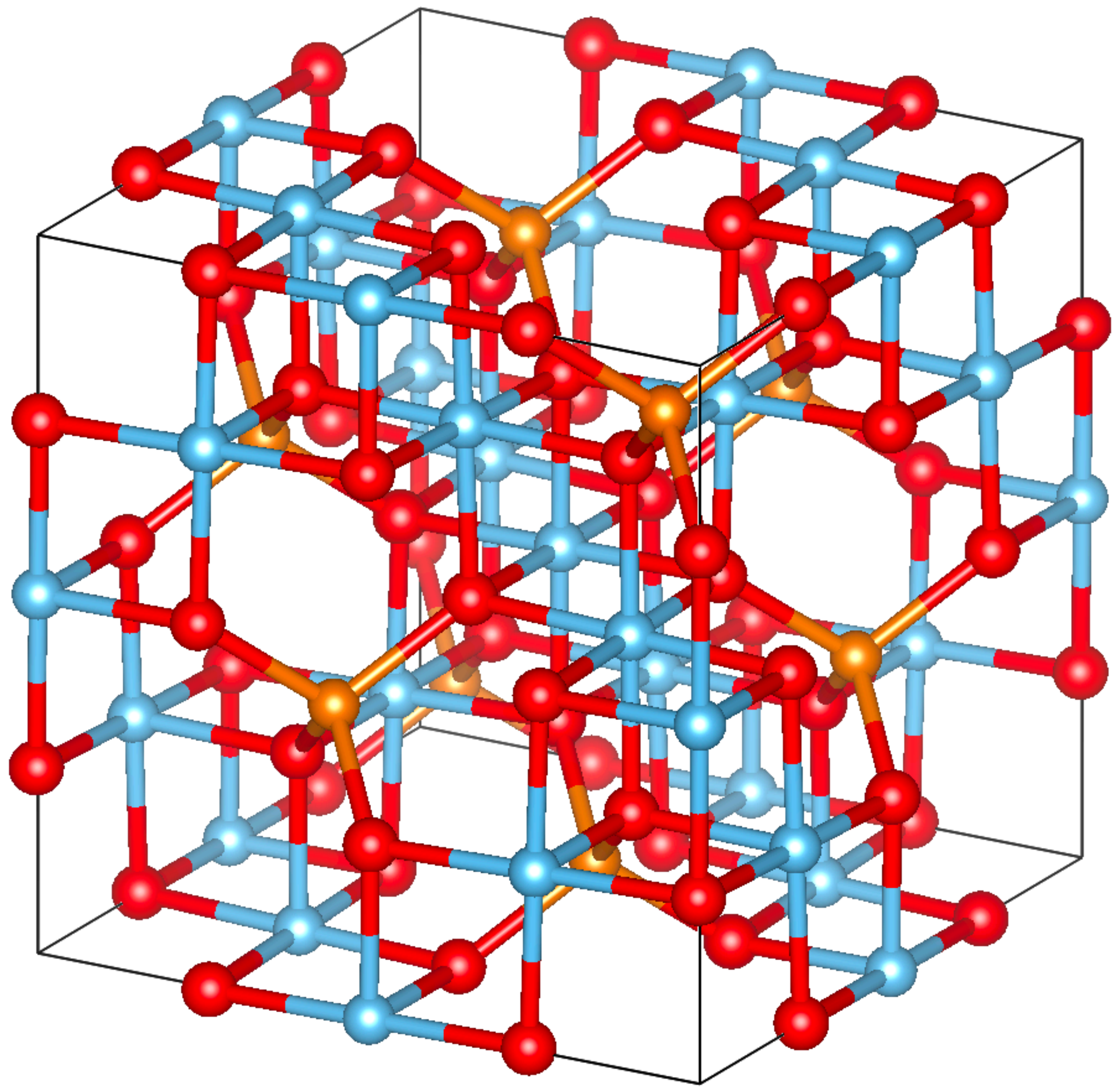
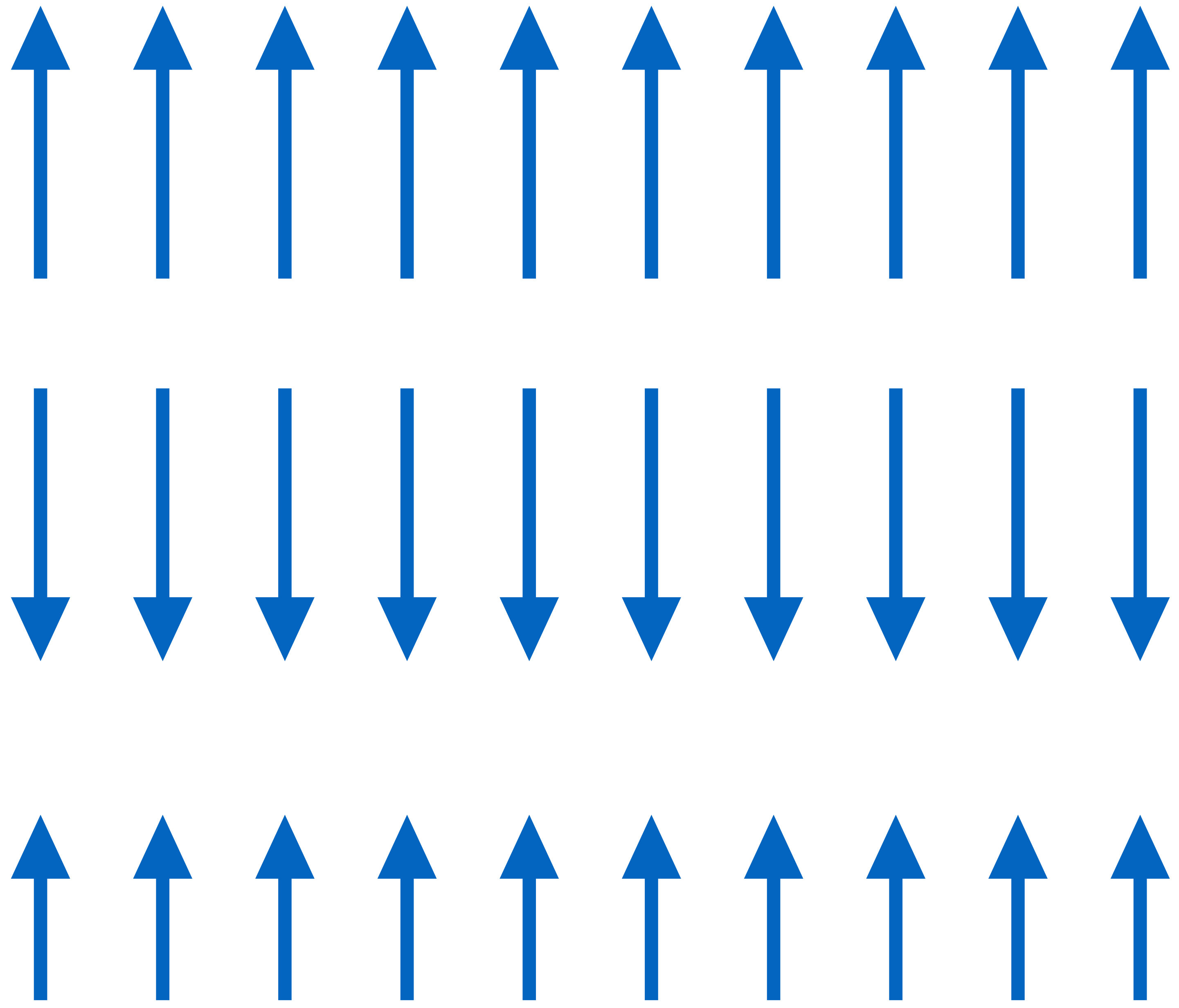
$\frac{1}{8} \times 64 = 8$ tetrahedral sites

8 are occupied by Fe^{3+}

Inverse spinel structure Fe_3O_4 (magnetite)



$$\boxed{(\mathrm{Fe}^{3+})_{\mathrm{tetra}}(\mathrm{Fe}^{2+}\mathrm{Fe}^{3+})_{\mathrm{octa}}\mathrm{O}_4^{2-}}$$

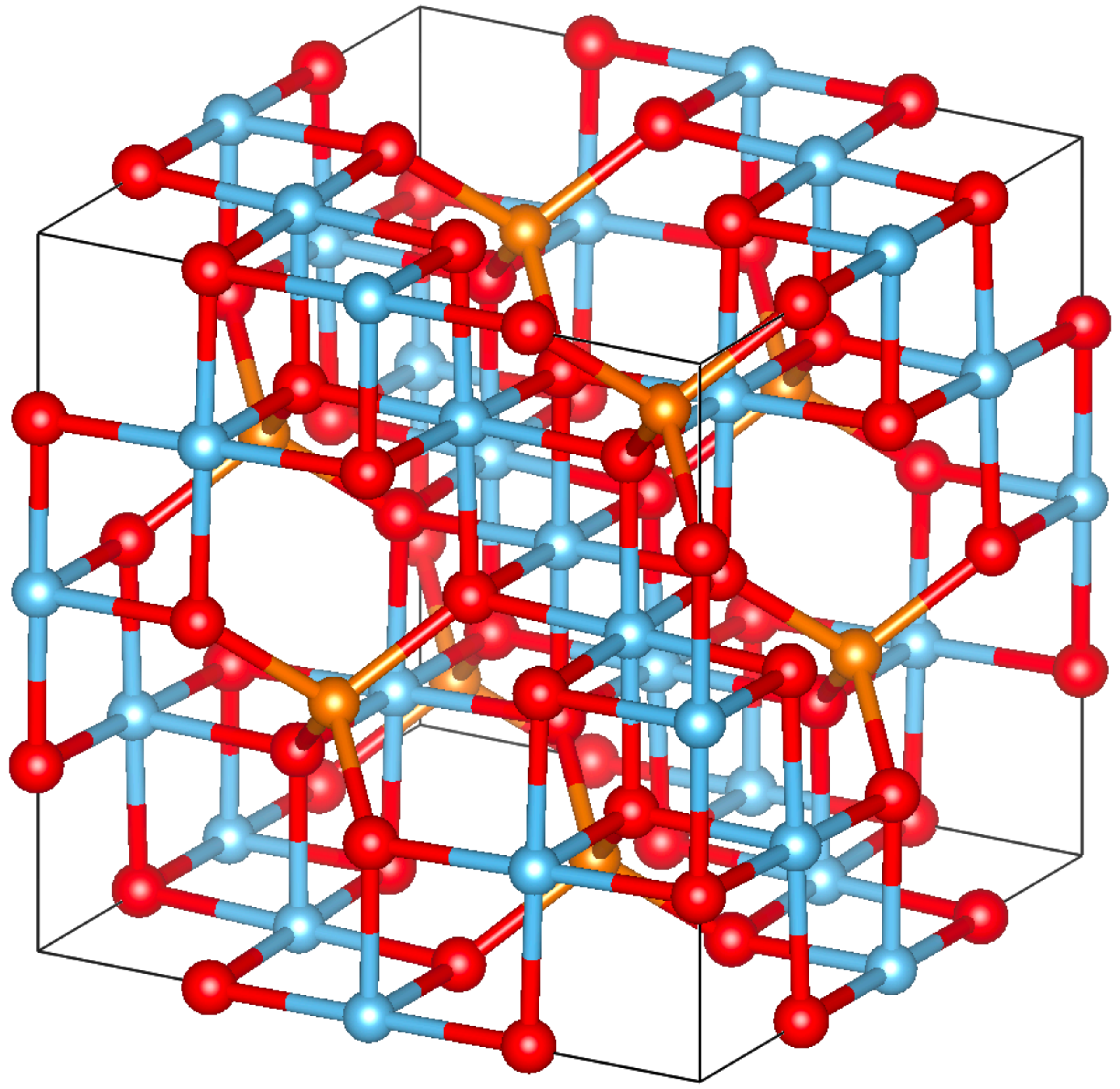


$(\mathrm{Fe}^{3+})_{\mathrm{tetra}}$: ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

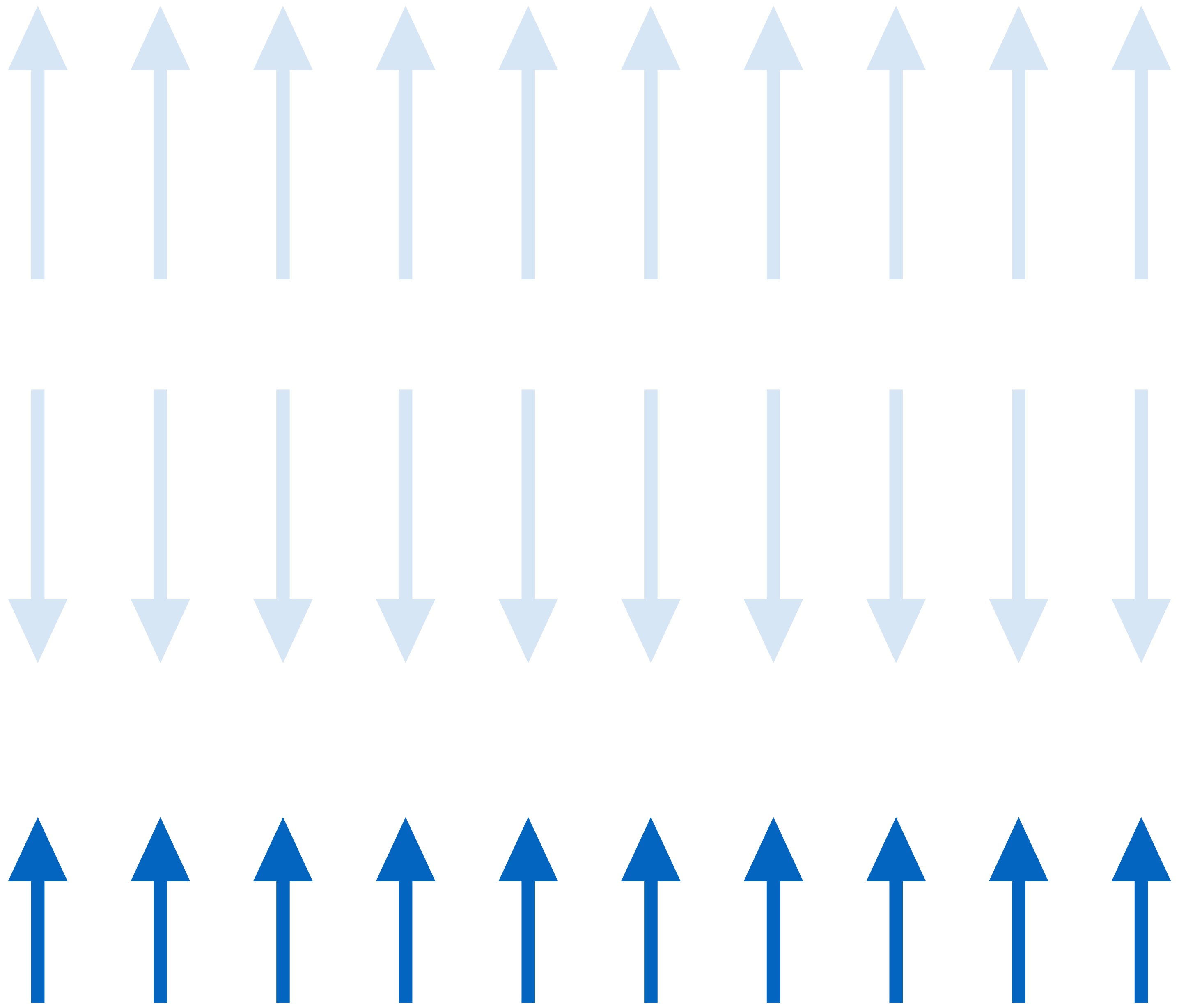
$(\mathrm{Fe}^{3+})_{\mathrm{octa}}$: ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

$(\mathrm{Fe}^{2+})_{\mathrm{octa}}$: ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

Inverse spinel structure Fe_3O_4 (magnetite)



ferrimagnet



$(\mathrm{Fe}^{3+})_{\mathrm{tetra}}$:

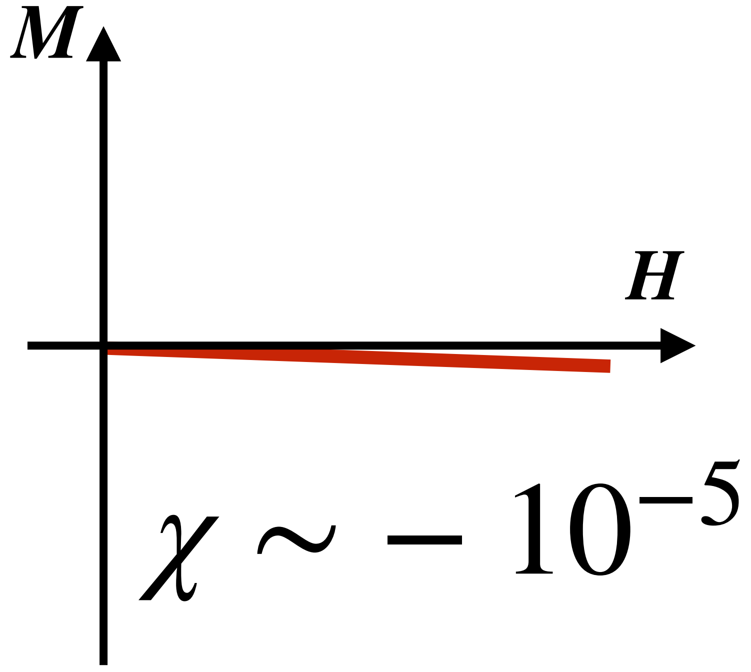
$(\mathrm{Fe}^{3+})_{\mathrm{octa}}$:

$(\mathrm{Fe}^{2+})_{\mathrm{octa}}$:

Classification of magnetic materials

diamagnetic

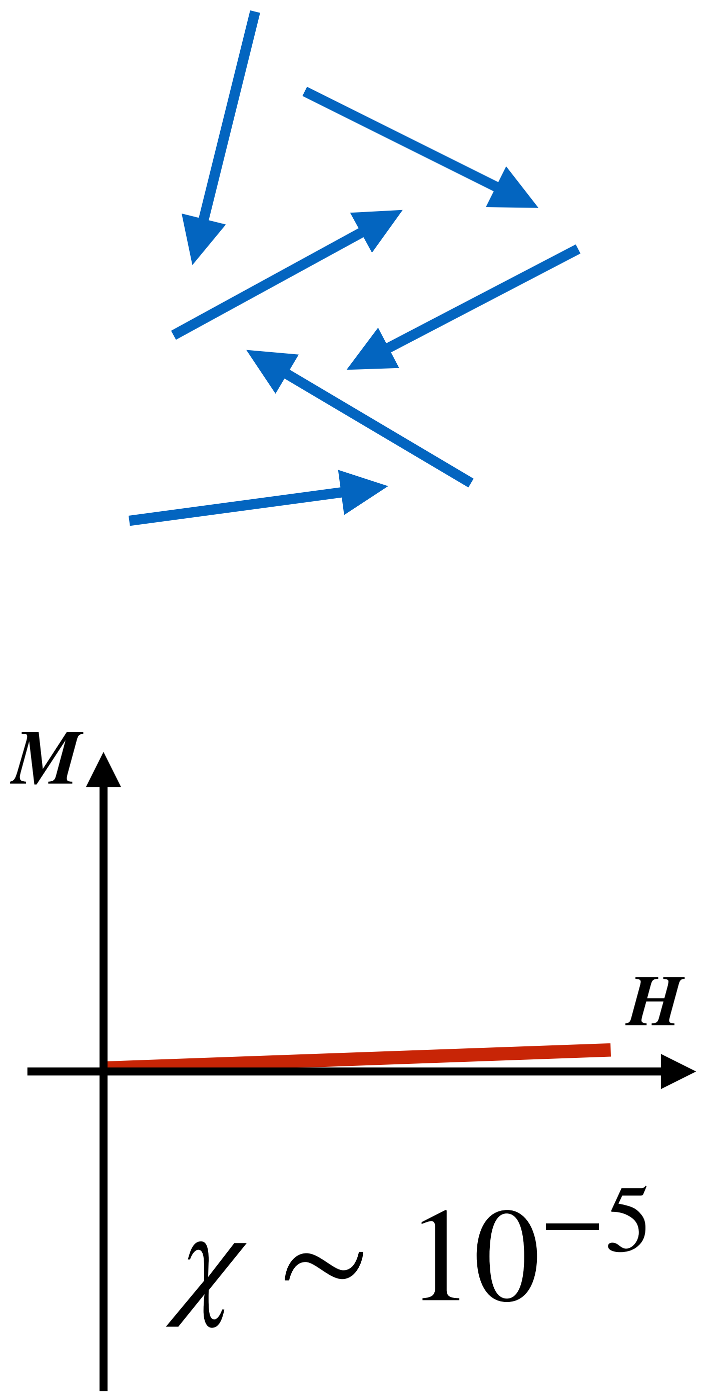
$M = 0$



$\chi \sim -10^{-5}$

copper, water

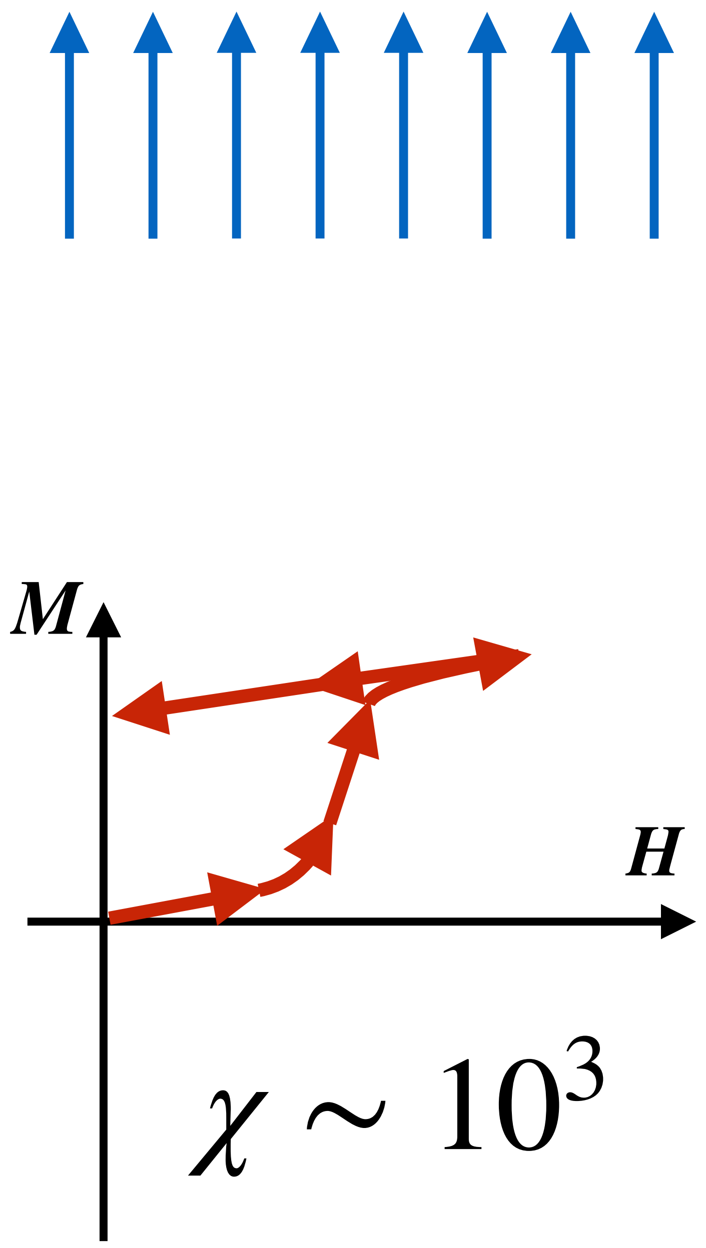
paramagnetic



$\chi \sim 10^{-5}$

aluminium, magnesium

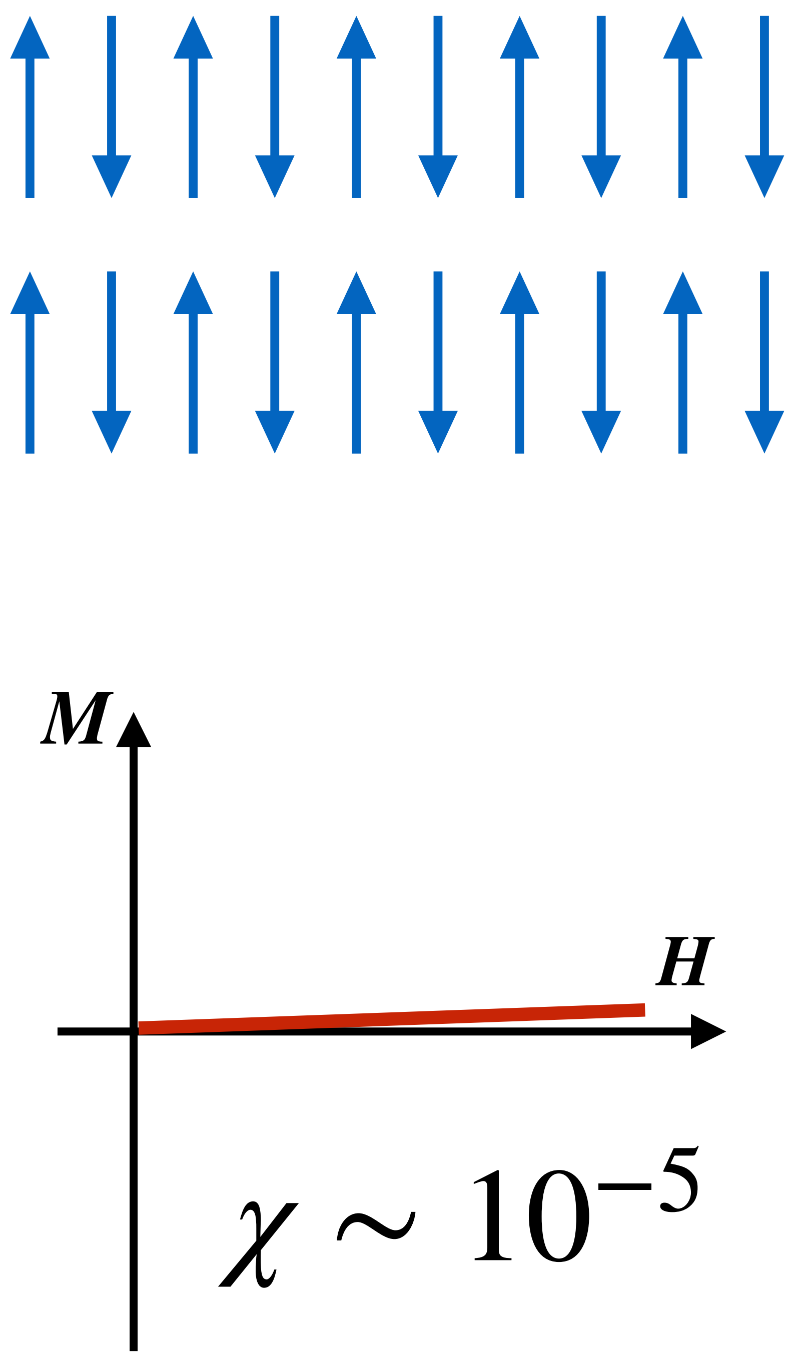
ferromagnetic



$\chi \sim 10^{3}$

iron, cobalt, nickel

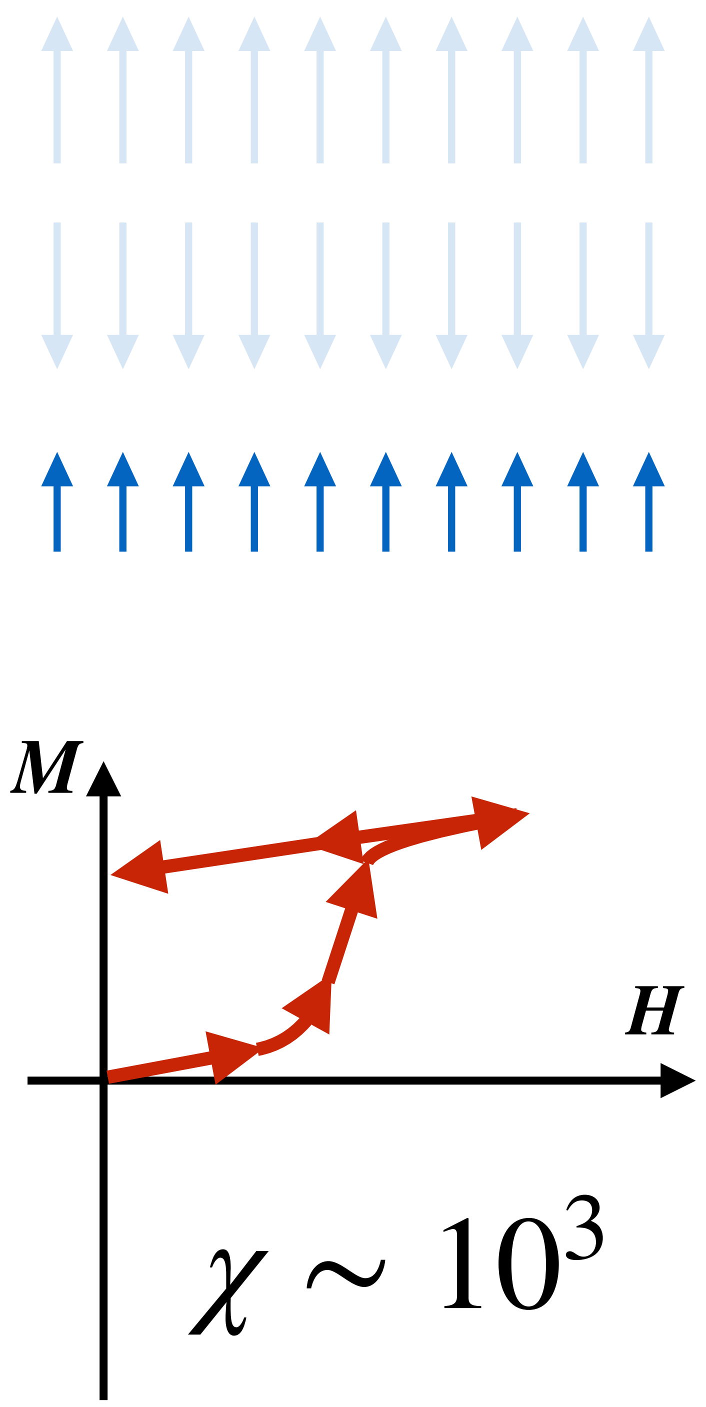
antiferromagnetic



$\chi \sim 10^{-5}$

FeMn, NiO

ferrimagnetic



$\chi \sim 10^{3}$

Fe_3O_4, $NiFe_2O_4$, $Y_3Fe_5O_{12}$

Supplementary slides

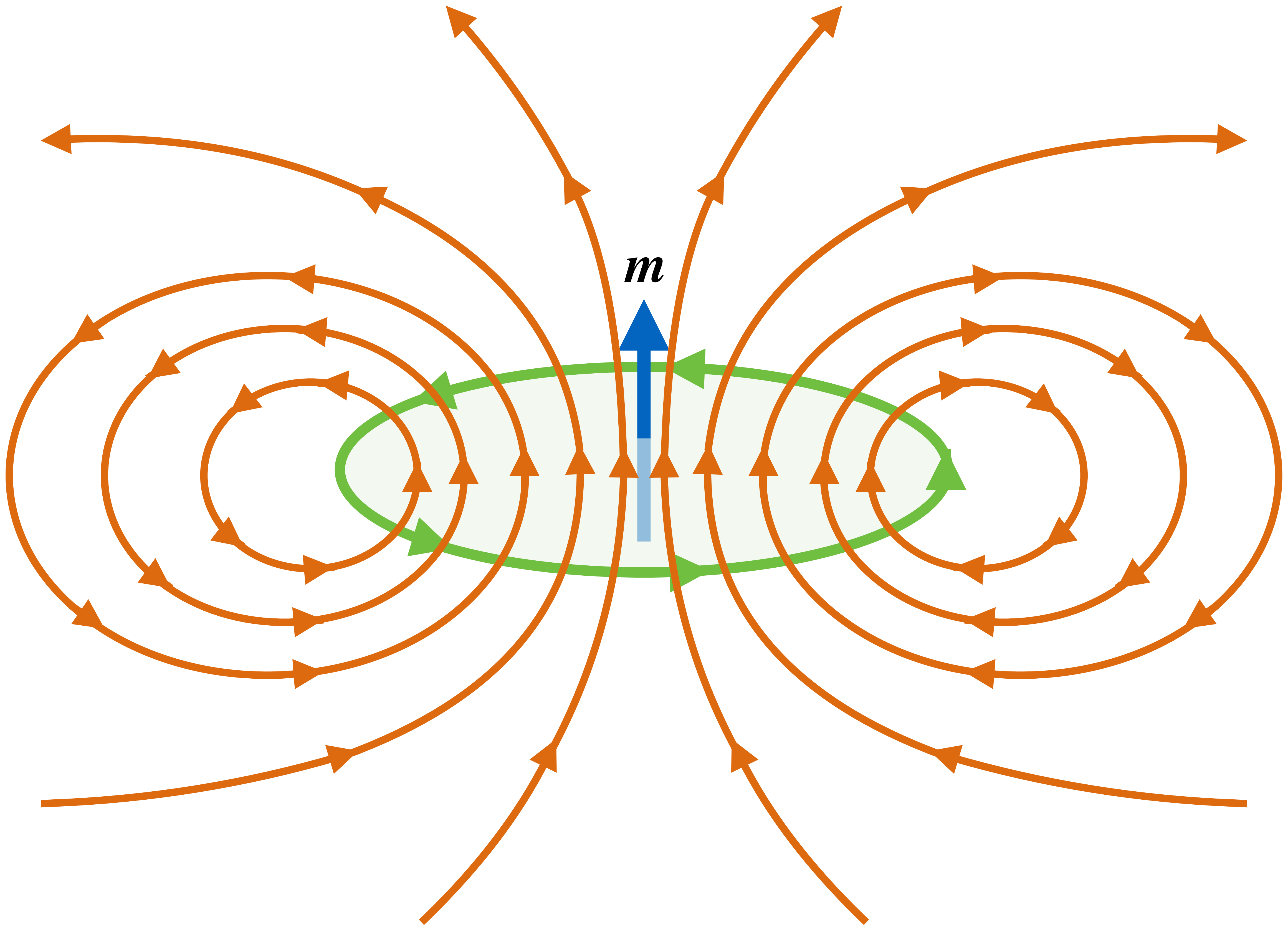
Magnetostatics: magnetic dipole

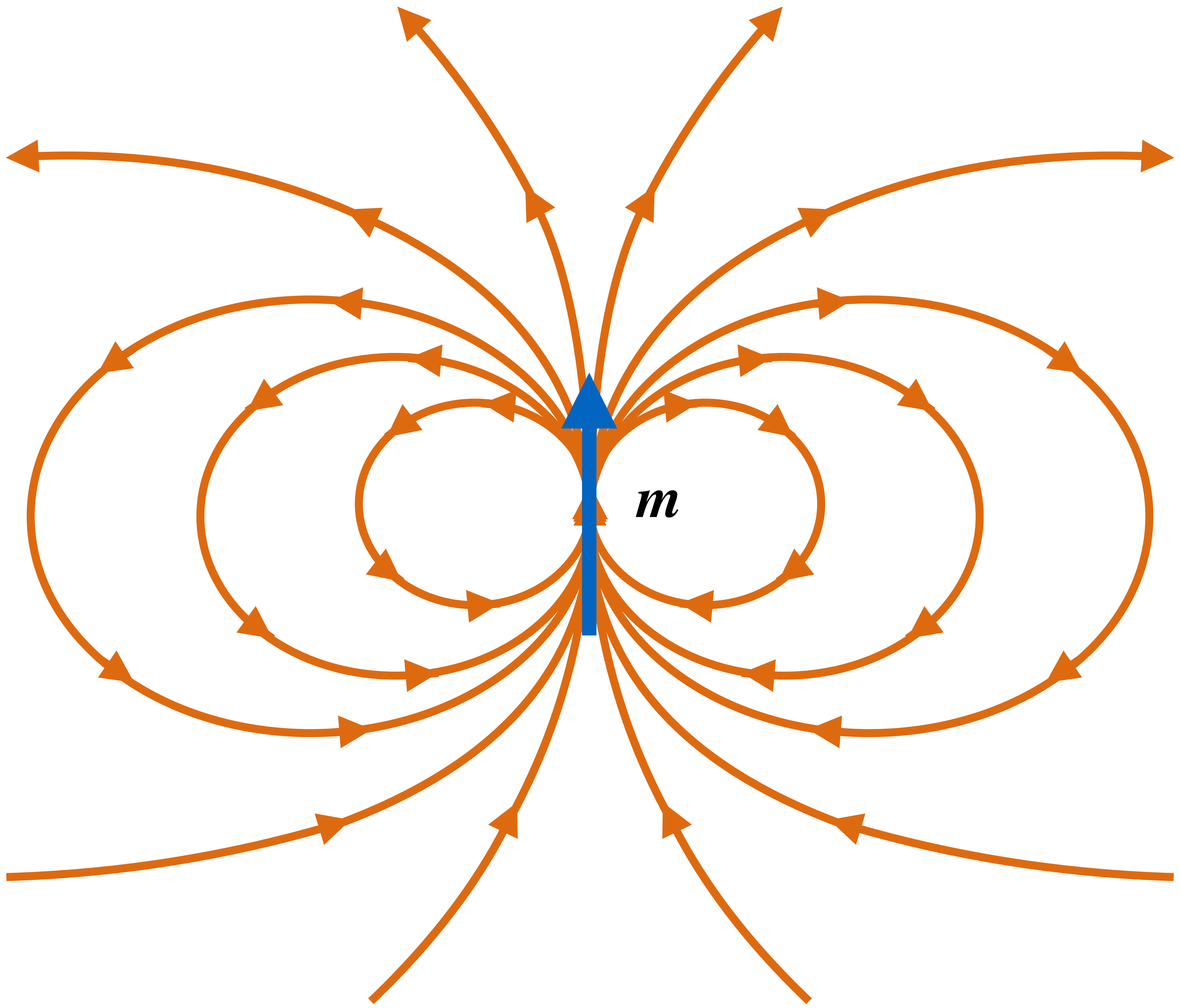
- Magnetic dipole: limit of a closed loop of electric current as the area is reduced to zero while keeping the magnetic moment constant

$$m = Ia = I'a'$$

Magnetostatics: magnetic dipole

- Magnetic dipole: limit of a closed loop of electric current as the area is reduced to zero while keeping the magnetic moment constant





Magnetostatics: magnetic induction

- Magnetic induction: response of a material to an applied magnetic field $\boldsymbol{H}$

$$\boxed{\boldsymbol{B} = \mu \boldsymbol{H} = \mu_0(\boldsymbol{H} + \boldsymbol{M})}$$

$\boldsymbol{B}$: magnetic induction $[\mathrm{T} = \mathrm{Wb\,m^{-2}} = \mathrm{kg\,s^{-2}A^{-1}}]$ $[\mathrm{Wb} = \mathrm{H\,A} = \mathrm{kg\,m^2s^{-2}A^{-1}}]$

$\boldsymbol{H}$: magnetic field $[\mathrm{Am^{-1}}]$

μ : permeability $[\mathrm{H\,m^{-1}}]$

μ_0 : permeability of free space $[1.26 \times 10^{-6}\,\mathrm{Hm^{-1}}]$

$\boldsymbol{M}$: magnetisation $[\mathrm{Am^{-1}}]$

Magnetostatics: magnetic induction

- Magnetic induction: response of a material to an applied magnetic field $\boldsymbol{H}$

$$\boxed{\boldsymbol{B} = \mu \boldsymbol{H} = \mu_0(\boldsymbol{H} + \boldsymbol{M})}$$